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We study contracting for a three-tier supply chain consisting of a buyer, a supplier, and a sub-supplier where disruptions of random length occur at the sub-supplier. As is common in supply chains, the buyer has a direct relationship with the supplier but not the sub-supplier; that is, the buyer has limited supply chain visibility. Both the supplier and the sub-supplier can reserve emergency capacity proactively to protect the supply chain from a disruption. We study how the buyer and the supplier can guarantee that the correct level of emergency capacity is reserved. Due to two types of inefficiencies—a special form of double marginalization and the substitution effect—the supply chain is misaligned in its decentralized form, leading to either under- or over-reservation of emergency capacity by the sub-supplier depending on the cost structure of the supply chain. The lack of visibility prevents the buyer from directly contracting with the sub-supplier to eliminate these inefficiencies. Yet, he can coordinate the supply chain through cascading: i.e., contracting with the supplier (using a value-based carrot-and-stick contract), who in turn contracts with the sub-supplier (using a cost-based carrot-and-stick or two-level wholesale price contract, depending on the cost structure of the supply chain). Although the sub-supplier is the source of limited visibility in the supply chain and is the party with private information, the supplier is the one that benefits from this limited visibility and is the party that receives information rent from the buyer.

Key words: multi-tier supply chain; disruption risk; asymmetric information; contract design

History: January 2018

1. Introduction

The 2011 Tohoku earthquake and the following tsunami had devastating social and environmental consequences in Japan ranging from over 15,000 deaths to nuclear accidents (NPA 2017). At the time, its total economic impact was estimated in the order of \$200-300 billion or roughly 4.5% of Japan's GDP (Ujikane 2011). As Japan houses many semiconductor and high-technology companies and their supply chains, companies such as Hitachi, Sony, Toshiba, and Texas Instruments suffered from supply chain disruptions. Texas Instruments, for example, lost approximately 10% of their annual revenue due to a 6-month plant closure (Nanto et al. 2011). The damage was not limited to the semiconductor and high-technology industries. When Toyota resumed production two weeks

after the earthquake and tsunami, almost two-thirds of its suppliers were still disrupted (Forbes 2011). At the time, analysts projected that as many as five million cars worldwide would not be built in 2011 due to the earthquake and tsunami related disruptions in automobile supply chains (Bunkley and Jolly 2011).

Intel Corporation was yet another company that was affected by the Tohoku earthquake. Although Intel does not have any manufacturing facilities in Japan, a significant part of suppliers and sub-suppliers for Intel's material supply chain (i.e., part of Intel's value network that concerns with the sourcing of raw materials and components required for production) are located in Japan. Historically, Intel had strong business continuity plans for its own manufacturing facilities and in many cases for its Tier 1 suppliers. However, Intel's visibility and control over the upstream parts of the supply chain (which was affected highly from the earthquake) was limited, opening Intel up to major disruption risks and supply chain management challenges. This paper is motivated by these challenges and is a part of a research collaboration developed in partnership with Intel.

1.1. Risk Management Challenges in Intel's Material Supply Chain

Raw materials and components in Intel's material supply chain can be grouped under four categories: micro-contamination-control equipment, high-density substrates, multilayer ceramic capacitors, and litho-chemicals (Figure 1). Next, we discuss *risk factors* in these four categories that inform our stylized model (Table 1).

Within these four categories, Intel sources from over 16,000 suppliers spanning more than 100 countries (Intel 2013), leading to a complex multi-tiered supply network. Due to this complexity, it is difficult for Intel to control their material supply chain beyond the company's Tier 1 supply base, which already includes about 250 suppliers. Therefore, Intel often delegates their sourcing either to third parties (such as two specialized distributors that source ~4,000 micro-contamination-control equipment from 400 sub-suppliers (Figure 1(a))) or to Tier 1 suppliers. Consequently, Intel has *limited control and visibility* over these sub-suppliers.

Strict intellectual property (IP) regulations are another main reason for lack of visibility in Intel's supply chain. For example, Intel's high-density-substrate (Figure 1(b)) and litho-chemicals (Figure 1(d)) suppliers use materials that involve highly protected IP sourced from their sub-suppliers, which thus must remain unknown. Although Intel is well connected with Tier 1 and in some cases Tier 2 suppliers of their high-density-substrate supply chain, they generally have *limited information about their sub-supplier base* beyond a certain point.

In addition to the challenges caused by limited control and visibility over sub-suppliers, Intel's ability to react to a disruption is also limited due to *high product specificity and strict quality standards* required to manufacture Intel's products. Therefore, proactive risk management strategies

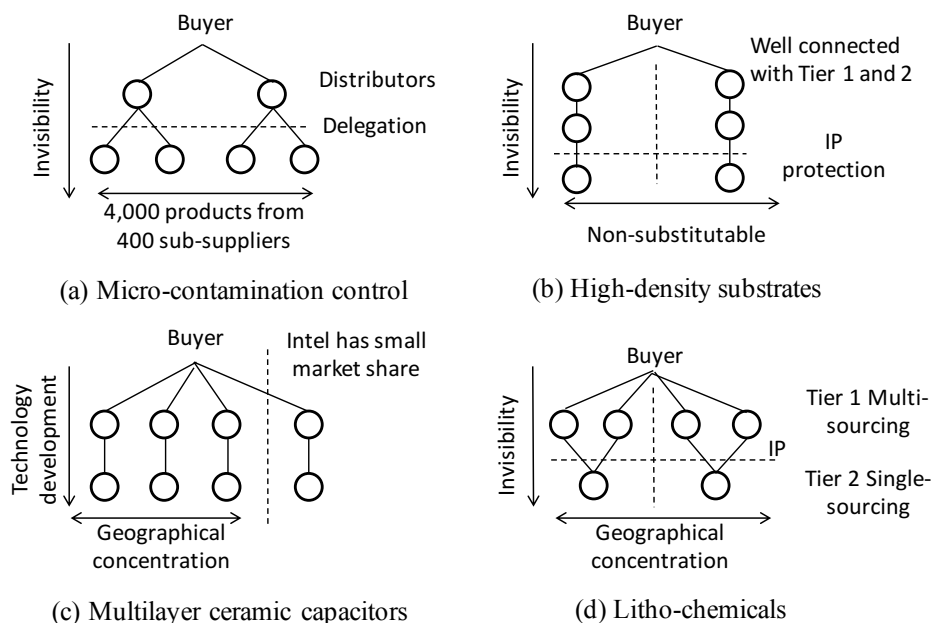


Figure 1 Different Risk Factors in Intel’s Material Supply Chain. In addition to the control and visibility issues (subfigures (a) and (b)), Intel faces additional risk factors, such as geographical concentration of supply base (subfigures (c) and (d)) and limited bargaining power (subfigure (c)).

are key to Intel’s success.

In this paper, we focus on supply chains with limited visibility beyond Tier 1 and with high product specificity¹ using a stylized three-tier supply chain consisting of a buyer, a Tier 1 supplier,

¹ There are other risk factors for Intel’s material supply chain. For example, most multi-layer ceramic capacitors suppliers (Figure 1(c)) are geographically concentrated in Japan, increasing supply risk for Intel.

Table 1 The risk factors in Intel’s material supply chain and how they translate to our model.

Real World Observation	Figure 1	Supply Chain Challenge	Model
Disruptions	a, b, c, d	Sub-suppliers tend to be under-prepared	Disruption of random length occurs at Tier 2
Intellectual Property	b, d	Invisibility aggravates supply chain coordination	3 players; limited visibility
Product Specificity	a, b, c, d	Short notice diversification is not possible	Periodic model where diversification has to be planned upfront via emergency capacity
Product Variety	a	Diversification is not practical for the buyer since sourcing is already delegated	Delegation to supplier
Geographic Concentration	c, d	Diversification is not effective for the buyer since protection is not guaranteed because of geographic concentration	Delegation to supplier
Small Market Share	c	Diversification is not economical for the buyer	Delegation to supplier

and a Tier 2 sub-supplier, who may suffer disruptions of random length. Although this stylized model does not capture any one of the categories in Intel's material supply chain, it does capture the underlying risk factors that these categories share (Table 1). We study two questions: (1) What is the optimal strategy for reserving emergency capacity at the supply base when the sup-supplier faces a disruption risk? What types of contracts can mitigate incentive misalignment between the supplier and the sub-supplier in this setting? (2) How can the buyer incentivize the supply base when he has limited information regarding the capacity reservation costs?

Analyzing the supplier and the sub-supplier first, we show that, due to two types of inefficiencies—a special form of double marginalization and the substitution effect—the supply chain is misaligned in its decentralized form, leading to either under- or over-reservation of emergency capacity by the sub-supplier depending on the cost structure of the supply chain. For under-reservation, we show that a cost-based carrot-and-stick contract can achieve incentive-compatible supply chain coordination but neither can penalties (e.g., Reuters 2010) nor subsidies (e.g., Babich 2010) alone. For over-reservation, a two-level wholesale price contract can achieve incentive-compatible supply chain coordination. Although the lack of visibility prevents the buyer from directly contracting with the sub-supplier to eliminate these inefficiencies, we show that the buyer can incentivize the supply chain through cascading: i.e., contracting with the supplier using a value-based carrot-and-stick contract, who in turn contracts with the sub-supplier using the proposed cost-based carrot-and-stick or two-level wholesale price contract depending on the cost structure of the supply chain. As such, an incentive-compatible supply chain coordination can be achieved. Although the sub-supplier is invisible and holds the private information, the supplier is the one who obtains information rent. Thus, the supplier benefits from a certain degree of invisibility in the supply chain.

1.2. Literature Review

Supply chain disruptions and risk management have been rigorously studied in operations management literature in recent years. We refer the reader to Snyder et al. (2016) for a review on disruptions and Tang (2006) for a review on supply chain risk management.

Early research on supply uncertainty focuses mainly on frequently occurring short-term interruptions such as machine breakdowns or short term unavailability of a supplier and study different forms of disruptions at a single firm distinguishing between random capacity (Ciarallo et al. 1994, Erdem and Özekici 2002), random disruption (Parlar and Perry 1996, Gürler and Parlar 1997, Moinzadeh and Aggarwal 1997, Arreola-Risa and DeCroix 1998), and random yield (Parlar and Wang 1993, Yano and Lee 1995, Agrawal and Nahmias 1997). More recently, the supply chain

aspect of disruptions is captured, and inventory (Tomlin 2006), diversification (Babich et al. 2007), supplier improvement (Wang et al. 2010), contracting (Chao et al. 2009), and the value of information on disruption risk (Tomlin 2009) are considered as effective risk management strategies. Our focus on limited supply chain visibility and high product specificity in a multi-tier supply chain set our research apart from the existing literature on supply chain disruptions and risk management.

In recent years, the operations management literature started to study multi-tiered supply chains. One stream of this literature investigates supply chain hierarchies—delegation versus control for multi-tiered supply chains (Kayış et al. 2013, Huang et al. 2017). This stream implicitly assumes that the sub-supplier is visible to the buyer; i.e., the buyer has the option of controlling the supplier or the sub-supplier. Besides the focus on disruptions, our model is different from the papers in this stream in that the buyer does not know the sub-supplier. Another stream of multi-tiered supply chains research considers supply network formation and typology under disruptions (Bakshi and Mohan 2017, Ang et al. 2017). Our focus on mechanism design and cascading adds to this stream.

The remainder of this paper is organized as follows. We describe our model and study three benchmark cases in Section 2. We tackle our first research question and analyze how the upstream supply chain can be coordinated in Section 3. We then focus on our second research question and study how cascading can be used in the multi-tier case in Section 4. In Section 5, we conclude our paper with a discussion on several risk management strategies for Intel’s material supply chain based on our analysis and observations. The details of the analysis for the benchmark cases are relegated to Appendix A. All proofs appear in Appendix B. We use x^+ to represent $\max\{0, x\}$.

2. Model Description and Benchmark Analysis

In this section, we describe our model and assumptions, and provide three benchmark cases.

2.1. Model

We study a stylized three-tier supply chain consisting of a buyer, a Tier 1 supplier, and a Tier 2 sub-supplier where disruptions of random length T occur at Tier 2 (Figure 2). To protect the supply chain against a disruption, the supplier and the sub-supplier can proactively reserve emergency capacity t_1 and t_2 , respectively. We assume that when there are no disruptions, demand is always satisfied. The emergency sources, which are perfectly reliable, are used as a contingency only when the supply at Tier 2 is disrupted. The reliability assumption is equivalent to assuming that disruptions at the sub-supplier and emergency sources never occur at the same time. Wagner et al. (2009) provide empirical evidence for positively correlated risk in the automobile industry, which is similar to what we observed in interviews with Intel supply chain managers about their geographically concentrated semiconductor supply chains. For example, three out of four multilayer

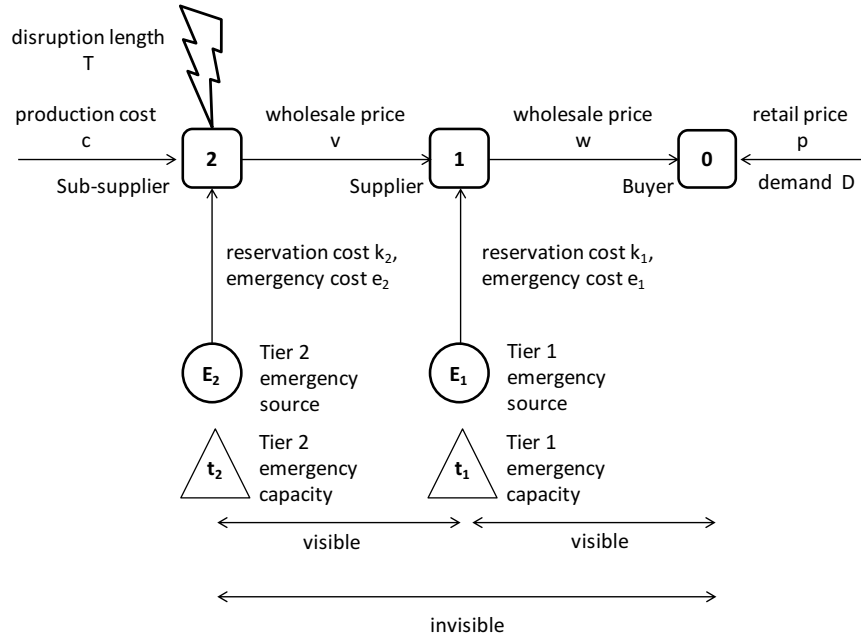


Figure 2 Three-Player Two-Emergency-Source Supply Chain. The supplier and the sub-supplier decide how much emergency capacity to reserve (t_1 and t_2) from their respective sources prior to the disruption. The buyer and the supplier design contracts (based on penalties and subsidies, or quantity discounts) to guarantee that the correct level of emergency capacity is reserved in the supply base.

ceramic capacitors suppliers (Figure 1(c)) are located in Japan. Thus, the reliability assumption requires enough independence between sources such as through geographical diversification; e.g., emergency sources may correspond to different locations of the sub-supplier.

We assume the disruption length T to be a random variable with distribution $F(T)$ and density $f(T)$, and with finite mean and variance. We model rare but disruptive events such as earthquakes or floods. Although the supply network of a large corporation, which combines multiple supply chains, may be disrupted frequently as noted by Snyder et al. (2016), we limit the number of disruptions in the planning horizon to one in order to represent the infrequency of disruptions for a given supply chain. Such single period models are common in the supply chain disruption literature (e.g., Dada et al. (2007), Federgruen and Yang (2008)).

Since our focus is on supply uncertainty, we assume demand D to be deterministic and constant, which is a common assumption in the literature on supply disruption (e.g., Parlar and Perry (1996), Swinney and Netessine (2009)). Since we assume demand and capacity to be deterministic, we represent all parameters in units of time, since the only uncertainty in the system is the length of the disruption. Without loss of generality, we set $D = 1$ and normalize all parameters accordingly. We denote the production cost as c and the retail price as p . The sub-supplier sells to the supplier at wholesale price v , and the supplier sells to the buyer at wholesale price w . We assume that

$c < v < w < p$ so that the supply chain is profitable for all players. The supplier's cost for reserving emergency capacity is k_1 and the sub-supplier's is k_2 , and emergency capacity can only be used for the length of the disruption at an additional cost of e_1 at the supplier and e_2 at the sub-supplier. We assume $k_1 + e_1 \geq k_2 + e_2$, i.e., the supplier's total reservation and execution cost for emergency capacity is higher than the sub-supplier's. Further, we assume $e_2 \leq e_1$, so that t_2 will always be used first. In order for the emergency sources to be economically feasible for the supplier and the sub-supplier, we assume that $e_1 < w$ and $e_2 < v$.

The supply chain partners have limited supply chain visibility. That is, players only know their direct supply chain partners: the buyer and the sub-supplier only know who the supplier is. This limited supply chain visibility for the buyer is associated with supply chains for high specificity products with IP protection, such as Intel's high-density substrates (Figure 1(b)) and litho-chemicals (Figure 1(d)), or when a buyer delegates the sourcing such as for Intel's micro-contamination-control equipment (Figure 1(a)). Therefore, setting a contract with the sub-supplier is not an option for the buyer.

The sequence of events is as follows. At the beginning of the planning horizon, the supplier and the sub-supplier decide how much emergency capacity to reserve. Next, the disruption is observed, and emergency capacity is used. If the disruption length exceeds emergency capacity, sales are lost for the portion of the disruption that exceeds emergency capacity. We assume that the disruption is observed by all players, which holds for major disruptive events such as earthquakes and floods.

2.2. Objective Functions

Since emergency capacity is only used during an actual disruption, the capacity decisions only affect profits during a disruption. Further, the reservation of emergency capacity at the beginning of the planning horizon creates costs for the supplier and the sub-supplier only. Therefore, without loss of generality, we can ignore components that are independent from the decision variable in our analysis. That is, we model profits during the disruption plus the cost for reserving emergency capacity, and as a result, mechanisms we study are additional to the ones during regular operations of the supply chain. We denote expected profits as Π_0 , Π_1 , and Π_2 for the buyer, the supplier, and the sub-supplier, respectively. As the emergency capacity decisions are made only by the suppliers, we first concentrate on the upstream supply chain:

$$\begin{aligned} \Pi_1(t_1, t_2) = & (w - v) \left[\int_0^{t_2} T dF(T) + t_2[1 - F(t_2)] \right] \\ & + (w - e_1) \left[\int_{t_2}^{t_1+t_2} (T - t_2) dF(T) + t_1[1 - F(t_1 + t_2)] \right] - k_1 t_1, \end{aligned} \quad (1)$$

$$\Pi_2(t_2) = (v - e_2) \left[\int_0^{t_2} T dF(T) + t_2[1 - F(t_2)] \right] - k_2 t_2. \quad (2)$$

The supplier's profit, $\Pi_1(t_1, t_2)$, depends on the supplier's own capacity decision t_1 as well as the sub-supplier's capacity decision t_2 . The supplier's profit margin during a disruption depends on the length of the disruption. Up to t_2 , the profit margin is $w - v$ with expected used capacity $\int_0^{t_2} T dF(T) + t_2[1 - F(t_2)]$. For the part of a disruption exceeding t_2 , the profit margin is $w - e_1$ with expected used capacity $\int_{t_2}^{t_1+t_2} (T - t_2) dF(T) + t_1[1 - F(t_1 + t_2)]$. The cost of reserving t_1 units of emergency capacity is $k_1 t_1$.

The sub-supplier's profit, $\Pi_2(t_2)$, depends only on the sub-supplier's own capacity decision t_2 . If the disruption is shorter than t_2 , then the sub-supplier is able to deliver all units requested by the supplier. This is reflected by the expectation $\int_0^{t_2} T dF(T)$. If the disruption is longer than t_2 , then the sub-supplier is able to deliver up to t_2 , which is reflected by $t_2[1 - F(t_2)]$. The cost of reserving t_2 units of emergency capacity is $k_2 t_2$.

Given a capacity reservation of t_1 and t_2 by the supply base, the buyer's profit, $\Pi_0(t_1, t_2)$, depends on the total capacity. That is, where in the supply chain the capacity is held does not change the buyer's profit:

$$\Pi_0(t_1, t_2) = (p - w) \left[\int_0^{t_1+t_2} T dF(T) + (t_1 + t_2)[1 - F(t_1 + t_2)] \right]. \quad (3)$$

Here, $p - w$ is the buyer's profit margin. If the disruption is shorter than $t_1 + t_2$, then the buyer is able to satisfy all demand during the disruption. This is reflected by the expectation $\int_0^{t_1+t_2} T dF(T)$. If the disruption is longer than $t_1 + t_2$, then the buyer is able to satisfy demand up to $t_1 + t_2$, which is reflected by $(t_1 + t_2)[1 - F(t_1 + t_2)]$. Note that since the buyer has limited visibility, he does not know the expected profit $\Pi_0(t_1, t_2)$; he can only calculate an expectation over the possible values t_2 can take based on the type of the sup-supplier.

2.3. Benchmark Cases

Table 2 displays the supplier's and the sub-supplier's emergency capacity reservations for three benchmark cases. The *base case* corresponds to a decentralized supply chain with wholesale price contracts between the buyer, the supplier, and the sub-supplier. The *centralized supply chain* is our first-best benchmark. Since limited supply chain visibility would not be an issue in the centralized supply chain and since it is always optimal to use Tier 2 emergency capacity before Tier 1 capacity, the objective function for the centralized supply chain simply equals to the sum of the three objective functions from the base case (Equations (1)-(3)). The *integrated-supplier benchmark* corresponds to a case where the supplier and the sub-supplier are coordinated and the upstream supply chain is centralized. The integrated supplier's objective function equals to the sum of the supplier's and the sub-supplier's objective functions from the base case (Equations (1) and (2)).

Table 2 Benchmark Cases (in increasing order of coordination). Note that, $x^+ = \max\{0, x\}$.

	Emergency Capacity	
	Supplier	Sub-supplier
Base Case	$t_1^* = \left[F^{-1} \left(\left[1 - \frac{k_1}{w-e_1} \right]^+ \right) - t_2^* \right]^+$	$t_2^* = F^{-1} \left(\left[1 - \frac{k_2}{v-e_2} \right]^+ \right)$
Integrated Supplier	$t_{1,I}^* = \left[F^{-1} \left(\left[1 - \frac{k_1}{w-e_1} \right]^+ \right) - t_{2,I}^* \right]^+$	$t_{2,I}^* = \begin{cases} F^{-1} \left(\left[1 - \frac{k_2}{w-e_2} \right]^+ \right) & \text{if } \frac{k_2}{w-e_2} \leq \frac{k_1}{w-e_1}, \\ F^{-1} \left(\left[1 - \frac{k_2-k_1}{e_1-e_2} \right]^+ \right) & \text{otherwise.} \end{cases}$
Centralized Supply Chain	$t_{1,C}^* = \left[F^{-1} \left(\left[1 - \frac{k_1}{p-e_1} \right]^+ \right) - t_{2,C}^* \right]^+$	$t_{2,C}^* = \begin{cases} F^{-1} \left(\left[1 - \frac{k_2}{p-e_2} \right]^+ \right) & \text{if } \frac{k_2}{p-e_2} \leq \frac{k_1}{p-e_1}, \\ F^{-1} \left(\left[1 - \frac{k_2-k_1}{e_1-e_2} \right]^+ \right) & \text{otherwise.} \end{cases}$

The limitations of the base case and the integrated-supplier case in securing adequate protection against disruptions justify Intel’s concerns about the risks that their material supply chain was exposed to. These limitations lead to our subsequent analysis of other types of contracts in two steps. First, we study how the upstream supply chain can be aligned in Section 3. We then refer to the aligned upstream supply chain as integrated supplier and study the downstream supply chain with integrated supplier using a backward induction argument in Section 4.

3. Coordinating the Decentralized Upstream Supply Chain

We first study the upstream supply chain and characterize how we can coordinate capacity decisions in the supply base. We start with identifying the inefficiencies in the supply base in Section 3.1. We then suggest contracts that would eliminate these inefficiencies in Sections 3.2 and 3.3.

3.1. Inefficiencies in the Upstream Supply Chain

We illustrate the results of Table 2 in Figure 3. In this figure, we fix k_2 , e_2 , e_1 , w , and v ($v \geq e_1$), and vary k_1 to investigate how the total emergency capacity as well as its mix between the supplier and the sub-supplier change for the integrated supplier (where the supplier and the sub-supplier are coordinated and the upstream supply chain is centralized) and the base case as the supplier’s emergency source becomes less economical compared to the sub-supplier’s.

The sub-supplier’s critical fractile in the base case is independent of k_1 . That is, the emergency capacity that the sub-supplier reserves does not change as we change the value of k_1 . However, the emergency capacity that the integrated supplier reserves through the sub-supplier’s emergency source depends on the relative value of k_1 with respect to k_2 , and (weakly) decreases as k_1 decreases. This difference in the sub-supplier’s emergency capacity causes two types of inefficiencies—double marginalization which is either countered or corroborated by the substitution effect. The manifestation of these inefficiencies on the emergency capacity is captured through a *volume misalignment* and *capacity mix distortion* as we discuss next.

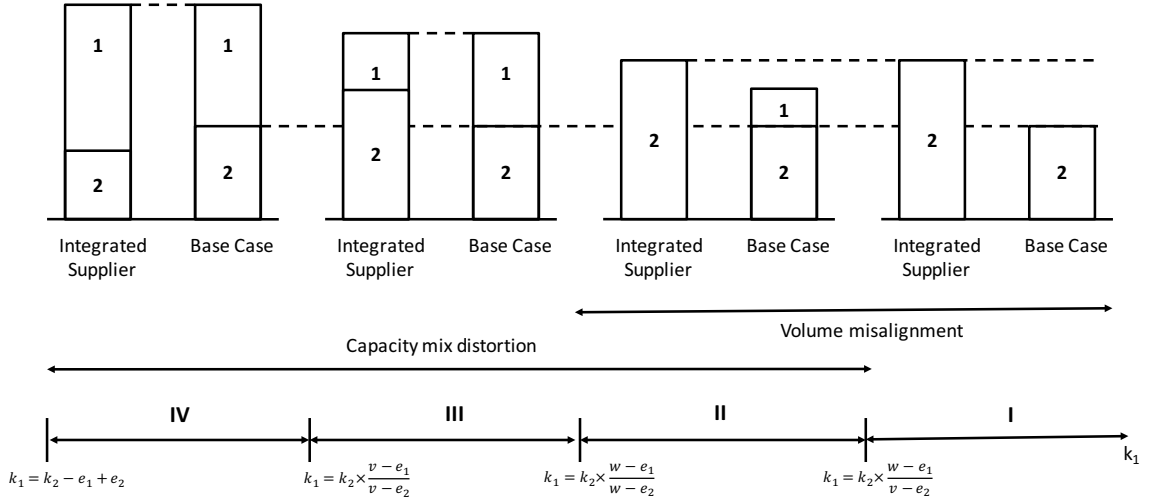


Figure 3 Capacity reservation in integrated supplier vs. base case as k_1 increases. Stacks 1 and 2 correspond to the emergency capacity reserved by the supplier and the sub-supplier, respectively, and bars represent the total emergency capacity in the supply base.

When k_1 is very high (Case I), although the supplier has an emergency source, that source is so expensive that it is never optimal to use it, even in the base case. Therefore, the supply chain practically has a single emergency source. In this case, the total capacity that the integrated supplier reserves is higher than that of the base case. That is, we observe a *volume misalignment* between the integrated supplier and the base case due to double marginalization.

When we decrease k_1 (Case II), the supplier's emergency source is still expensive for the integrated supplier. However, in the base case, this emergency source becomes a feasible option for the supplier to deal with the disruption and compensate for the sub-supplier's under-reservation. In this case, the volume misalignment decreases; however, the emergency inventory is misplaced in the supply chain; that is, we observe a *capacity mix distortion* due to the substitution effect.

As we continue to decrease k_1 (Case III), the supplier can now effort to reserve even more emergency capacity from his source, and, the volume misalignment is eliminated. However, the supply chain suffers from a higher capacity mix distortion. Finally, for very low values of k_1 (Case IV), it is optimal for the integrated supplier to shift most of the emergency capacity to the supplier. However, since the sub-supplier makes profit from the emergency capacity he delivers, he strives to substitute the supplier's emergency capacity with his own under the base case. Once again, even though the volume misalignment is eliminated, the supply chain suffers from a capacity mix distortion; this time in the reverse direction.

Coordination in the supply base requires both the quantity and the mix of the emergency capacity to be the same as in the integrated supplier case. Since the supply chain is not coordinated in the

base case due to double marginalization and the substitution effect, we propose one of two contracts. For an under-reserving sub-supplier, we propose a carrot-and-stick contract in Section 3.2, which combines penalties and subsidies to incentivize the sub-supplier to reserve *more* emergency capacity. For an over-reserving sub-supplier, we propose a two-level wholesale price contract in Section 3.3, which incentivizes and compensates the sub-supplier to reserve *less* emergency capacity than in the base case. These two contracts guarantee that the emergency capacities reserved by the supplier and the sub-supplier are equivalent to their corresponding quantities for the integrated supplier.

3.2. The Cost-Based Carrot-and-Stick Contract

The carrot-and-stick contract combines a subsidy k that the supplier pays to the sub-supplier per unit of emergency capacity reserved by the sub-supplier with a penalty s that the sub-supplier has to pay to the supplier when the sub-supplier is not able to fulfill the supplier's order:

$$\Pi_{1,ks}(t_1, t_2) = \Pi_1(t_1, t_2) + s \int_{t_2}^{\infty} (T - t_2) dF(T) - kt_2, \quad (4)$$

$$\Pi_{2,ks}(t_2) = \Pi_2(t_2) - s \int_{t_2}^{\infty} (T - t_2) dF(T) + kt_2. \quad (5)$$

Equations (4) and (5) are based on Equations (1) and (2), respectively, with the addition of subsidy and penalty terms. In particular, the subsidy term kt_2 is subtracted from the supplier's expected profit function and added to the sub-supplier's and the penalty term $s \int_{t_2}^{\infty} (T - t_2) dF(T)$ is added to the supplier's expected profit function and subtracted from the sub-supplier's. Proposition 1 characterizes the emergency capacity reservations under the carrot-and-stick contract.

PROPOSITION 1. *Under a carrot-and-stick contract with subsidy k and penalty s , the supplier's and the sub-supplier's respective optimal emergency capacities are*

$$t_{1,ks}^* = \left[F^{-1} \left(\left[1 - \frac{k_1}{w - e_1} \right]^+ \right) - t_{2,ks}^* \right]^+,$$

$$t_{2,ks}^* = F^{-1} \left(\left[1 - \frac{k_2 - k}{v - e_2 + s} \right]^+ \right).$$

The carrot-and-stick contract can coordinate the upstream supply chain when the sub-supplier is under-reserving in the base case; i.e., under Cases (I)-(III) in Figure 3. In Proposition 2, we study the coordinating parameters that are incentive compatible.

PROPOSITION 2. *For $\frac{k_2 - k_1}{e_1 - e_2} < \frac{k_2}{v - e_2}$, a carrot-and-stick contract with a subsidy $k \geq \underline{k}(s)$ and penalty $\underline{s} \leq s \leq \bar{s}$ can achieve incentive-compatible coordination of the upstream supply chain, where*

$$k(s) = \begin{cases} k_2 \times \frac{w - v - s}{w - e_2} & \text{if } \frac{k_2}{w - e_2} \leq \frac{k_1}{w - e_1} \\ \frac{(v - e_2 + s)k_1 - (v - e_1 + s)k_2}{e_1 - e_2} & \text{otherwise} \end{cases},$$

$$\begin{aligned} \underline{k}(s) &= \{k(s) : \Pi_{2,ks}(t_{2,ks}^*) = \Pi_2(t_2^*)\}, \\ \underline{s} &= \{s : \Pi_{1,k(s)s}(t_{1,C}^*, t_{2,C}^*) = \Pi_1(t_1^*, t_2^*)\}, \\ \bar{s} &= \begin{cases} w - v & \text{if } \frac{k_2}{w-e_2} \leq \frac{k_1}{w-e_1}, \\ \frac{(v-e_1)k_2 - (v-e_2)k_1}{k_1 - k_2} & \text{otherwise.} \end{cases} \end{aligned}$$

3.3. The Two-Level Wholesale Price Contract

Next, we consider a contract that is designed to coordinate the upstream supply chain under Case IV in Figure 3; i.e., when the sup-supplier is over-reserving capacity in the base case. The two-level wholesale price contract is defined by three parameters: a price break t_w , a wholesale price w_1 , which is paid during disruptions up to a length t_w , and a wholesale price $w_2 < w_1$, which is paid for any portion of a disruption that exceeds t_w . The lower wholesale price w_2 after the price break lowers the sub-supplier's optimal emergency capacity. The higher wholesale price w_1 before the price break helps to satisfy the sub-supplier's incentive compatibility condition, i.e., makes sure that the sub-supplier's profit is as high as in the base case.

Let $\bar{w} := (w_1, w_2, t_w)$. The resulting profit functions for the supplier and the sub-supplier depend on the relative value of t_2 with respect to t_w . If $t_2 \leq t_w$, then $(\Pi_{1,\bar{w}}(t_1, t_2), \Pi_{2,\bar{w}}(t_2)) = (\Pi_1(t_1, t_2), \Pi_2(t_2))|_{v=w_1}$. If $t_2 > t_w$, then

$$\begin{aligned} \Pi_{1,\bar{w}}(t_1, t_2) &= (w - w_1) \left[\int_0^{t_w} T dF(T) + t_w [1 - F(t_w)] \right] \\ &\quad + (w - w_2) \left[\int_0^{t_2} (T - t_w) dF(T) + (t_2 - t_w) [1 - F(t_2)] \right] \\ &\quad + (w - e_1) \left[\int_{t_1+t_2}^{t_w+t_2} (T - t_2) dF(T) + t_1 [1 - F(t_1 + t_2)] \right] - k_1 t_1, \end{aligned} \quad (6)$$

$$\begin{aligned} \Pi_{2,\bar{w}}(t_2) &= (w_1 - e_2) \left[\int_0^{t_w} T dF(T) + t_w [1 - F(t_w)] \right] \\ &\quad + (w_2 - e_2) \left[\int_{t_w}^{t_2} (T - t_w) dF(T) + (t_2 - t_w) [1 - F(t_2)] \right] - k_2 t_2. \end{aligned} \quad (7)$$

For $t_2 \leq t_w$, the price break is not reached. Since there is only one wholesale price, this corresponds to the base case with w_1 replacing v . For $t_2 > t_w$, the price break is reached. For the supplier, there are now three different profit margins. Until the price break, the profit margin is $(w - w_1)$. After that, the profit margin for the portion of the disruption that exceeds t_w is $(w - w_2)$ for disruptions shorter than t_2 . For longer disruptions, the profit margin for the portion of the disruption that exceeds t_2 is $(w - e_1)$. For the sub-supplier, there are two profit regions. For disruptions up to t_w , the profit margin is $(w_1 - e_2)$. For longer disruptions, for the portion of the disruption that exceeds t_w , the profit margin is $(w_2 - e_2)$. Proposition 3 characterizes the supplier's and the sub-supplier's emergency capacity reservations under the two-level wholesale price contract.

PROPOSITION 3. Under a two-level wholesale price contract with $\bar{w} = (w_1, w_2, t_w)$, the supplier's and the sub-supplier's respective optimal emergency capacities are

$$t_{1,\bar{w}}^* = \left[F^{-1} \left(\left[1 - \frac{k_1}{w - e_1} \right]^+ \right) - t_{2,\bar{w}}^* \right]^+,$$

$$t_{2,\bar{w}}^* = \begin{cases} F^{-1} \left(\left[1 - \frac{k_2}{w_1 - e_2} \right]^+ \right) & \text{if } F^{-1} \left(\left[1 - \frac{k_2}{w_1 - e_2} \right]^+ \right) \leq t_w, \\ F^{-1} \left(\left[1 - \frac{k_2}{w_2 - e_2} \right]^+ \right) & \text{if } F^{-1} \left(\left[1 - \frac{k_2}{w_2 - e_2} \right]^+ \right) > t_w, \\ t_w & \text{otherwise.} \end{cases}$$

We observe that with medium to high price breaks t_w , the sub-supplier's capacity is determined by w_1 and all capacity is sold for w_1 , which makes the sub-supplier's emergency capacity even larger than in the base case since $w_1 > v$. With a low price break t_w , on the other hand, the sub-supplier's capacity is determined by w_2 , and capacity is sold for w_1 up to t_w and for w_2 afterwards. Thus, the two-level wholesale price contract can coordinate the upstream supply chain only under a low price break t_w . In Proposition 4, we study the coordinating parameters.

PROPOSITION 4. For $\frac{k_2 - k_1}{e_1 - e_2} > \frac{k_2}{v - e_2}$, a two-level wholesale price contract with (w_1, w_2, t_w) can achieve incentive-compatible coordination of the upstream supply chain, with

$$t_w < F^{-1} \left(\left[1 - \frac{k_2}{w_2 - e_2} \right]^+ \right), \quad (8)$$

$$w_2 = e_2 + k_2 \times \frac{e_1 - e_2}{k_2 - k_1}, \quad (9)$$

$$w_1 \in [w_1, \bar{w}_1], \quad (10)$$

where $w_1 = \{w_1 : \Pi_{2,\bar{w}}(t_{2,C}^*) = \Pi_2(t_2^*)\}$, $\bar{w}_1 = \{w_1 : \Pi_{0,\bar{w}}(t_{1,C}^*, t_{2,C}^*) = \Pi_1(t_1^*, t_2^*)\}$, and $\bar{w} = (w_1, w_2, t_w)$.

Conditions (8) and (9) guarantee a low price break and a discounted wholesale price that would align the optimal capacity of the sub-supplier in the base case with the integrated supplier's. Condition (10) assures incentive compatibility and determines how profits are shared. For $w_1 = w_1$, the supplier receives all the gains and for $w_1 = \bar{w}_1$, the sub-supplier receives all the gains from supply chain coordination. Between these extreme points, there is a continuum of share.

To lower the sub-supplier's emergency capacity decision compared to the base case, the wholesale price that determines the sub-supplier's capacity decision must be lower than in the base case. Thus, $v > w_2$ is needed. In addition, to compensate for the lowered profit margin after the price break, the wholesale price before the break must be higher than in the base case, i.e., $w_1 > v > w_2$, which is satisfied by the conditions in Proposition 4.

4. Coordinating the Downstream Supply Chain with Integrated Supplier through Cascading

In this section, we shift our focus back to the entire supply chain, and study the buyer's strategy to align the incentives when, due to invisibility, he cannot have the means to engage with the Tier 2 sub-supplier directly. In such a case, we propose cascading as a measure to coordination. Since we have already shown how incentives in the upstream supply chain can be aligned in an incentive-compatible manner, we consider the upstream supply chain as a single, integrated supplier. We denote the integrated supplier as Player I' .

4.1. The Buyer's Problem

The integrated supplier's decision is how much capacity to reserve at Tier 1 and Tier 2. We assume that the buyer can observe the integrated supplier's Tier 1 capacity $t_{1'}$. This can be achieved by auditing, which is common in many industries. For example, Intel conducted 142 on-site audits in 2013 (Intel 2013). Similarly, the Tier 1 costs are known to the buyer. However, due to invisibility, the Tier 2 capacity $t_{2'}$ and the associated costs cannot be observed by the buyer.

Contracts we studied in Section 3 are based on the suppliers' costs. However, basing a contract on the integrated supplier's cost is impossible for the buyer under invisibility, when some of the integrated supplier's cost, i.e., the costs of Tier 2, are private information. Therefore, we propose a value-based carrot-and-stick contract, which is based on the *value for the buyer* rather than the cost of the integrated supplier.

Recall the buyer's profit function (Equation (3)):

$$\begin{aligned}\Pi_0(t_{1'}, t_{2'}) &= (p - w) \left[\int_0^{t_{1'} + t_{2'}} T dF(T) + (t_{1'} + t_{2'})[1 - F(t_{1'} + t_{2'})] \right] \\ &= (p - w) \left[\int_0^t T dF(T) + t[1 - F(t)] \right] = \Pi_0(t).\end{aligned}$$

Note that the buyer's profit is a function of $t = t_{1'} + t_{2'}$; i.e., the buyer is only concerned about the total emergency capacity in the supply chain rather than the actual capacity mix, which is a key condition for coordination. For the buyer, the marginal value of capacity during a disruption is $\frac{\partial}{\partial t} \Pi_0(t) = (p - w) [1 - F(t)]$. As expected, this marginal value is always positive since the cost for reserving capacity is paid by the integrated supplier. To align this mismatch between costs and benefits, we suggest a mechanism that incorporates the marginal profit of the buyer to the integrated supplier's decision.

We define $C_S(t) = (p - w) \left[\int_0^t T dF(T) + t[1 - F(t)] \right]$ as a value-based subsidy and $C_P(t) = -(p - w) \left[\int_t^\infty T dF(T) - t[1 - F(t)] \right]$ as a value-based penalty. While both of these measures could coordinate the supply chain, a subsidy is incentive incompatible for the buyer while a penalty is incentive incompatible for the integrated supplier. Therefore, we propose a similar carrot-and-stick mechanism, i.e., a combination of penalties and subsidies, as in Section 3.

4.2. The Value-Based Carrot-and-Stick Contract

Let $C_\alpha(t) = \alpha C_S(t) + (1 - \alpha)C_P(t)$, $\alpha \in [0, 1]$, be the value-based carrot-and-stick contract, where the integrated supplier receives $C_\alpha(t)$ from the buyer. While the definition allows $C_\alpha(t) < 0$, for incentive compatibility $C_\alpha(t) > 0$ must hold. Adjusting the objective functions of the integrated supplier and the buyer with $C_\alpha(t)$ leads to:

$$\begin{aligned}\Pi_{0,C}(t) &= \Pi_0(t) - C_\alpha(t), \\ \Pi_{I',C}(t_{1'}, t_{2'}) &= \Pi_{I'}(t_{1'}, t_{2'}) + C_\alpha(t).\end{aligned}$$

The following proposition shows that an incentive-compatible coordination is always possible with a value-based carrot-and-stick contract.

PROPOSITION 5. *With a value-based carrot-and-stick contract $C_\alpha(t)$, $\alpha \in [\underline{\alpha}, \bar{\alpha}]$:*

- (i) *A profit maximizing integrated supplier chooses $t_{1,C}^*$ and $t_{2,C}^*$.*
- (ii) *The buyer's profit is higher than in the base case for $\alpha < \bar{\alpha}$, and is equal to the base case profit for $\alpha = \bar{\alpha}$, where $\bar{\alpha} = \{\alpha : \Pi_{0,C_\alpha}(t_{1,C}^* + t_{2,C}^*) = \Pi_0(t_{1'}^* + t_{2'}^*)\}$.*
- (iii) *The integrated supplier's profit is higher than in the base case for $\alpha > \underline{\alpha}$, and is equal to the base case profit for $\alpha = \underline{\alpha}$, where $\underline{\alpha} = \{\alpha : \Pi_{I',C_\alpha}(t_{1,C}^*, t_{2,C}^*) = \Pi_{I'}(t_{1'}^*, t_{2'}^*)\}$.*
- (iv) *$0 < \underline{\alpha} < \bar{\alpha} < 1$ always holds.*

The parameter α determines the penalty-subsidy ratio of $C_\alpha(t)$ and thus how supply chain profit gains from coordination are shared between the buyer and the integrated supplier. For large values of α , the subsidy is predominant, which allocates the majority of profit gains to the integrated supplier. For $\underline{\alpha}$, the integrated supplier is indifferent to the base case. For small values of α , the penalty is predominant, which allocates the majority of profit gains to the buyer. For $\bar{\alpha}$, the buyer is indifferent to the base case. Any value of α in between allocates the additional profits due to coordination between the integrated supplier and the buyer arbitrarily.

Although the buyer prefers $\underline{\alpha}$ since he benefits from all profit gains due to coordination, it is clear that he cannot achieve this as $\underline{\alpha}$ depends on the integrated supplier's base case profit, which is unknown to the buyer due to invisibility and private information of the Tier 2 sub-supplier. Next, let us assume a case where the integrated supplier reports the total capacity t as well as his base case costs (and thus profit). As the buyer can observe Tier 1 capacity $t_{1'}$, such a report allows the buyer to determine the reserved Tier 2 capacity $t_{2'}$. Therefore, e_2 gives enough information about the base case costs/profits. However, there are two issues with the integrated supplier's report that we need to consider: (1) untruthful reporting of the reserved emergency capacity and (2) untruthful reporting of the cost.

Under-reporting of the capacity cannot be optimal for the integrated supplier. Over-reporting of the capacity can be eliminated with an additional large penalty that is enforced when over-reporting has been detected. Thus, truthful reporting of capacity can be achieved and the integrated supplier's capacity decision is centrally optimal by construction of $C_\alpha(t)$ as desired.

In contrast, untruthful reporting of the cost can be profitable for the integrated supplier. While different costs can lead to the same Tier 2 capacity, profits for the integrated supplier can be different. Thus, there is moral hazard for the integrated supplier to untruthfully report costs that maximize his profit. Since the buyer tries to leave the integrated supplier with his base case profit, the integrated supplier will report Tier 2 costs that maximize his implied base case profit. For a given capacity level, the integrated supplier's base case profit increases in e_2 . Therefore, when asked to report Tier 2 costs the integrated supplier will report the highest possible $e_2 < e_1$. Accounting for this situation, the integrated supplier can set $\tilde{\alpha}(t)$ as stated in the following proposition; that is, ask the buyer to pay an information rent to guarantee truthful reporting of his costs.

PROPOSITION 6. *The integrated supplier reserves the centrally optimal capacity and receives maximum incentive-compatible information rent with*

$$\tilde{\alpha}(t) = \min \left(\bar{\alpha}, \frac{(p-w) \int_t^\infty T dF(T) - (w-e_1) \int_{\hat{t}_0(t)}^t T dF(T)}{(p-w)\mathbb{E}[T]} \right),$$

where $\hat{t}_0(t)$ is the maximum plausible base case capacity.

With this value-based carrot-and-stick contract that includes information rent of the supplier, the buyer can always coordinate the supply chain. Basing penalties and subsidies on his value rather than the sub-supplier's costs allows the buyer to cascade his risk management strategy, since he only contracts with the supplier and not the sub-supplier. While cascading allows supply chain coordination under invisibility, the value of information is equal to the information rent obtained by the supplier. It is interesting to note that although the sub-supplier is invisible and holds the private information, the supplier obtains the information rent. Thus, the supplier benefits from a certain degree of invisibility in the supply chain.

5. Conclusion

Motivated by Intel's risk management challenges in their material supply chain, this research studies how a buyer may work with his Tier 1 supplier to manage supply risk in upper tiers of his supply chain. In a setting where the suppliers can reserve emergency capacity proactively, an important concern is the buyer's limited visibility and control over the upper parts of the supply chain. Using a stylized three-tier supply chain consisting of a buyer, a Tier 1 supplier, and a Tier 2

sub-supplier, we first show that the supplier can use either a cost-based carrot-and-stick contract or a two-level wholesale price contract to coordinate capacity reservation with the sub-supplier. These contracts not only eliminate a special form of double marginalization but also manage the substitution effect that cause inefficiencies by leading to either under- or over-reservation of the sub-supplier depending on the cost structure of the supply chain. For under-reservation, we show that a cost-based carrot-and-stick contract can achieve incentive-compatible supply chain coordination. For over-reservation, a two-level wholesale price contract can achieve incentive-compatible supply chain coordination. We then study the buyer’s problem and show that a value-based carrot-and-stick contract combined with the contracts between the supplier and the sub-supplier will allow for an efficient risk cascading mechanism. Our results suggest that, for supply chains with disruption risk, different combinations of penalties and subsidies are necessary to align incentives. Generally, neither subsidies nor penalties alone can align the supply chain.

Our stylized model does not fully represent the structure of any one of the categories in Intel’s material supply chain; yet, it captures the salient risk challenges that Intel faces. Thus, our analysis and observations help us suggest several risk management strategies for Intel’s material supply chain as summarized in Figure 4.

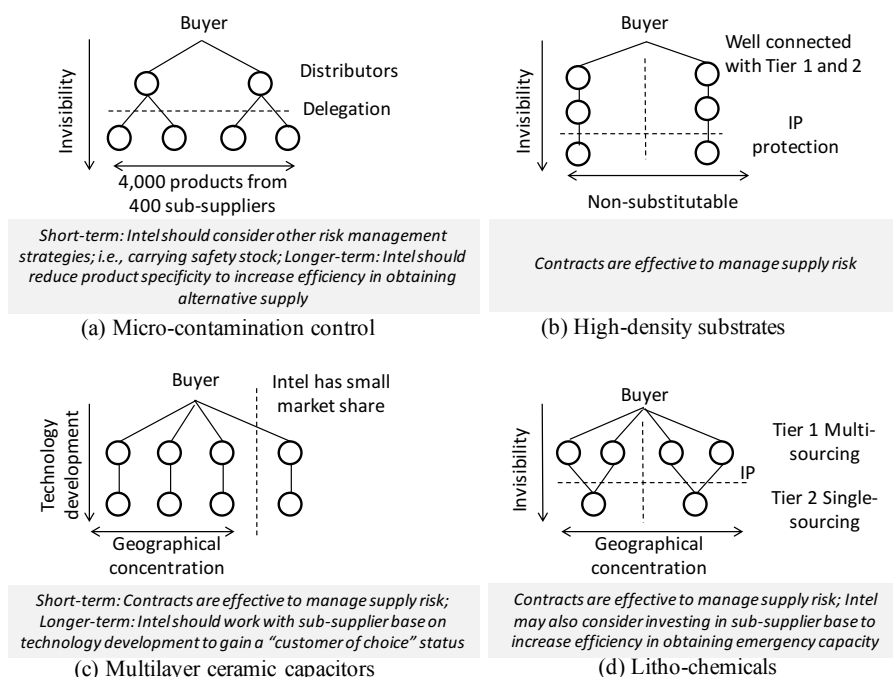


Figure 4 Risk management strategies for Intel's material supply chain.

The proposed carrot-and-stick contract can help Intel risk manage their material supply chain efficiently and effectively, especially for high-density substrates (Figure 4(b)) and litho-chemicals

(Figure 4(d)). These materials are highly significant for Intel constituting over 90% of their annual spending on materials. As a result, Intel has strong supplier relationships with high-density substrates and litho-chemicals suppliers; for high-density substrates, some suppliers even have dedicated Intel factories. Given the importance of high-density substrates and litho-chemicals for Intel's business, and the already strong relationships with Tier 1, the primary focus should be to add protection at Tier 2. In our model, Case III describes this situation the best since both base case and integrated supplier intend capacity at both tiers (Figure 3). In the past, Intel chose not to enforce existing penalty contracts with these suppliers in lieu of maintaining relationships. Therefore, introducing a subsidy (in addition to existing penalties) can indeed be a more effective way of managing risks; with added benefits of aligning supply chains and further strengthening relationships. Such a change in contract structure can easily be justified by the importance of these two categories of materials and Intel's recent shift towards more collaborative risk management strategies in their value chains (Peng et al. 2012, Kempf et al. 2013). For litho-chemicals, Intel should also consider investing in sub-supplier base to reduce geographical concentration and increase the efficiency in obtaining emergency capacity.

Although Intel usually has more bargaining power compared to their suppliers, this is not always the case. For example, Intel's market share in their multilayer ceramic capacitors supply chain (Figure 4(c)) is only 3-5%, therefore, their market power is quite limited. The proposed carrot-and-stick contract can still help Intel manage risk for such materials. It may be in Intel's best interest to use technology development to increase their bargaining power and negotiate preferential treatment, a "customer of choice" status, during disruptions. Within our modeling framework, Case II in Figure 3 describes this scenario the best since Intel wants to increase capacity at both Tier 1 and Tier 2.

Despite the benefits, risk management through cascading is not the answer to all of Intel's challenges and Intel cannot employ a one-size-fits-all strategy for all types of materials in their material supply chain. For example, Intel's annual spending on micro-contamination control materials (Figure 4(a)) accounts for only 2% of their total spending. Further, this spending is spread across over 400 sub-suppliers. Therefore, Intel is a small buyer for many of these sub-suppliers and actively managing relationships would be costly for Intel. Providing additional capacity at the distributor level is similarly challenging due to the high product variety. Incentivizing sub-suppliers to add capacity can be an option that corresponds to Case I of our model. However, for such materials, easier to manage measures such as increasing safety stock levels and protecting the supply chain with inventory might be an effective risk management alternative.

At its inception, the goal of this research was to incentivize sub-suppliers to better prepare for disruptions. In Case IV, however, sub-suppliers over-reserve capacity. Thus, sub-suppliers need to be incentivized to reserve less capacity, opposite to what we intended and observed in Intel's material supply chain. While Case IV does not apply to any of the Intel supply chains that we studied, our model is more general by including Case IV, which makes our results relevant even beyond the motivating case study. In addition, although currently this is not Intel's practice, it is of Intel's interest to think about product design in order to lower the emergency capacity costs, especially for low market share supplies. Such an approach would generate advantages in terms of managing risk with alternative resources.

While our research has been motivated by Intel's risk management challenges, our results apply more generally to multi-tiered risk management. Specifically, our focus on limited supply chain visibility and high product specificity in a multi-tier supply chain contributes to the literature on supply chain disruptions and effective risk management strategies. Our focus on mechanism design and cascading adds to the literature on multi-tier supply chains. For multi-tier supply chains, the effects of delegation versus control have been studied in the academic literature (e.g., Kayış et al. (2013)). We show that, despite being frequently studied, control may not be a feasible option, especially when the buyer has limited visibility and control over the upper parts of the supply chain. Given that limited visibility has been a major concern for stakeholders in supply chains recently, understanding how a firm may use delegation effectively under such a setting makes an important theoretical and practical contribution.

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Appendix A: Additional Results

We begin by considering three benchmark cases. The decentralized supply chain with wholesale price contracts between the buyer, the supplier, and the sub-supplier is our base case. The centralized supply chain is our first-best benchmark. Finally, we consider an integrated supplier, which we define as the centralized upstream supply chain.

A.1. Base Case

Proposition A1, derived from the first-order conditions of Equations (1) and (2), shows the supplier's and the sub-supplier's optimal capacity decisions.

PROPOSITION A1. [Base case] *The supplier's and the sub-supplier's respective optimal emergency capacities are as given in Table 2.*

Proof: The supplier's and the sub-supplier's profits in the base case are given in Equations (1) and (2), respectively. The supplier's decision variable is t_1 . The first derivative of the supplier's objective function with respect to t_1 leads to

$$\frac{\partial}{\partial t_1} \Pi_1(t_1, t_2) = -k_1 + (w - e_1) [1 - F(t_1 + t_2)].$$

The second derivative, $\frac{\partial^2}{\partial t_1^2} \Pi_1(t_1, t_2) = -(w - e_1) f(t_1 + t_2) \leq 0$, shows concavity of the supplier's objective function in t_1 . The supplier's capacity level depends on the sub-supplier's decision; i.e., the supplier reserves emergency capacity so that the total emergency capacity is $t_1 + t_2 = F^{-1} \left(\left[1 - \frac{k_1}{w - e_1} \right]^+ \right)$.

The first derivative of the sub-supplier's objective function is

$$\frac{\partial}{\partial t_2} \Pi_2(t_2) = -k_2 + (v - e_2) [1 - F(t_2)].$$

The second derivative, $\frac{\partial^2}{\partial t_2^2} \Pi_2(t_2) = -(v - e_2) f(t_2) \leq 0$, shows concavity of the sub-supplier's objective function. Thus, the sub-supplier reserves emergency capacity $t_2 = F^{-1} \left(\left[1 - \frac{k_2}{v - e_2} \right]^+ \right)$. \square

The supplier only reserves extra capacity when $\frac{k_1}{w - e_1} < \frac{k_2}{v - e_2}$, i.e., when the supplier's ratio of reservation cost to profit margin is lower than the sub-supplier's. We refer to the case where capacity is reserved at both tiers as a mixed strategy.

A.2. Centralized Supply Chain

Since $e_2 \leq e_1$ by assumption (i.e., it is always optimal to use Tier 2 emergency capacity before Tier 1 capacity), the objective function for the centralized supply chain then equals the sum of the three objective functions from the base case (Equations (1) - (3)):

$$\begin{aligned} \Pi_C(t_1, t_2) = & -k_1 t_1 - k_2 t_2 + (p - e_2) \left[\int_0^{t_2} T dF(T) + t_2 [1 - F(t_2)] \right] \\ & + (p - e_1) \left[\int_{t_2}^{t_1 + t_2} (T - t_2) dF(T) + t_1 [1 - F(t_1 + t_2)] \right]. \end{aligned} \quad (\text{A1})$$

The first two terms correspond to the costs of reserving emergency capacity; the last two terms correspond to expected profits from using emergency capacity during a disruption. Optimizing Equation (A1) over Tier 1 and Tier 2 emergency capacity decisions leads to the following capacity strategy.

PROPOSITION A2. [Centralized supply chain] *Optimal emergency capacities are as given in Table 2.*

Proof: Partial first derivatives of the objective function in Equation (A1) with respect to t_1 and t_2 are $\frac{\partial}{\partial t_1} \Pi_C(t_1, t_2) = -k_1 + (p - e_1) [1 - F(t_1 + t_2)]$ and $\frac{\partial}{\partial t_2} \Pi_C(t_1, t_2) = -k_2 + (e_1 - e_2) [1 - F(t_2)] + (p - e_1) [1 - F(t_1 + t_2)]$. Consequently, the Hessian matrix is:

$$H_{\Pi_C}(t_1, t_2) = \begin{bmatrix} -(p - e_1) f(t_1 + t_2) & -(p - e_1) f(t_1 + t_2) \\ -(p - e_1) f(t_1 + t_2) & -(e_1 - e_2) f(t_2) - (p - e_1) f(t_1 + t_2) \end{bmatrix}.$$

Note $\det(H_{\Pi_C}(t_1, t_2)) = (-(p - e_1)f(t_1 + t_2))(-(e_1 - e_2)f(t_2) - (p - e_1)f(t_1 + t_2)) - (-(p - e_1)f(t_1 + t_2))^2 > 0$ and the second partial derivatives are non-positive. That is, the objective function is jointly concave.

Setting first derivatives to zero and substituting leads to $t_1^* + t_2^* = F^{-1}\left(\left[1 - \frac{k_1}{p - e_1}\right]^+\right)$ and $t_2^* = F^{-1}\left(\left[1 - \frac{k_2 - k_1}{e_1 - e_2}\right]^+\right)$. For $\frac{k_2 - k_1}{e_1 - e_2} < \frac{k_1}{p - e_1}$, t_1 would have to be negative to satisfy the first-order conditions. Therefore, we eliminate t_1 from the objective function for this case to achieve $\Pi_C(t_1, t_2) = -k_2 t_2 + (p - e_2) \left[\int_0^{t_2} T dF(T) + t_2 [1 - F(t_2)] \right]$. Since $\frac{\partial}{\partial t_2} \Pi_C(t_1, t_2) = -k_2 + (p - e_2) [1 - F(t_2)]$ and $\frac{\partial}{\partial t_2} \Pi_C(t_1, t_2) = -(p - e_2) f(t_2)$, $\Pi_C(t_1, t_2)$ is maximized for $t_2 = F^{-1}\left(\left[1 - \frac{k_2}{p - e_2}\right]^+\right)$ when $\frac{k_2 - k_1}{e_1 - e_2} < \frac{k_1}{p - e_1}$. \square

We say an emergency source i is more economical when $\frac{k_i}{p - e_i} \leq \frac{k_{-i}}{p - e_{-i}}$, i.e., when its ratio of reservation cost to profit margin is smaller. The critical fractile $\frac{k_2 - k_1}{e_1 - e_2}$ can be viewed as an indicator of the total cost difference between the two alternative emergency sources as it is the ratio of the differences between the reservation and emergency costs. Large values of $\frac{k_2 - k_1}{e_1 - e_2}$ mean that the supplier's emergency source is more economical than the sub-supplier. Conversely, small values of $\frac{k_2 - k_1}{e_1 - e_2}$ mean that the sub-supplier's emergency source is more economical than the supplier's. That is, for $\frac{k_2}{p - e_2} \leq \frac{k_1}{p - e_1}$ and $\frac{k_2}{p - e_2} < 1$, it is centrally optimal to reserve capacity only at Tier 2. For $\frac{k_2}{p - e_2} > \frac{k_1}{p - e_1}$ and $\frac{k_2 - k_1}{e_1 - e_2} > 1$, it is centrally optimal to reserve capacity only at Tier 1; this case is not feasible in our setting due to our cost assumptions. Finally, for $1 > \frac{k_2}{p - e_2} > \frac{k_1}{p - e_1}$, a mixed capacity strategy is centrally optimal.

A.3. Integrated Supplier

We now study the centralized upstream supply chain, i.e., when the supplier and the sub-supplier are coordinated. The centralized upstream supply chain is equivalent to an integrated supplier.

$$\begin{aligned} \Pi_I(t_1, t_2) = & -k_1 t_1 - k_2 t_2 + (w - e_2) \left[\int_0^{t_2} T dF(T) + t_2 [1 - F(t_2)] \right] \\ & + (w - e_1) \left[\int_{t_2}^{t_1 + t_2} (T - t_2) dF(T) + t_1 [1 - F(t_1 + t_2)] \right]. \end{aligned} \quad (\text{A2})$$

Equation (A2) is equivalent to Equation (A1) except for a lower profit margin. Optimizing Equation (A2) gives the following optimal emergency capacities, which can be interpreted similar to Proposition A2.

PROPOSITION A3. [Integrated supplier] *Optimal emergency capacities are as given in Table 2.*

Proof: This proof directly follows from the proof of Proposition A2 by replacing p with w . \square

Appendix B: Proofs

Proof of Proposition 1: The supplier's and the sub-supplier's profits with the carrot-and-stick contract are as given in Equations (4) and (5), respectively. The supplier's decision variable is t_1 . Since the second and third terms are independent of the supplier's decision in Equation (4), the supplier's optimal decision is equivalent to the one in the base case; i.e., the total emergency capacity again is $t_1^* + t_2^* = F^{-1}\left(\left[1 - \frac{k_1}{w - e_1}\right]^+\right)$.

The first derivative of the sub-supplier's objective function is $\frac{\partial}{\partial t_2} \Pi_{2,ks}(t_2) = \frac{\partial}{\partial t_2} \Pi_2(t_2) + s [1 - F(t_2)] + k = -(k_2 - k) + (v - e_2 + s) [1 - F(t_2)]$. The second derivative, $\frac{\partial^2}{\partial t_2^2} \Pi_{2,ks}(t_2) = -(v - e_2 + s) f(t_2) \leq 0$, shows concavity of the sub-supplier's objective function. Thus, the sub-supplier will reserve emergency capacity $t_2^* = F^{-1}\left(\left[1 - \frac{k_2 - k}{v - e_2 + s}\right]^+\right)$. \square

Proof of Proposition 2: The optimal Tier 2 capacity in the integrated supplier setting is

$$t_{2,I}^* = \begin{cases} F^{-1} \left(\left[1 - \frac{k_2}{w-e_2} \right]^+ \right) & \text{if } \frac{k_2}{w-e_2} \leq \frac{k_1}{w-e_1}, \\ F^{-1} \left(\left[1 - \frac{k_2-k_1}{e_1-e_2} \right]^+ \right) & \text{otherwise} \end{cases}$$

and under a carrot-and-stick contract is

$$t_{2,ks}^* = F^{-1} \left(\left[1 - \frac{k_2-k}{v-e_2+s} \right]^+ \right)$$

according to Propositions A.3 and 1, respectively.

From $t_{2,I}^* = t_{2,ks}^*$, we get

$$k(s) = \begin{cases} k_2 \times \frac{w-v-s}{w-e_2} & \text{if } \frac{k_2}{w-e_2} \leq \frac{k_1}{w-e_1}, \\ \frac{(v-e_2+s)k_1 - (v-e_1+s)k_2}{e_1-e_2} & \text{otherwise} \end{cases}.$$

Subsidy $k(s)$ is decreasing in s . It is easy to see that $k_2 \times \frac{w-v-s}{w-e_2}$ decreases in s . Furthermore, $\frac{(v-e_2+s)k_1 - (v-e_1+s)k_2}{e_1-e_2}$ decreases in s since $k_2 \geq k_1$ must hold when $\frac{k_2}{w-e_2} > \frac{k_1}{w-e_1}$ as $e_2 \leq e_1$ by assumption.

Since k cannot be negative,

$$s \leq \bar{s} = \begin{cases} w-v & \text{if } \frac{k_2}{w-e_2} \leq \frac{k_1}{w-e_1}, \\ \frac{(v-e_1)k_2 - (v-e_2)k_1}{k_1-k_2} & \text{otherwise} \end{cases}.$$

Supply chain profits $\Pi_I(t_1, t_2)$ are jointly concave (see proof of Proposition A.3). Since $t_{1,I}^* = t_{1,ks}^*$, it follows that $\Pi_I(t_{1,I}^*, t_2)$ increases in t_2 up to $t_{2,I}^*$. Since $t_{2,ks}^* = F^{-1} \left(\left[1 - \frac{k_2-k}{v-e_2+s} \right]^+ \right)$ increases both in k and s , it further follows that $\Pi_I(t_{1,I}^*, t_{2,ks}^*)$ increases both in k and s up to $k = k(s)$ and $s = \bar{s}$.

We now show that there always exists a \underline{s} with corresponding coordinating $k(s)$ such that $\Pi_{1,ks}(t_{1,C}^*, t_{2,C}^*) = \Pi_1(t_1^*, t_2^*)$. For any $k(s)$ and s ,

$$\Pi_{1,ks}(t_{1,C}^*, t_{2,C}^*) = (w-v) \int_0^{t_{2,C}^*} T dF(T) + (w-e_1) \int_{t_{2,C}^*}^{t_{1,C}^* + t_{2,C}^*} T dF(T) + s \int_{t_{2,C}^*}^{\infty} T dF(T).$$

Note that $\Pi_{1,ks}(t_{1,C}^*, t_{2,C}^*)$ increases in s and $\Pi_1(t_1^*, t_2^*)$ is independent of s . Thus, the difference $\Pi_{1,ks}(t_{1,C}^*, t_{2,C}^*) - \Pi_1(t_1^*, t_2^*)$ increases in s and is smallest for $s = 0$. We have $\Pi_{1,ks}(t_{1,C}^*, t_{2,C}^*) - \Pi_1(t_1^*, t_2^*)$ as

$$-(v-e_1) \int_{t_2^*}^{t_{2,C}^*} T dF(T) + s \int_{t_{2,C}^*}^{\infty} T dF(T) - [(w-e_1)[1-F(t_1^*+t_2^*)] - (v-e_1)[1-F(t_2^*)]].$$

To show that for $s = 0$, $\Pi_{1,ks}(t_{1,C}^*, t_{2,C}^*) - \Pi_1(t_1^*, t_2^*) \leq 0$, note that the first term is non-positive because $v > e_1$ and the second term disappears at $s = 0$. To show that the last term is non-positive, we consider two cases.

- For $\frac{k_2}{w-e_2} \leq \frac{k_1}{w-e_1}$, $t_1^* = 0$ and thus

$$-[(w-e_1)[1-F(t_1^*+t_2^*)] - (v-e_1)[1-F(t_2^*)]] = -[(w-e_1)[1-F(t_2^*)] - (v-e_1)[1-F(t_2^*)]] \leq 0.$$

- For $\frac{k_2}{w-e_2} > \frac{k_1}{w-e_1}$ and $\frac{k_2}{v-e_2} \geq \frac{k_2-k_1}{e_1-e_2}$,

$$\begin{aligned} k_2(e_1-e_2) \geq (k_2-k_1)(v-e_2) &\iff (v-e_2)k_1 - (v-e_1)k_2 \geq 0 \iff \frac{(v-e_2)k_1 - (v-e_1)k_2}{v-e_2} \geq 0 \\ &\iff -[(w-e_1)[1-F(t_1^*+t_2^*)] - (v-e_1)[1-F(t_2^*)]] \leq 0. \end{aligned}$$

For \bar{s} , $\Pi_{1,ks}(t_{1,C}^*, t_{2,C}^*) - \Pi_1(t_1^*, t_2^*) \geq 0$. Therefore, by the mean value theorem, there must exist \underline{s} such that $\Pi_{1,ks}(t_{1,C}^*, t_{2,C}^*) = \Pi_1(t_1^*, t_2^*)$. The desired result follows since $\Pi_{1,ks}(t_{1,C}^*, t_{2,C}^*)$ increases in s .

We have

$$\begin{aligned}\Pi_{2,ks}(t_{2,ks}^*) &= -(k_2 - k)t_{2,ks}^* + (v - e_2) \left[\int_0^{t_{2,ks}^*} T dF(T) + t_{2,ks}^*[1 - F(t_{2,ks}^*)] \right] - s \int_{t_{2,ks}^*}^{\infty} (T - t_{2,ks}^*) dF(T) \\ &= (v - e_2) \int_0^{t_{2,ks}^*} T dF(T) - s \int_{t_{2,ks}^*}^{\infty} T dF(T). \\ \Pi_2(t_2^*) &= -k_2 t_2^* + (v - e_2) \left[\int_0^{t_2^*} T dF(T) + t_2^*[1 - F(t_2^*)] \right] = (v - e_2) \int_0^{t_2^*} T dF(T).\end{aligned}$$

Note that $t_{2,ks}^*$ increases in k , and $\Pi_{2,ks}(t_{2,ks}^*)$ increases in $t_{2,ks}^*$. It follows that $\Pi_{2,ks}(t_{2,ks}^*)$ increases in k . For $k = 0$ and $s > 0$, $\Pi_{2,ks}(t_{2,ks}^*) - \Pi_2(t_2^*) < 0$. As $\lim_{k \rightarrow k_2} \Pi_{2,ks}(t_{2,ks}^*) = (v - e_2) \int_0^{\infty} T dF(T) > (v - e_2) \int_0^{t_2^*} T dF(T) = \Pi_2(t_2^*)$, we get $\lim_{k \rightarrow k_2} \Pi_{2,ks}(t_{2,ks}^*) - \Pi_2(t_2^*) > 0$.

By the mean value theorem it follows, that for every s there exist a $\underline{k}(s) \in [0, k_2]$, for which $\Pi_{2,ks}(t_{2,ks}^*) - \Pi_2(t_2^*) = 0$ meaning $\Pi_{2,ks}(t_{2,ks}^*) = \Pi_2(t_2^*)$. The desired result follows since $\Pi_{2,ks}(t_{2,ks}^*)$ increases in k . \square

Proof of Proposition 3: The supplier's profit under a two-level wholesale price contract (Equation (6)) is

$$\Pi_{1,\bar{w}}(t_1, t_2) = \begin{cases} \begin{aligned} & -k_1 t_1 + (w - w_1) \left[\int_0^{t_2} T dF(T) + t_2[1 - F(t_2)] \right] \\ & + (w - e_1) \left[\int_{t_2}^{t_1+t_2} (T - t_2) dF(T) + t_1[1 - F(t_1 + t_2)] \right] \end{aligned} & \text{if } t_2 \leq t_w, \\ \begin{aligned} & -k_1 t_1 + (w - w_1) \left[\int_0^{t_w} T dF(T) + t_w[1 - F(t_w)] \right] \\ & + (w - w_2) \left[\int_{t_w}^{t_2} (T - t_w) dF(T) + (t_2 - t_w)[1 - F(t_2)] \right] \\ & + (w - e_1) \left[\int_{t_2}^{t_1+t_2} (T - t_2) dF(T) + t_1[1 - F(t_1 + t_2)] \right] \end{aligned} & \text{otherwise.} \end{cases}$$

We take the first derivative of the supplier's objective function with regard to t_1 , which leads to $\frac{\partial}{\partial t_1} \Pi_{1,\bar{w}}(t_1, t_2) = -k_1 + (w - e_1)[1 - F(t_1 + t_2)]$. The second derivative, $\frac{\partial^2}{\partial t_1^2} \Pi_{1,\bar{w}}(t_1, t_2) = -(w - e_1)f(t_1 + t_2) \leq 0$, shows concavity of the supplier's objective function in t_1 . The supplier reserves emergency capacity so that the total emergency capacity is $t_1^* + t_2^* = F^{-1} \left(\left[1 - \frac{k_1}{w - e_1} \right]^+ \right)$. Thus,

$$t_{1,\bar{w}}^* = \left[F^{-1} \left(\left[1 - \frac{k_1}{w - e_1} \right]^+ \right) - t_{2,\bar{w}}^* \right]^+.$$

From Equation (7), the sub-supplier's profit under a two-level wholesale price contract is

$$\Pi_{2,\bar{w}}(t_2) = \begin{cases} \begin{aligned} & -k_2 t_2 + (w_1 - e_2) \left[\int_0^{t_2} T dF(T) + t_2[1 - F(t_2)] \right] \end{aligned} & \text{if } t_2 \leq t_w, \\ \begin{aligned} & -k_2 t_2 + (w_1 - e_2) \left[\int_0^{t_w} T dF(T) + t_w[1 - F(t_w)] \right] \\ & + (w_2 - e_2) \left[\int_{t_w}^{t_2} (T - t_w) dF(T) + (t_2 - t_w)[1 - F(t_2)] \right] \end{aligned} & \text{otherwise.} \end{cases} \quad (\text{A3})$$

The first derivative of the sub-supplier's objective function is

$$\frac{\partial}{\partial t_2} \Pi_{2,\bar{w}}(t_2) = \begin{cases} -k_2 + (w_1 - e_2)[1 - F(t_2)] & \text{if } t_2 \leq t_w, \\ -k_2 + (w_2 - e_2)[1 - F(t_2)] & \text{otherwise,} \end{cases}$$

and the second derivative is

$$\frac{\partial^2}{\partial t_2^2} \Pi_{2,\bar{w}}(t_2) = \begin{cases} -(w_1 - e_2)f(t_2) & \text{if } t_2 \leq t_w, \\ -(w_2 - e_2)f(t_2) & \text{otherwise.} \end{cases}$$

Since $w_1 > w_2 > e_2$, the second derivative is negative. Thus, the sub-supplier reserves

$$t_{2,\bar{w}}^* = \begin{cases} F^{-1} \left(\left[1 - \frac{k_2}{w_1 - e_2} \right]^+ \right) & \text{if } F^{-1} \left(\left[1 - \frac{k_2}{w_1 - e_2} \right]^+ \right) \leq t_w, \\ F^{-1} \left(\left[1 - \frac{k_2}{w_2 - e_2} \right]^+ \right) & \text{if } F^{-1} \left(\left[1 - \frac{k_2}{w_2 - e_2} \right]^+ \right) > t_w, \\ t_w & \text{otherwise.} \end{cases}$$

□

Proof of Proposition 4: From Proposition 3 and conditions (8) and (9), we have $t_{2,\bar{w}}^* = F^{-1} \left(\left[1 - \frac{k_2}{w_2 - e_2} \right]^+ \right) = F^{-1} \left(\left[1 - \frac{k_2 - k_1}{e_1 - e_2} \right]^+ \right)$, i.e., the supply chain is coordinated. For a fixed t_w , $\Pi_{1,\bar{w}}(t_{1,C}^*, t_{2,C}^*)$ decreases and $\Pi_{2,\bar{w}}(t_{2,C}^*)$ increases in w_1 . With $w_1 = \{w_1 : \Pi_{2,\bar{w}}(t_{2,C}^*) = \Pi_2(t_2^*)\}$ and $\bar{w}_1 = \{w_1 : \Pi_{1,\bar{w}}(t_{1,C}^*, t_{2,C}^*) = \Pi_1(t_1^*, t_2^*)\}$, $w_1 \geq \bar{w}_1$ is the sub-supplier's and $w_1 \leq \bar{w}_1$ is the supplier's participation constraint. Thus, condition (10), $w_1 \in [\bar{w}_1, w_1]$, guarantees incentive compatibility. □

Proof of Proposition 5: By definition, we have

$$\begin{aligned} C_\alpha(t) &= \alpha C_S(t) + (1 - \alpha) C_P(t) \\ &= \alpha(p - w) \left[\int_0^t T dF(T) + t[1 - F(t)] \right] - (1 - \alpha)(p - w) \left[\int_t^\infty T dF(T) - t[1 - F(t)] \right] \\ &= (p - w) \left[\alpha \mathbb{E}[T] - \int_t^\infty T dF(T) + t[1 - F(t)] \right]. \end{aligned}$$

Thus, $\Pi_{I',C}(t_{1'}, t_{2'}) = \Pi_{I'}(t_{1'}, t_{2'}) + C_\alpha(t)$ equals

$$\begin{aligned} \Pi_{I',C}(t_{1'}, t_{2'}) &= -k_1 t_{1'} - k_2 t_{2'} + (w - e_1) \left[\int_{t_{2'}}^{t_{1'} + t_{2'}} (T - t_{2'}) dF(T) + t_{1'} [1 - F(t_{1'} + t_{2'})] \right] \\ &\quad + (w - e_2) \left[\int_0^{t_{2'}} T dF(T) + t_{2'} [1 - F(t_{2'})] \right] \\ &\quad + (p - w) \left[\alpha \mathbb{E}[T] - \int_{t_{1'} + t_{2'}}^\infty T dF(T) + (t_{1'} + t_{2'}) [1 - F(t_{1'} + t_{2'})] \right]. \end{aligned}$$

First partial derivatives of $\Pi_{I',C}(t_{1'}, t_{2'})$ are

$$\begin{aligned} \frac{\partial}{\partial t_{1'}} \Pi_{I',C}(t_{1'}, t_{2'}) &= -k_1 + (p - e_1) [1 - F(t_{1'} + t_{2'})], \\ \frac{\partial}{\partial t_{2'}} \Pi_{I',C}(t_{1'}, t_{2'}) &= -k_2 + (e_1 - e_2) [1 - F(t_{2'})] + (p - e_1) [1 - F(t_{1'} + t_{2'})]. \end{aligned}$$

First (and in turn second) partial derivatives are the same as in the centralized case. Thus, we conclude that the integrated supplier reserves the centrally optimal capacity and the supply chain is aligned.

For $\alpha = 0$, $C_0(t) = -(p - w) \left[\int_t^\infty T dF(T) - t[1 - F(t)] \right] < 0$. Thus, $\Pi_{I',C_0}(t_{1,C}^*, t_{2,C}^*) = \Pi_{I'}(t_{1,C}^*, t_{2,C}^*) + C_0(t_{1,C}^* + t_{2,C}^*) < \Pi_{I'}(t_{1,C}^*, t_{2,C}^*) \leq \Pi_1(t_{1'}^*, t_{2'}^*)$. As the integrated supplier's profit is lower than in the base case, the buyer's profit must be higher.

For $\alpha = 1$,

$$\begin{aligned} C_1(t) &= (p - w) \left[\int_0^{t_2} T dF(T) + \int_{t_2}^{t_1 + t_2} T dF(T) + (t_1 + t_2) [1 - F(t_1 + t_2)] \right] \\ &= (p - w) \left[\int_0^{t_2} T dF(T) + t_2 [1 - F(t_2)] + \int_{t_2}^{t_1 + t_2} T dF(T) + t_1 [1 - F(t_1 + t_2)] - t_2 [F(t_1 + t_2) - F(t_2)] \right] \\ &= (p - w) \left[\int_0^{t_2} T dF(T) + t_2 [1 - F(t_2)] \right] + (p - w) \left[\int_{t_2}^{t_1 + t_2} (T - t_2) dF(T) + t_1 [1 - F(t_1 + t_2)] \right]. \end{aligned}$$

Thus, the terms of $\Pi_{I',C_1}(t_1, t_2)$ can be rearranged to get

$$-k_1 t_1 - k_2 t_2 + (p - e_1) \left[\int_{t_2}^{t_1+t_2} (T - t_2) dF(T) + t_1 [1 - F(t_1 + t_2)] \right] + (p - e_2) \left[\int_0^{t_2} T dF(T) + t_2 [1 - F(t_2)] \right],$$

which is the same as the profit function of the centralized supply chain. That is, the integrated supplier's profit is higher than in the base case. Independent of the integrated supplier's decision, the buyer's profit $\Pi_{0,C_1}(t_1, t_2) = 0$ meaning $\Pi_{0,C_1}(t_1, t_2) < \Pi_0(t_1^*, t_2^*)$.

As $\frac{\partial}{\partial \alpha} C_\alpha(t) = (p - w)\mathbb{E}[T]$, the integrated supplier's profit increases and the buyer's profit decreases in α . By the mean value theorem, there exist $0 < \underline{\alpha}$, for which the integrated supplier's profit is equal to his base case profit, and $\bar{\alpha} < 1$, for which the buyer's profit is equal to his base case profit. \square

Proof of Proposition 6: Assume that the integrated supplier is asked to report his total capacity t and his base case capacity \hat{t}_0 . Tier 1's capacity, \tilde{t}_1 , is observable. Further, assume the non-trivial case of $t > F^{-1} \left(\left[1 - \frac{k_1}{p-e_1} \right]^+ \right)$. For $t = F^{-1} \left(\left[1 - \frac{k_1}{p-e_1} \right]^+ \right)$, the publicly known Tier 1 parameters determine α . Then,

$$\alpha(t, \hat{t}_0, \tilde{t}_1) = \frac{(p-w) \int_t^\infty T dF(T) - (w - e_2(t, \hat{t}_0, \tilde{t}_1)) \int_{\hat{t}_0}^t T dF(T)}{(p-w)\mathbb{E}[T]}$$

leads to profit for the integrated supplier that is equal to his base case profit implied by the reported reserved and base case capacities: $\Pi_{I',C_{\alpha(t, \hat{t}_0, \tilde{t}_1)}}(t_1, t_2) = (w - e_2(t, \hat{t}_0, \tilde{t}_1)) \int_{\hat{t}_0}^{t_2} T dF(T)$.

Using $t = F^{-1} \left(\left[1 - \frac{k_2(t, \hat{t}_0, \tilde{t}_1)}{p-e_2(t, \hat{t}_0, \tilde{t}_1)} \right]^+ \right)$ and $\hat{t}_0 = F^{-1} \left(\left[1 - \frac{k_2(t, \hat{t}_0, \tilde{t}_1)}{w-e_2(t, \hat{t}_0, \tilde{t}_1)} \right]^+ \right)$, i.e.,

$$k_2(t, \hat{t}_0, \tilde{t}_1) = (p - e_2(t, \hat{t}_0, \tilde{t}_1)) [1 - F(t)] = (w - e_2(t, \hat{t}_0, \tilde{t}_1)) [1 - F(\hat{t}_0)],$$

we can solve for $e_2(t, \hat{t}_0, \tilde{t}_1)$:

$$e_2(t, \hat{t}_0, \tilde{t}_1) = \frac{w[1 - F(\hat{t}_0)] - p[1 - F(t)]}{F(t) - F(\hat{t}_0)}.$$

The first derivative of $e_2(t, \hat{t}_0, \tilde{t}_1)$ with respect to \hat{t}_0 is $\frac{\partial}{\partial \hat{t}_0} e_2(t, \hat{t}_0, \tilde{t}_1) = -\frac{(p-w)f(\hat{t}_0)[1-F(t)]}{[F(t)-F(\hat{t}_0)]^2} < 0$. In addition,

$$\begin{aligned} [(p-w)\mathbb{E}[T]] \frac{\partial}{\partial \hat{t}_0} \alpha(t, \hat{t}_0, \tilde{t}_1) &= \frac{\partial}{\partial \hat{t}_0} \left[-(w - e_2(t, \hat{t}_0, \tilde{t}_1)) \int_{\hat{t}_0}^t T dF(T) \right] \\ &= -\frac{(p-w)f(\hat{t}_0)[1-F(t)]}{[F(t)-F(\hat{t}_0)]^2} \int_{\hat{t}_0}^t T dF(T) + (w - e_2(t, \hat{t}_0, \tilde{t}_1)) \hat{t}_0 f(\hat{t}_0) \\ &< -\frac{(p-w)f(\hat{t}_0)[1-F(t)]}{[F(t)-F(\hat{t}_0)]^2} \hat{t}_0 [F(t) - F(\hat{t}_0)] + (w - e_2(t, \hat{t}_0, \tilde{t}_1)) \hat{t}_0 f(\hat{t}_0) \\ &= -\hat{t}_0 f(\hat{t}_0) \left[(p-w) \frac{1-F(t)}{F(t)-F(\hat{t}_0)} - (w - e_2(t, \hat{t}_0, \tilde{t}_1)) \right] \\ &= -\hat{t}_0 f(\hat{t}_0) \left[(p-w) \frac{w - e_2(t, \hat{t}_0, \tilde{t}_1)}{p - e_2(t, \hat{t}_0, \tilde{t}_1) - w + e_2(t, \hat{t}_0, \tilde{t}_1)} - (w - e_2(t, \hat{t}_0, \tilde{t}_1)) \right] = 0. \end{aligned}$$

Consequently, $\alpha(t, \hat{t}_0, \tilde{t}_1)$ decreases in \hat{t}_0 , and, thus, the integrated supplier will report \hat{t}_0 as small as possible. Furthermore, since $e_2(t, \hat{t}_0, \tilde{t}_1)$ decreases in \hat{t}_0 , the integrated supplier will report \hat{t}_0 such that $e_2(t, \hat{t}_0, \tilde{t}_1) = e_1$. Adding $\bar{\alpha}$ as the buyer's incentive compatibility constraint, and replacing $e_2(t, \hat{t}_0^*, \tilde{t}_1)$ with e_1 and \hat{t}_0 with $\hat{t}_0(t)$ leads to

$$\bar{\alpha}(t) = \min \left(\bar{\alpha}, \frac{(p-w) \int_t^\infty T dF(T) - (w - e_1) \int_{\hat{t}_0(t)}^t T dF(T)}{(p-w)\mathbb{E}[T]} \right).$$

\square