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**Household Location and Schools in Metropolitan Areas
with Heterogeneous Suburbs: Tiebout, Alonso, and Government Policy**

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Household Location and Schools in Metropolitan Areas with Heterogeneous Suburbs: Tiebout, Alonso, and Government Policy^{*}

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Abstract

An important element in considering school finance policies is that households are not passive but instead respond to policies. Household behavior is especially important in considering how households affect the spatial structure of metropolitan areas where different jurisdictions incorporate bundles of advantages and disadvantages. This paper adds richness to existing urban models by incorporating multiple workplace locations, alternative public services by jurisdiction (school qualities), and voter-determined school expenditure. In our general equilibrium model of residential location and community choice, households base optimizing decisions on commuting costs, school quality, and land rents. The resulting equilibrium has heterogeneous communities in terms of income and tastes for schools. This basic model is used to analyze a series of conventional policy experiments, including school district consolidation and district power utilization. The important conclusion within our range of simulations is that welfare falls for all families with the restrictions on choice that are implied by these approaches.

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1 Introduction

A unique feature of the U.S. education system is the high degree of both funding and control granted to local governments. As a result, school choice is inextricably tied to residential location decisions. This organization has been lauded for its responsiveness to individual demands and for the potential of increased school accountability. On the other hand, it also introduces potential inequities by tying funding decisions to local ability to pay. These conflicting views have made debates about school financing a regular item on both legislative and judicial agendas.

The complexities of analyzing the interaction of location and schooling are well known. When local citizens control taxing and spending decisions and when the quality of schools depends on the peer group, school quality is an endogenous outcome that depends on aggregate individual choices. With local funding of schools through a property tax, housing prices and the tax base also become endogenous and potentially strongly influenced by any governmental policies that affect the funding formula. Finally, residential location, while potentially responsive to school quality, is also strongly influenced by job location and journey to work.

With a few exceptions, analyses of school finance policies have generally ignored one or more of these features of household choices of schools and homes. As a result, analysis of policy alternatives for school finance and operations is likely to be severely distorted.

This paper integrates the essential features of schools and location. In a general equilibrium framework, heterogeneous families (in terms of income and tastes) seek out an optimal residential location and workplace based on commuting costs, wages, and school quality. They also vote on local taxes, yielding variations in school spending that, along with peer influences, produce variations in school quality. The general equilibrium aspects are especially important, because housing prices vary with demand and with governmental policy.

This model is used to analyze the impacts of two alternative school finance policies designed to increase the equity in schooling by reducing the reliance on local property taxes. First, district power equalization – a commonly proposed remedy for unequal tax bases and varying school spend-

ing – is put into the general equilibrium framework where families react to the altered locational advantages. Second, district consolidation and full state funding for school districts are considered from the perspectives of school outcomes and of individual welfare.

We find that the resulting impacts of school policy are very different than previously thought. Relying on our parameterizations (and a variety of sensitivity analyses surrounding these), both of these policies lead to reduced welfare for *all* households regardless of income or taste for schooling. Full state funding does narrow educational disparities by incomes, but it does so at the cost of lowered achievement for all students. Additionally, these policies set in motion a series of adjustments that significantly change housing rents and support for schools.

The next section places our work into the context of existing research. We then provide the theoretical formulation for our basic model with decentralized employment, household demand for schools, and governmental support for education through voter choices of taxes. This model is calibrated for a benchmark case that provides the basis for evaluating the impact of significant changes in the financing of schools. The policy changes, while within the range of observed governmental decisions, are large enough that the general equilibrium nature of the problem cannot be ignored – and indeed is the motivation for this work.

2 Existing Literature

Urban location and local public finance have been built on two artificially separated streams of literature, namely urban residential location models and Tiebout models of community choice. In urban location theory, a household’s residential location is determined by the trade-off between accessibility and space. The pioneer of this approach was Alonso (1964) with his simple but instructive model of the land market, modelling later followed by a great deal of theoretical and empirical work by Muth (1969), Mills (1972), Kain (1975), and others. (See the reviews in Straszheim (1987) and Fujita (1989)). These models are mainly concerned with how to model residential choice where the driving force is workplace accessibility and commuting costs. By ignoring local public goods and services, they do not address many policy questions, but they

do provide baseline predictions about locational choices. The typical model predicts that higher income people live in suburban areas, although Glaeser, Kahn, and Rappaport (2008) suggest that the observed behavior has more stratification than can be supported by the models. Their explanation focuses on public transportation, but they also ignore other public services including schooling, as emphasized here.

In Tiebout models of community choice, on the other hand, households care (only) about local public goods and vote with their feet to shop for the community which best satisfies their preferences. This literature has evolved from the central insight of Tiebout (1956) and builds upon the analytical framework developed in Ellickson (1971). The most influential studies from this approach have been conducted by Epple, Filimon, and Romer (1984, 1993), who have also introduced politics into the model. (See also the additions by Epple and Romano (1998, 2003) and the review by Ross and Yinger (1999)). This literature concludes that households will stratify into communities by their income and tastes and that each community will attract a specific household type. This is an important shortcoming of these models, given empirically that communities tend to be quite heterogeneous in terms of income (see, for example, Pack and Pack 1977, 1978).

Some prior work has addressed the problem of homogeneous communities in Tiebout models. Epple and Platt (1998) introduce households that differ both by income and by tastes and show that there is income heterogeneity within communities because of these preference differences. In their model, they concentrate on residential location decisions where different communities provide differing amounts of local redistribution of income. Their results lead to an interpretation of the resulting communities as a central city (with redistribution) and suburban locations (without redistribution), but location (or accessibility) per se is not important.¹

The urban location model and the Tiebout model have each attempted to abstract from reality in order to concentrate on a specific feature of interest. However, the general conflicts

¹ With a single community characteristic (the amount of local redistribution), the distribution of tastes yields an equilibrium with communities that have a mixture of income, but the same type of individual (denominated by income and taste) will only be found in a single community. As described below, when there are multiple motivations for living in a community, the same type of household can be found in different communities in equilibrium.

with the gross empirical data are severe, suggesting that the models may not provide reliable indications of the comparative statics and of how policy interacts with location. An innovative paper by de Bartolome and Ross (2003) suggests that combining the two modelling perspectives may provide a more realistic portrait of urban location. Their monocentric city model that includes fiscal motivation of jurisdictions and majority voting can produce income mixing in a central city and suburban ring, although households all consume a common amount of land – something in conflict with standard urban location models. Nechyba (2000, 2003), taking a different route, develops a general equilibrium model that is calibrated on pre-existing heterogeneity of income and housing. A review of alternative modelling approaches is provided by Epple and Nechyba (2004) and Nechyba (2006).²

This paper builds on these various strands. The objective is construction of a model with sufficient richness to capture the basic reality of urban spatial structure and the key elements of governmental policy interventions.³ We build on our prior work that developed a general equilibrium model of household location and school demand in a model with centralized employment that has two competing school districts (Hanushek and Yilmaz 2007).⁴ That work suggests that the interaction of accessibility and local public goods is indeed very important for understanding the spatial structure of residential locations.⁵ But it falls short in two fundamental ways. First, it fails

²In a recent paper, Epple, Gordon, and Sieg (2010) take a different approach to merging location models and Tiebout models. They consider equilibrium of households and housing suppliers in a metropolitan area and develop a set of sufficient conditions that justify estimation of multi-community equilibrium models while ignoring intra-community variation in amenities. Empirically they view each parcel as having a single amenity value that combines travel times to the center of the city (Pittsburgh) and specific high school attendance zone.

³This analysis concentrates on the long run equilibrium for the residential location of households. As such it ignores any of the short run dynamics or of the interactions with the macroeconomy; cf Leung (2004).

⁴One innovation in that work that is important to understand the interplay of accessibility and the demand for public goods is to move away from the models with the traditional circular central city and donut shaped suburb. Hanushek and Yilmaz (2007) consider a metropolitan area divided into two equal halves, say by a river down the middle. All employment, however, remains in the center of the metropolitan area. The more common depiction of a monocentric city has a circular central city surrounded by a donut shaped suburban ring; see de Bartolome and Ross (2003) or Cassidy, Epple, and Romer (1989). In this traditional formulation, it is not possible to observe locations with the same accessibility but different school qualities. Note that our cities have some similar structure in that there is "ring-separation" of different household types within each jurisdiction.

⁵A key element of the equilibrium is the heterogeneity of both income and tastes in both communities. Necessarily, with four types of individuals, there must be mixing. But this solution shows that neither income groups nor tastes groups form homogeneous communities. Moreover, if we make income and tastes perfectly correlated (as is implicit in most theoretical investigation of locational choice), we still find heterogeneity of income across communities (not shown). The key is that the varied components of the decision making lead to trade-offs in terms of accessibility and

to characterize the continuum of school quality that is observed within virtually all metropolitan areas in the United States. Second, it provides an inadequate basis for policy considerations – specifically school finance policy here – where the key element is the heterogeneity of communities and the behavioral responses of households to governmental changes in the spatial attractiveness of individual jurisdictions.

Here we move to consideration of multiple workplace centers with three separate school districts. Among other things, this exercise incorporates a fundamental empirical feature of today’s urban landscape, i.e., the importance of suburbs and of their heterogeneity. In 1940, 32.8 percent of the U.S. population lived in central cities of metropolitan areas, with less than half that many (15.8 percent) in suburban areas; by 2000, 30.3 percent lived in central cities while fully half of the U.S. population was found in suburbs (Hobbs and Stoops 2002). Moreover, looking at a constant set of metropolitan areas, Kim (2007) notes that central cities began declining absolutely in population after 1950 as population density gradients flattened significantly. These changes in the spatial structure of metropolitan areas paralleled the decentralization of business and industry. While centralized employment and undifferentiated suburban locations are convenient and tractable from a modelling perspective, they lack realism in describing current metropolitan areas.

Decentralized employment models, which move away from imposing radial symmetry of locations, produce income and taste mixing within communities and clearly show the development of competing (and heterogeneous) suburban jurisdictions. The fundamental model provides a benchmark for subsequent policy experiments. Of particular importance here is the reaction of households to common changes in the financing rules for schools. Households have alternative margins of adjustment to changed incentives, and the resulting equilibria are significantly different than predicted by traditional models.

Introducing the complexity of multiple employment centers and heterogeneous households comes at a cost. First, analytical solutions of the models are impossible, and we must turn to calibrated school quality. If we compare households of a given skill type and taste for education, we see that some are willing to accept lower quality schools in order to gain better access to work and vice versa.

general equilibrium solutions. Second, given this analytical approach, we must rely upon both specific assumptions about key elements such as the utility functions of households and specific parameterizations of utilities, transportation costs, and employment surfaces. As a result, while we can readily illustrate the qualitative importance of considering a richer formulation of household behavior, uncertainty remains about how to generalize the results to any real policy deliberation. Generalizing the policy conclusions will require an expanded set of underlying formulations and parameter specifications.

3 Formal Model of Household Decisionmaking

The outline of our structure is easy to describe. Urban location and public good preferences are integrated with a calibrated general equilibrium model of community choice. Households that differ in both income and tastes commute to their workplaces, facing both pecuniary and time costs in commuting. There are decentralized employment centers (a central city and two subcenters) and three local school districts. Education, the sole public good, is provided through local schools and is financed through property taxes determined by majority voting. The production function for education models quality as a combination of peer group effects and spending. Both location and school quality affect housing prices, and thus both influence taxes and mobility. Households maximize utility through choice of workplace, residential location, and housing and in equilibrium have no incentive to move to an alternate jurisdiction.

The primary innovation in our analysis is the introduction of more realistic decision problems for households that in turn provides a basis for analyzing how exogenous policy changes affecting the attractiveness of different locations impacts overall outcomes. Households choose residential and job locations and housing quality based upon price, accessibility, and school quality attributes. Specifically, individuals differ in both tastes for schooling and in income and are motivated in their locational decisions by wages available at different employment centers, land prices, commuting costs, and the tax price-quality bundle of schools. They also vote on school funding (always with the option of voting with their feet if the collective decision is not satisfactory).

Introducing these multiple decision margins permits calibrating the model to urban structures closer to what we observe – multiple competing jurisdictions within a metropolitan area where communities differ in housing prices, school quality, and taxes and yet the populations of each are heterogeneous in terms of income and underlying tastes. Having such a benchmark is particularly important for policy simulations that mirror a variety of place-based proposals such as changing the financing of schools.

3.1 Location with Decentralized Employment

The starting point for our modelling is the prior empirical observation that some 60 percent of population of metropolitan areas is found in multiple suburban jurisdictions, which in turn reflects the movement of jobs to more decentralized urban locations.⁶ While it is possible to illustrate the trade-offs of accessibility and local amenities (here schools) in a model with centralized employment (Hanushek and Yilmaz 2007), the limited variation between jurisdictions constrains the range of policy interventions that can be realistically considered. We embed our analysis of household locational decisions within a model of a metropolitan area with three employment centers and three separate local school districts. This expansion from the more common central-city/suburban split provides a richness of alternatives while still being analytically tractable.

We begin with a flat featureless plane that has exogenously determined jurisdictions. As shown in Map 1, the area has two suburban workplaces, namely the West Suburban Center (w) and the East Suburban Center (e), as well as a Central City (cc). This jurisdictional structure permits disentangling accessibility and school quality. This city is a stylized representation of the many cities on water boundaries such as Chicago, Cleveland, or New Orleans.⁷

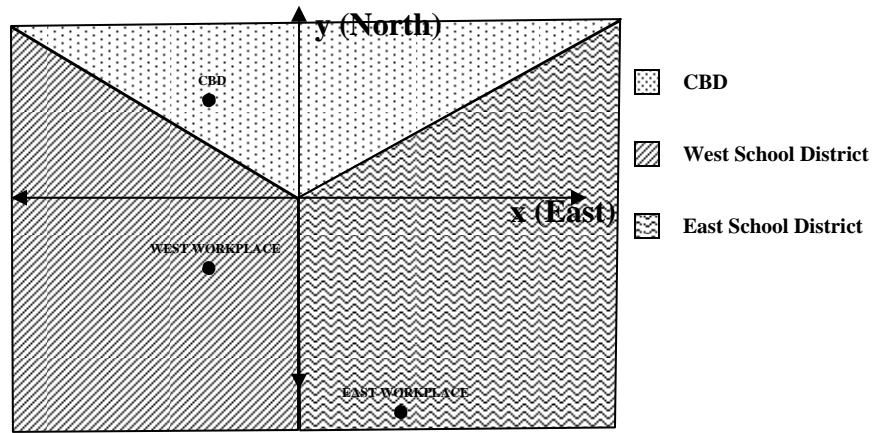
Firms are located at points, take up no space, and have no taxable property.⁸ Each jurisdiction also contains a school district (named after its employment center).

⁶See, for example, the discussion of patterns of American cities in Glaeser and Kahn (2001, 2004). The incorporation of decentralized employment into urban modelling is explored in depth in White (1976, 1999).

⁷As Rose(1989) points out, half of the 40 most populous metropolitan areas were bound by the Pacific Ocean, Atlantic Ocean, the Gulf of Mexico, or the Great Lakes.

⁸Note that ignoring the impact of commercial and industrial property on the tax base ignores an important feature of local finance (see Ladd (1975), Fischel (2006)), but it would not change the qualitative nature of our results.

Map 1: City Map



— Map 1 about here —

An alternative might be a generalization of a monocentric city with a circular central city surrounded by a donut shaped suburban rings. With this structure, the radial symmetry permits straightforward analytical solutions of location where it is necessary only to trace locational choices along any ray from the employment center. In its simplest form, it has also motivated a large number of empirical analyses of urban form that are based on estimating household density functions and price gradients emanating from the center (see, for example, Mills(1972), Rose(1989), and Kim(2007)).

For our purposes, however, this circular structure is problematic. For analytical purposes, all locations close to the employment center are served by a common school district, making it difficult to see the separate influences of location and school quality. Additionally, there are empirical reasons to consider alternative depictions. The circular city is more of an analytical convenience than a realistic portrayal of American cities. The variety of cities that result from natural boundaries such as lakes, rivers, and mountains or from historical development patterns makes the stylized "von Thunen pattern" more a simplifying device than an accurate generalization of city structures (see Rose 1989). Moreover, while it is possible to correct estimation of density gradients for missing quadrants (see Mills(1972) or Rose(1989)), the simple depiction fails in a significant number of metropolitan areas.⁹

Labor market. We concentrate on residential and schooling choices and take wages as exogenous. At each workplace (l), there are both high wage jobs (paid to skilled workers, s) and low wage jobs (paid to unskilled workers, u) such that ($w_s^l > w_u^l$). (All skilled workers are perfectly substitutable, as are all low wage workers). Wages of both skilled and unskilled workers vary across suburban centers $sub \in \{w, e\}$ depending on their locations relative to the Central City. Suburban wages are less than their counterparts in the CC (i.e., $w_s^{cc} > w_s^{sub}$ and $w_u^{cc} > w_u^{sub}$), because of the

⁹Kim(2007) describes a number of situations where the standard depiction does not work including, importantly, the significant numbers of U.S. metropolitan areas with multiple central cities or other anomalies. Bertaud and Malpezzi (2003) also find a number of international cities are inaccurately described by smooth density gradients.

wage gradient induced by the lesser need to commute and the larger and cheaper houses around these places.¹⁰

Households and preferences. One member of each household works and makes all the economic decisions in the house. Each household has one pupil attending school, although the crucial element is that all children in a household attend the same school district. Households place different values on the quality of education a jurisdiction provides. Some value education more (high valuation types, H), some less (low valuation types, L). Therefore, we have four different types of households in the city $i \in \{SL, SH, UL, UH\}$. The different valuations could reflect differences in inherent tastes or could be induced by underlying differences in family size. A low valuation household may, for example, be a household with no children at home, but it nevertheless recognizes that school quality is still relevant because it will be capitalized into rents and housing values. The metropolitan area is closed in the sense that there is a set population of each of four types of households. Since we have four types of households, three workplaces and three school districts, there are $4 \times 3 \times 3 = 36$ different household-by-residence-by-workplace outcomes, although some types may be absent in equilibrium.

The preferences for a type i household is represented by a Cobb-Douglas utility function given by $U(\alpha_i, \eta_i; q, s, z, t) = q_j^{\alpha_i} s^{\eta_i} z^\gamma t^\delta$, where $\alpha_i + \eta_i + \gamma + \delta = 1$, q_j is the quality of education in community $j \in \{cc, w, e\}$, $s > 0$ is the lot size, $z > 0$ is the numeraire composite commodity, and $t \in [0, 24]$ is leisure. $\alpha_i \in \{\alpha_H, \alpha_L\}$ is the taste parameter for education and, $\eta_i \in \{\eta_H, \eta_L\}$ is the taste parameter for lot size. The normalization of the parameters to sum to one is done to differentiate high valuation and low valuation households with the same income. To see this, consider two households, one low and one high valuation, with the same income. In this formulation, a high valuation type enjoys education (lot size) more (less) than a low valuation type (i.e. $\alpha_H > \alpha_L$ and $\eta_H < \eta_L$), and as will be shown later, a high valuation type prefers a higher

¹⁰In locational models, the wage gradient is frequently derived after the residential price gradient is found. A complete model would, however, simultaneously solve both the household and employer locational problem along with rents and wages – a task beyond our capacity. In our model with exogenous wages and employment, the rent gradient adjusts to wages, because wages will enter into the value of accessibility.

property tax rate (i.e., more school expenditure) than a low valuation type.

Accessibility and Budget Constraint. Consider a type i household who is trying to decide where to work and live. The area has a dense radial transportation system from each workplace, and the worker of every household commutes daily from his residence to his workplace. But a household could have a residence differing from his/her workplace. For instance, a household commuting to his/her workplace in the cc , could reside in West School District due to a better education, tax advantages, or less commuting distance. There is both a time and Euclidean distance component to the commuting cost. Formally, commuting requires $a/2$ dollars per mile and $b/2$ hours per mile. The time endowment for the household is 24 hours. The budget constraint of the household with a workplace l and school district j , at a location (x,y) on the xy plane (i.e. on the map) is given by

$$z_{ijl}(x, y) + (1 + \tau_j)R(x, y)s_{ijl}(x, y) + w_i^l t_{ijl}(x, y) = Y_i^l(r) = 24w_i^l - (a + bw_i^l)r \quad (1)$$

where r is the distance¹¹ to workplace l , τ_j is the property tax rate, $R(x, y)$ is the equilibrium rent per unit of land at the coordinate (x,y) on the map, that is paid to a landlord for his land in community j . Notice that this formulation suggests that households sell all available time to employers and buy back some leisure at the prevailing market wage rate. The household's problem could be summarized by picking up a workplace, a school district and residential location (i.e. $(x, y) \in R^2$), a lot size, the amount of leisure, and the composite commodity simultaneously at given prices.

Land Market. Following Alonso (1964), we assume a competitive land market in which households bid for land and absentee land owners offer the land to the highest bidder. For any given location, nondevelopment is an option and the land is left for agricultural use if the households cannot outbid the agricultural use, which has a fixed bid of r_a .

In equilibrium, the advantages and disadvantages at different workplaces and school districts/locations are capitalized into prices. Identical households obtain the same utility level. The bid-rent function captures this feature and allows us to calculate rents at different locations/school

¹¹ $r = \sqrt{(x - x_l)^2 + (y - y_l)^2}$, where (x_l, y_l) is the location of the workplace l on the map. Note that the set of locations with the same distance to workplace l is a circle.

districts. In a standard way, we can define the bid-rent function of the household, which shows the household's willingness to pay for a residence at a location holding utility at a fixed level as:¹²

$$\Psi(l, j, r, u_i, q_j, \tau_j) = \max_{s, z, t} \left\{ \frac{Y_i^l(r) - z - w_i^l t}{(1 + \tau_j)s} \mid U(l, j, r, \alpha_i, \eta_i; q, s, z, t) = u_i \right\} \quad (2)$$

$$= \frac{k_i^{1/\eta_i}}{(1 + \tau_j)(w_i^l)^{\delta/\eta_i}} q_j^{\alpha_i/\eta_i} Y_i^l(r)^{\frac{\eta_i + \gamma + \delta}{\eta_i}} u_i^{-1/\eta_i} \quad (3)$$

First, consider households with, say, workplace l and school district j. The relative steepness of bid-rent functions for different types by distance determines the spatial ordering of household locations (rings) around the employment center. Our model relies on the single crossing of bid-rent functions that needs to be confirmed.¹³ We find that poor (high valuation) households have a steeper bid-rent curve than rich (low valuation) households, and the spatial ordering of households turns out to be Unskilled High, Unskilled Low, Skilled High and Skilled Low, as we move away from the employment center. See the appendix for a detailed discussion. If we compare households with a workplace l in each school district, we expect to see the bid-rent levels and the ring sizes are altered due to quality of education and tax package differences as well as taste for education differences, but the ordering of household types remains the same. The implication is that households with workplace l occupy rings around the employment center, but those rings have a different radius as we move to another school district. See the picture at the end of appendix. In school district j, we have three different ring sets originating from three employment centers. Therefore, at any given location in school district j, the landlord receives $4 \times 3 + 1 = 13$ implicit offers corresponding to the four different types of households with three possible different workplaces, and agricultural use. The highest bid gets the land. As we move away from a center, bid-rent curves go down. Since the

¹²In the explicit formulation, $k_i = \frac{\eta_i^{\eta_i} \gamma^\gamma \delta^\delta}{(\eta_i + \gamma + \delta)(\eta_i + \gamma + \delta)}$.

¹³Numerically, we show that for any pair of bid-rent functions originating from workplace l, one of them is always steeper (larger slope in absolute value) than the other at any intersection point. Since bid-rent functions are continuous, it must be the case that they intersect once, and one of them is steeper than the other one. It also implies households are segregated by distance, and identical household types locate in rings around the employment center. See Fujita (1999). Moreover, at the intersection point, the relative slopes are independent of quality of education/tax packages. i.e. the spatial order is valid in any school district.

highest bid gets the land, a location closer to an employment center is more likely to be occupied by a households with a job in that employment center.

We can identify the equilibrium location (both residential and workplace) of households and equilibrium market rents in our closed city, once we know the households' utilities. Market rent, $R(x, y)$ is the upper envelope of the equilibrium bid-rent curves $\Psi_i(j, l, r, u_i^*, \cdot)$ for all household types $i \in \{SL, SH, UL, UH\}$, all workplaces $j \in \{cc, w, e\}$, and the agricultural rent line. Needless to say, in equilibrium, if type i households are present in more than one jurisdiction, they should get the same utility wherever they are so that nobody has an incentive to switch workplace, school district/location, or consumption pattern

Taxes and schools. Our real interest is to examine the interaction of school quality and location. For most interesting analysis, we must turn to a full general equilibrium model.

From a household's point of view, each jurisdiction is characterized by the quality of education and property tax rate pair (q_j, τ_j) it provides. Education in community $j \in \{cc, w, e\}$ is financed through property taxes on residential land. Each jurisdiction's local government spends all tax revenue on education. Then, the government budget constraint in school district j is

$$E_j = \tau_j \bar{R}_j = \tau_j \frac{\int_{(x,y) \in j} \int_{R(x,y) > r_a} R(x,y) dx dy}{N_j} \quad (4)$$

where N_j is the population, E_j is the expenditure per pupil, and \bar{R}_j is the tax base per pupil in school district j . Note that the agricultural land does not pay any property taxes (i.e. integration over $R(x, y) > r_a$).¹⁴

Characterizing the relationship between the quality of education and the expenditure on schools has proven difficult (Hanushek 2003). Here, we emphasize the interaction of peers and spending in determining quality. Specifically, we characterize quality as being determined by:

$$q_j = \phi_j(N_L^j, N_H^j) E_j \quad (5)$$

where N_L^j and N_H^j are the number of low educational valuation and high educational

¹⁴When the land is left for agriculture in equilibrium, $R(x, y) = r_a$.

valuation households in school district j , respectively.¹⁵ The peer group effect function, $\phi_j(\cdot)$, which has a natural interpretation of determining the efficiency of spending, is given by

$$\phi_j(N_L^j, N_H^j) = c_1 + c_2 \exp(-c_3 \frac{N_L^j}{N_H^j}) \quad (6)$$

where c_1 , c_2 , and $c_3 > 0$ are constants. Notice that ϕ_j , the efficiency of schools in jurisdiction j , is increasing in high valuation households and decreasing in low valuation households. The value of peer group effect is between c_1 and $c_1 + c_2$. Two arguments can be made to justify this kind of peer group effect. The first argument is based on the classical peer-group effect: the more my neighbor knows, the more I can learn from him. The second argument is that high valuation households are more involved in how schools operate such as taking a part in the schooling process as board members or simply continuously watching over school decisions. This involvement is presumed to lead to a more efficient use of resources.

The property taxes are determined by majority voting in each school district. Following Epple et al. (1983, 1984, 1993), we assume that voters are myopic in the sense that they do not consider that their decision about (q_j, τ_j) will influence land prices, populations, efficiency of the schooling system, etc.¹⁶ Their vote will reflect their tax preferences (τ_j) that come from maximizing indirect utility as in:

$$\max_{\tau_j} V(\cdot) = \frac{k_i}{R(x, y)^{\eta_i} (1 + \tau_j)^{\eta_i} (w_i^l)^{\delta}} q_j^{\alpha_i} Y_i^l(r)^{\eta_i + \gamma + \delta} \quad \text{subject to} \quad q_j = \phi_j(\cdot) E_j \quad (7)$$

$$E_j = \tau_j \bar{R}_j$$

Solving this problem yields the preferred tax rate for type i household with a workplace l and residence at (x, y) , $\tilde{\tau}_i = \frac{\alpha_i}{\eta_i - \alpha_i}$.¹⁷ The preferred tax rate is a direct function of the household's valuation of the importance of schooling. Since there are only two valuation types for households, there are two possible preferred tax rates in the economy, and high valuation types have a higher

¹⁵This structure would produce qualitatively similar results if the peer effects depended on the relative concentrations of the different income groups.

¹⁶For a model of voters with perceptions of capitalization and capital gains, see Yinger (1982, 1985).

¹⁷It is implicitly assumed $\eta_i > \alpha_i$. This assumption also guarantees the single peaked preference requirement for the existence of majority voting equilibrium.

preferred tax rate ($\tilde{\tau}_{SH} > \tilde{\tau}_{SL}$ and $\tilde{\tau}_{UH} > \tilde{\tau}_{UL}$). Also, the more they value housing and spend on it (higher η), the lower the property tax rate they prefer.

Timing of Decisions. The timing of events would be as follows: At the beginning of each period, households make workplace and school district/location decisions with the expectation that the last period's education and property tax packages would prevail in the current period. Once they move in, they are stuck. They vote for the property tax rate in their school district of residence. The public good and tax rate package might be different from what they expected, but they have chosen the community for that period. At the beginning of the next period, they update their expectations and events start over again. In analyzing this model, we impose a requirement that all local governments budget are balanced. Equilibrium occurs when, regardless of their workplace or residential location or school district, households of the same type attain the same utility level regardless of residence (i.e. a type i household gets u_i^* everywhere).

Definition. *An equilibrium is a set of utility levels, quality of education and property tax rate for each school district, the spatial distribution of households over workplaces and school districts such that:*

- *Given prices, households pick a workplace, a school district, a location, lot size, leisure, and composite commodity to maximize their utility.*
- *Different household types in three employment centers bid for each location. The land at a location is developed for the highest bidder if the highest bid exceeds the fixed non-urban purpose bid, agricultural rent. Otherwise, it is not developed.*
- *The city is a polycentric city, and jobs are offered by firms located at the CC or two sub-centers. Wages in the workplaces are exogenously determined. The city has a dense radial transportation system around workplaces. Households commute to workplaces. Commuting has both pecuniary and time costs.*
- *We have identical treatment of identical households. Regardless of their residential location,*

Parameter	Value	Parameter	Value
α_H	0.019	η_H	0.048
α_L	0.016	η_L	0.051
γ	0.187	δ	0.747
a	\$1.10	b	0.1 hrs
w_s^{cc}	\$18.30	w_u^{cc}	\$10.70
w_s^w	\$17.60	w_u^w	\$10.30
w_s^e	\$17.90	w_u^e	\$10.50

Table 1: Calibration Parameters

workplace or school district, households of the same type attain the same utility level.

- *The metropolitan area is a closed city and contains three school districts, each of which operates its own schools. Moreover, land is owned by absentee landlords.*
- *The local public good, education, is produced through a production function defined by peer characteristics and school spending, where spending is financed through local property taxes on residential land as determined by majority voting in each school district.*
- *Labor and land markets clear.*
- *The local government budget balances in school districts.*

3.1.1 Calibration

The central parameter values for the functional forms used in the model follow those displayed in Table 1.

These parameters relate the equilibrium choices of households to relevant aggregate statistics. Recall that the household spends $\frac{\eta_H}{\eta_H + \gamma + \delta}$, $\frac{\gamma}{\eta_H + \gamma + \delta}$, and $\frac{\delta}{\eta_H + \gamma + \delta}$ percent of his net income $Y(r)$ on land, the composite commodity, and leisure, respectively.¹⁸ U.S. average weekly hours of persons working full time are about 40 hours¹⁹, and the average annual earnings (for individuals 18-years old or more) is \$22,154 for high school educated workers and \$38,112 for college graduate workers in 1997. These figures suggest the hourly wages in the CC for unskilled and skilled workers

¹⁸For the calculations we begin with the skilled, high valuation type (SH).

¹⁹The statistical facts, unless otherwise indicated, come from the Statistical Abstract of the United States, 1998.

should be calibrated as $w_u \approx \$10.70/\text{hour}$ and $w_s \approx \$18.30/\text{hour}$, respectively. The wages at the suburban center are lower so that a higher fraction of jobs are located at the CC (46.7%).²⁰ The share of leisure (nonwork time) in the household's budget is $\frac{\delta}{\eta_H + \gamma + \delta} = 1 - \frac{40w_s}{24 \times 7 \times w_s} \approx 0.762$. The data on average annual expenditures of some selected MSAs suggest that a household spends about 20 percent of his income on shelter. Therefore, we set the budget share of composite commodity and land as $\frac{\gamma}{\eta_H + \gamma + \delta} = (1 - 0.762) \times 0.8 \approx 0.1904$ and $\frac{\eta_H}{\eta_H + \gamma + \delta} = (1 - 0.762) \times 0.2 \approx 0.0476$, respectively. Recall that the preferred tax rate for a type i household is given by $\tilde{\tau}_i = \frac{\alpha_i}{\eta_i - \alpha_i}$ and we had two possible preferred tax rates, one for high valuation and another for low valuation type households. The one for high (low) valuation type is set to be about 1.7 percent (1.1 percent), out of the value of a house, which is the present value of rents generated by the house annually. These relationships provide sufficient information by which to calibrate $\alpha_H, \alpha_L, \eta_H, \eta_L, \gamma, \delta$.

Since the most common practise of commuting in the U.S. is by car, pecuniary commuting cost per round trip mile is based on the cost of owning and operating an automobile. In 1997, pecuniary cost per mile was 53.08 cents, suggesting a pecuniary commuting cost of $a = \$1.1$ per round trip mile. Assuming the commuting speed is 20 miles per hour within the city, the time cost of commuting per round trip mile is set to be $b = 0.1$ hours per mile.

The population of the city is set to be 3,000,000 households, which implies approximately a population density of 5,185 households per square mile.²¹ Approximately, 40 percent of the total population is assumed to be skilled worker households. Moreover, 30 percent of skilled households are assumed to be low valuation types. For the unskilled households, 70 percent are assumed to be low valuation types. The agricultural rent bid r_a is set to be \$8,897 per acre per year.²²

The metropolitan area is large, and we have a inelastic supply of residential property in

²⁰Inlandfeldt (1992) reports that wages decline by approximately one percent per mile of distance from the CBD. Here we consider distance from the central city employment center.

²¹The median population per square mile of cities with 200,000 or more population was 3,546 in 1992. Source: County and City Data Book, 1994.

²²Parameters of the education production function are set to be $c_1 = 5.819$, $c_2 = 3.975$, $c_3 = 0.461$, so that (q_j, τ_j) preferences of households in different jurisdictions are consistent with (q_j, τ_j) pairs that induce the underlying population distribution. With these parameter values for peers, a school district of all residents with low (high) valuation has a productivity of 5.819 (9.794), a 68 percent difference in productivity for the higher peer group.

Variable	CC	West	East
<i>Quality of Education</i>	40.1	59.9	29.9
<i>Tax rate</i>	1.3%	1.66%	1.3%
<i>Expenditure per pupil per year</i>	\$1,968	\$2,273	\$1,876
<i>Efficiency</i>	7.43	9.63	5.82
<i>Average monthly gross rent per acre</i>	\$3,815	\$4,054	\$3,399

Table 2: The characteristics of communities in equilibrium.

any direction in the metropolitan area. For calibration, we divide the metropolitan area into fine grids (i.e. fine two dimensional squares) and find the residential location pattern over those grids. Usually, in a monocentric city set up, polar coordinates are preferred because of radial symmetry. Due to the interaction of concentric rings originating from three employment centers, the symmetry is broken, and we use a cartesian coordinate system. The cost is the evaluation of double integrals, as opposed to one dimensional integrals in polar coordinates. In a rich set up with decentralized workplaces, it is not possible to talk about the uniqueness of equilibrium analytically. We rely on computational methods to find all equilibria.²³

3.1.2 Basic Results with Decentralized Employment

The simulation results for the benchmark model are given in Table 2, Table 3, and Figures 2 and 3. In the base model, the West School District is the best in terms of the education it provides, while the East is the worst. The West School District attracts mostly high valuation type households. These high valuation households put more pressure on the schools and make schools more productive and efficient. The East School District is the opposite, attracting households with a low valuation of education and resulting in low quality schools. The majority voting outcome of tax rates in West (East) School district are the preferred tax rates of high (low) valuation type households. To be precise, tax rates for the West and both the East and CC School Districts are 1.66 percent and 1.3 percent, respectively. Average rents attain their peak value in the West School District and their lowest value in the East.

²³See the appendix for computational details. With benchmark parameters, we identified two more equilibria. We report the one with the highest welfare due to space considerations. The discussions and findings for the benchmark model remain valid for the other equilibria as well.

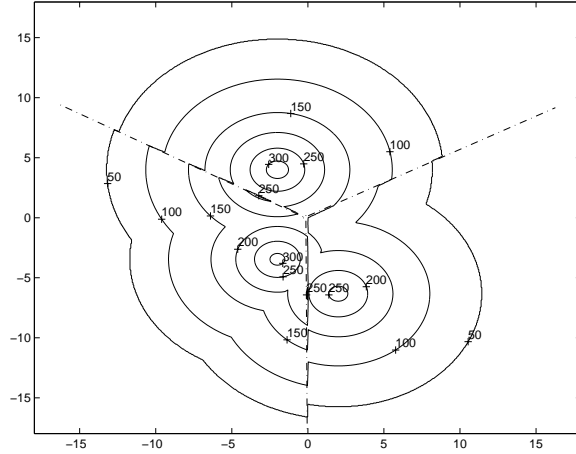


Figure 2: ISO-RENT CURVES: *Iso-rent curves have been shown. Notice the rings around the CC extending to West and East School Districts. These are in-commuters. Also see the jump in rents around the school district borders where in-commuters live. This is due to efficiency/quality of education difference between districts. The rent is the gross rent (i.e. $(1 + \tau_j)R(x, y)$) per acre per day.*

The finding seems to be counterintuitive. After all, the east suburban center offers higher wages than west suburban center, and higher wages are expected to be capitalized into housing prices. However, the quality of education is also capitalized into housing prices. As a matter of fact, what we see is the better education in the west school district being directly capitalized into housing prices. Moreover, the West School District with the highest property taxes and land prices spends more on education than the other communities in the metropolitan area.

Figure 2 shows iso-rent curves for the metropolitan area. As sites get close to employment centers, we observe a monotonic increase in rents with three local maxima around workplaces. More importantly, we see the capitalization of higher quality of education and higher wages. The rents are higher in the West School District with the best education, compared to the East School District with the worst education. The West School District also provides a much better education than the CC but their rents are almost the same, reflecting the higher wages and greater employment accessibility in the central city.

The iso-rent rings around the CC center extend to both the West and East School districts, reflecting the presence of in-commuters. The rings around CC center in the west school district

Type/Workplace	Residence			
	CC	West	East	All
<i>Skilled Low:CC</i>	3.6		1.2	4.8
<i>Skilled High:CC</i>	10.4	6.3	0.3	17
<i>Unskilled Low:CC</i>	19.6		0.5	20.1
<i>Unskilled High:CC</i>	1.5	3.2		4.7
<i>Skilled Low:West</i>				
<i>Skilled High:West</i>		6.3		6.3
<i>Unskilled Low:West</i>		2.6	1.0	3.6
<i>Unskilled High:West</i>		11.1		11.1
<i>Skilled Low:East</i>			7.3	7.3
<i>Skilled High:East</i>		4.4	0.2	4.6
<i>Unskilled Low:East</i>		0.6	17.8	18.4
<i>Unskilled High:East</i>		2.1		2.1
	35.1	36.6	28.3	100

Table 3: Equilibrium percentage distribution of households across communities.

are different from those in the CC school district. Although households in both rings have got a job at the CC center, we see the capitalization of better education and tax package in the west school district. For in-commuters, the rents at locations the same distance to the CC employment are highest in the West School District and lowest in the East. There are big jumps in rents as we cross into west school district from the CC school district. We also see the rings around the East employment center extending to the West School District, showing the presence of households with a job at the East employment center residing in the West School District, again to enjoy a better education.

In equilibrium, all poor households with their steeper bid-rent curves locate closer to their workplaces than rich households with flatter bid-rent curves; within each skill group, high valuation households locate closer to the workplace than low valuation households. As a result, households of each type form a concentric ring, or zone, around the workplaces, and zones for all household types are ranked by the distance from the workplaces in the order of steepness of their bid rent functions. No agricultural land should be left inside the urban fringe. At the locations in between workplaces, we see the interference of concentric rings around workplaces.

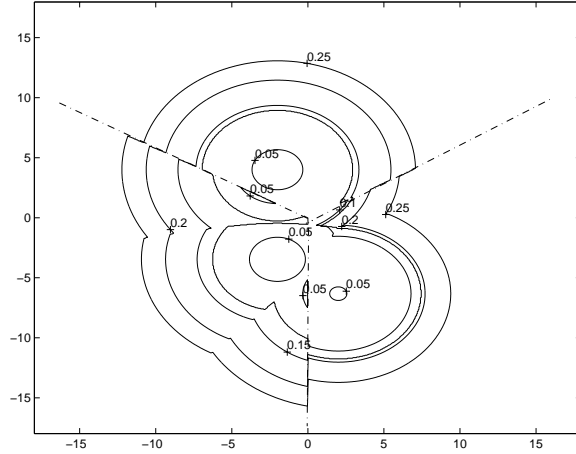


Figure 3: ISO-LOT SIZE CURVES: *Iso-lot size curves have been shown. The pattern is quite similar to that of iso-rent curves.*

Although the central city offers 46.6 percent of all jobs available, a much smaller fraction of households, 35.1 percent, resides in the CC in equilibrium (Table 3). The east suburban center (32.4%) offers more jobs than west suburban center (20.7%). The west suburban center can be thought of as a bedroom community with employment concentrated in services for the population. Besides, high valuation households disproportionately reside in the West School district that provides the best education while commuting to work in the CC ($6.3\% + 3.2\% = 9.5\%$). Also, there are some households (2 percent) with a job in the CC and residence at the East School district. We do not see any out-commuters (i.e. residents of the CC getting to and from their workplace at any suburban employment center). Also, note the presence of some households with a workplace at East (West) and residence at West (East), either to commute less or to provide a better education to their children.

Iso-lot size curves are drawn in Figure 3. The pattern is quite similar to iso-rent curves, and similar arguments can be made. The lot sizes increase monotonically with distance from employment centers and have local troughs at workplaces. Consistent with empirical evidence in the U.S., the rich reside in bigger houses away from their workplaces. Once again, we clearly see two effects: Holding distance to workplaces constant, houses in the West School District are smaller than houses in the East and CC School Districts. This is due to higher rents resulting

from the capitalization of better education. Also, observe the rings around the CC extending to the West and East School Districts. In the west, households with a job in the CC accept having a smaller house and/or commute more to provide their children with a better education. Residential densities follow a pattern analogous to rents.

Finally, the benchmark results vividly demonstrate the utility of moving to the multi-district model with decentralized employment. The two suburban areas not only serve very heterogeneous populations but also have very different patterns of school quality and taxes. In fact, the East district provides the worst schools in the area, but this is compensated for by having the lowest rents.

4 Alternative School Finance Policies

Since the late 1960s and the California court case of *Serrano v. Priest*, courts and legislatures have been concerned with potential inequities in the provision of schooling arising from the differential ability of some districts to raise funds for schools. The focus has been the use of local property taxes to fund schools. The central argument is that differences in the tax base between wealthy and poor districts result in "discrimination on the basis of the wealth of ones' neighborhoods," because wealthy districts could more easily raise funds for schools (Coons, Clune, and Sugarman (1970)). Any such funding discrepancies would then lead to poorer schools and lower educational outcomes for poor children. The history of school finance discussions throughout this period involves the interaction of courts and legislatures to move the funding of schools away from local property taxes to some alternative revenue plan (see Murray, Evans, and Schwab (1999), Fischel (2006), and Hanushek and Lindseth (2009)).

The school finance debate is motivated by student achievement but most attention in both the courts and in legislatures has quickly moved from achievement to policies focusing on revenues and expenditures for local districts. The policy discussion is often framed by an implicit assumption that income, property tax base, and school outcomes are very highly correlated across

school districts such that poor people – who buy less housing – necessarily live in jurisdictions with a lower property tax base and end up with poorer schools.²⁴ What is generally missing is an understanding of the equilibrium nature of residential decisions and the resulting patterns of school outcomes – just the focus of the analysis developed here. Similarly, it is virtually never discussed in the policy debates that, if the method of funding the schools is altered, the choices of households will also shift, leading to a different pattern of locational and distribution of schooling outcomes.²⁵

The prior models frame the issues well. First, we see that incomes and school outcomes are not perfectly related because households, even with the same income and tastes, are balancing different outcomes that are modelled simply here by valuing both school outcomes and access to employment. Thus, some households who highly value schools will optimally live in districts with low quality schools. Others who place a low value on schools are quite happy to live in a district that has low tax rates and low value of housing, allowing them to consume better bundles of access and housing. Second, houses of the same size and employment access command different rents when school quality is capitalized into the value.²⁶ As a result, the property tax base is endogenous, depending on the equilibrium choices of households. In particular, some households willingly pay more for any specific quality of housing than they would need to in order to buy better schools.

In each of these cases, not only the interpretation of equity but also the implications for educational outcomes and individual well-being become more interesting and more realistic. Additionally, because of the reaction of households to changed opportunities, the results of alterations in funding of schools are not easily predicted from partial equilibrium analyses that assume no adjustments by households. Simply put, policies that explicitly favor districts with concentrations of poor people may or may not lead to better outcomes for poor families.

This section analyzes a series of alternative school finance policies, representing variants of

²⁴Another element that leads to differences in property tax bases but that does not reflect individual incomes is the presence of commercial and industrial property, which can be taxed to pay for local schools (see Ladd 1975). Of course, the way that these interact with households and schools depends upon the equilibrium choices of households (Fischel 2006).

²⁵See Hoxby (2001), Fischel (2006).

²⁶The capitalization of school quality and other locational amenities is well-documented in Oates (1969), Black (1999), Weimer and Wolkoff (2001), and Gibbons and Machin (2008).

policies that have been discussed or implemented in the recent period of school funding. We start with district power equalization, an alternative funding plan that compensates districts that have a low tax base. Subsequently, we consider full state funding, which in our simplified metropolitan area is fiscally equivalent to district consolidation.

4.1 District Power Equalization

When households in different school districts can choose the amount spent on schools, they tend to sort into communities with concentrations of households having similar demands – just as suggested by Tiebout (1956) – and spending will vary across districts. In the models here, these decisions are amplified by peer influences on the efficiency of school spending and thus on school quality. An alternative approach to purely local property taxes is district power equalization.²⁷ A portion of the funding in many states is based on a version of this. The central idea is a variable matching grant from the state that equalizes the per student revenue yield across varying tax bases for any property tax rate chosen by the district. It explicitly does not call for equal spending among districts, only that all districts are able to realize the same revenues from the same tax effort. (Note that this is not the case in the benchmark, where the CC and the East districts apply the same tax rate but collect varying revenues because the capitalization of school quality and location yields varying tax bases). As is well known, however, the implications for spending patterns depend centrally on the behavior of households in setting taxes and choosing locations and in general these choices will yield spending that is correlated with wealth(see, for example, Feldstein (1975)).

Table 4 shows the equilibrium outcome of a move to finance through district power equalization. With revenue and spending choices under district power equalization, the West district again disproportionately attracts people who value schooling highly, but the largest impact is a significant fall in quality in the CC schools. The West School District is the most efficient school district, since it is home mostly to high valuation households. We also see the effect of access and

²⁷This was introduced into the debates by Coons, Clune and Sugarman (1970) and has been a perennial candidate for funding, largely because it was identified as being "wealth neutral." It is variously called guaranteed tax base, district power equalization, or wealth neutrality.

Variable	CBD	West	East
<i>Quality of Education</i>	31.8	58	37.3
<i>Tax rate</i>	1.3%	1.66%	1.3%
<i>Expenditure per pupil per year</i>	\$1,957	\$2,186	\$1,993
<i>Efficiency</i>	5.93	9.68	6.83
<i>Average monthly gross rent per acre</i>	\$3,675	\$4,044	\$3,543

Table 4: Equilibrium characteristics of communities after district power equalization.

Average School Quality		
	Benchmark	Power Equalization
<i>Skilled Residents</i>	46.2	44.5
<i>Unskilled Residents</i>	43.3	42.2
<i>High Valuation Families</i>	54.4	52.4
<i>Low Valuation Families</i>	36.0	35.2

Table 5: School quality with power equalization.

wages on rents. Rents in the East remain below those in the CC, even though the tax rates and school quality are essentially the same.

If we look at the comparisons of school quality in Table 5, we see that the implications for different types of households is not as simple as prior analyses have suggested. In the simple partial equilibrium setting that motivates much of the discussion of school finance policy, equalizing the ability to raise money is typically seen as a way of improving the education of kids from poor families. But, as the table shows, the average quality of schools for the unskilled residents falls – as it does for *all* family types. The range of schooling outcomes across household types narrows but only slightly.

The most severely hit group is Skilled High valuation households (who have to share part of the capitalized rents from high school quality with other groups). Nonetheless, each group finds that the pre-policy equilibrium yielded better schools. There is a slight narrowing of the gap between low income (unskilled) and high income (skilled) families, but it comes as the cost of poorer outcomes overall.

As Feldstein (1975) previously indicated, this program does not sever the relationship between a community’s expenditure per pupil and its wealth (here measured by rents). Communities

with the same property tax rates, as in our simulation, might end up with different quality of schools, and tax rates also vary by wealth.

4.2 Full State Funding

One obvious way to reduce the variation in spending (the objective of many court and legislative decisions) is simply to raise the share of spending that is provided by the state. This is precisely the history of school funding, as the state share of educational funding has gone from 30 percent in 1940 to 40 percent in 1970 to 50 percent in 2000 (U.S. Department of Education 2008).²⁸

The extreme is full state funding, where all local choice in funding decisions is eliminated. A close relative of full state funding is school district consolidation where taxes and spending are equalized across merging districts.²⁹

The two policies are conceptually somewhat different. Full state funding can still work with separate school districts that make their own educational decisions, while consolidation assumes both common funding and common administration. Additionally, consolidation does not have to be done at the state level but instead can be done at lower levels such as the county-wide school districts seen in many southern states. Nonetheless, in our metropolitan area analysis we do not distinguish between full state funding and local consolidation. While there has been prior analysis of district consolidation, the implications both for welfare and for school quality remain uncertain.³⁰

This section first explores the consequences of school district consolidation. The CC, West, and East School Districts are consolidated under the name, Greater City School District. For

²⁸Note that the federal government currently provides 9 percent of total revenues (U.S. Department of Education 2008). These revenues are largely distributed in a compensatory manner that will provide larger funding to districts with more poor people. Thus, even with a flat amount of full state funding there would be variation across districts. States, however, also have compensatory programs that would amplify such variations.

Because the federal share has increased since the passage of the Elementary and Secondary Education Act of 1965, the local share of funding has fallen by even more than the rise in state shares.

²⁹In fact, the history of U.S. schooling in the 20th century was one of consolidation of districts. At the beginning of World War II, there were over 115,000 school districts, but this fell to less than 15,000 today.

³⁰The nature of voluntary consolidation (Brasington 1999) and the potential cost savings from school district consolidation (Duncombe and Yinger, 2002) have been previously considered. Calabrese, Cassidy, and Epple (2002) analyze consolidation within the context of a political model and suggest that voters as a group are unlikely to support further consolidation, although they suggest the welfare aspects of consolidation are ambiguous.

Variable	CC	West	East
<i>Quality of Education</i>	51.1	31	34.7
<i>Tax rate</i>	1.3%	1.3%	1.3%
<i>Expenditure per pupil per year</i>	\$1,936	\$1,936	\$1,936
<i>Efficiency</i>	9.63	5.84	6.55
<i>Average monthly gross rent per acre</i>	\$4,046	\$3,439	\$3,516

Table 6: The characteristics of communities after school district consolidation.

consolidation, however, we require that students attend their neighborhood schools, which follow the boundaries of the prior school districts. Thus, consolidation and full state funding are both modelled as a policy of common spending and tax policy across all of the neighborhoods/districts.³¹

This imposition of common spending does not, however, imply that outcomes are the same. The metropolitan area moves from the benchmark to a new equilibrium, which is described by Tables 6, and 7, and Figure 4. One striking feature of the new equilibrium is that, although all jurisdictions spend the same amount of money on education, they end up with providing different qualities of education. In a significant change from the prior benchmark, the CC School district offers the best education, while the West School District is the worst. What has happened?

Consolidation eliminates the ability of residents to choose the tax-spending policy that they prefer, thus implicitly elevating the role of workplace accessibility in their decision making. The property tax rate prevailing at equilibrium is, not surprisingly, the preferred tax rate of low valuation households who represent the majority of households. The driving force in the school outcomes is the impact of peers on schooling outcomes, and high valuation families systematically move together to the more accessible CC district (see Table 7).

We also see the capitalization of better education into rents in Figure 4. The rents drop as we cross the school district boundary between CC and west school district, where in-commuters live.

³¹District consolidation with a common decision structure across all schools in the consolidated district may, for example, employ compensatory schemes to re-direct funds to one or more of the prior (pre-consolidated) districts or it could pursue various "compensatory" assignment policies within the consolidated districts. We do not look at these potential within-district policies.

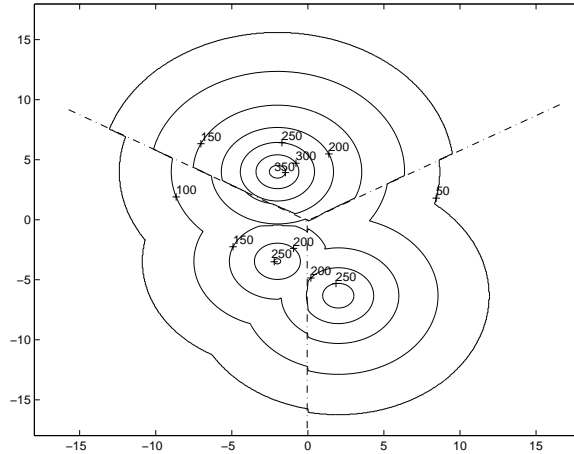


Figure 4: ISO-RENT CURVES: *Iso-rent curves after School district consolidation have been shown.*

The overall residential population of the CC increases, since the CC now also offers the best education in addition to offering higher wages than other workplaces in the metropolitan area. We do not see any significant change in worker/resident population in the East. The major worker/resident movement to CC comes from West. The exclusive bedroom suburb no longer offers the advantages that it did previously when local choice entered into spending decisions. Even so, we still do not have perfect stratification by incomes or tastes.

It is now useful to put this policy change into perspective. Table 8 again investigates directly what has happened to the school quality across the different groups of the population. Similar to district power equalization, there is a reduction in the outcome gap by income, but it follows from dramatic decreases in school quality. By attempting equalization, the full state funding plan actually harms school quality.

4.3 Summary of Welfare Changes

The key to these calculations is that we have covered the most commonly advocated policies. These are not the only approaches, but they are the most relevant.³² The results are striking.

³²Two other potential policies complete the full range of options. First, the state or courts could simply declare equal spending across districts. Second, the state could declare a common tax rate. Neither are entirely realistic options because, with differences in the (capitalized) home values, these policies would not address the tax capacity problem that has motivated much of the discussion to date.

	Residence Neighborhood/district			
Type/workplace	CC	West	East	All
<i>Skilled Low:CC</i>		2.2%	0.7%	2.9%
<i>Skilled High:CC</i>	19.2%	1.2%	1.2%	21.6%
<i>Unskilled Low:CC</i>	3.3%	4.2%	0.3%	7.8%
<i>Unskilled High:CC</i>	17.9%			17.9%
<i>Skilled Low:West</i>		2.8%		2.8%
<i>Skilled High:West</i>		0.4%		0.4%
<i>Unskilled Low:West</i>		12.5%	1%	13.5%
<i>Unskilled High:West</i>				
<i>Skilled Low:East</i>		1.8%	4.6%	6.4%
<i>Skilled High:East</i>		0.6%	5.4%	6%
<i>Unskilled Low:East</i>		3.1%	17.6%	20.7%
<i>Unskilled High:East</i>				
	40.4%	28.8%	30.8%	100%

Table 7: The percentage distribution of households across communities after school district consolidation.

	Average School Quality	
	Benchmark	Consolidation
<i>Skilled Residents</i>	46.2	41.7
<i>Unskilled Residents</i>	43.3	39.3
<i>High Valuation Families</i>	54.4	47.8
<i>Low Valuation Families</i>	36.0	33.9

Table 8: School quality with full state funding (district consolidation).

First, paralleling the actions of legal cases surrounding school funding, spending and tax rates are equalized as an objective measure of actions to improve the equity of the system. Second, aggregating across the schools attended by the different groups, we see more equality in educational outcomes *at the cost of lower the quality of schools for all groups*. As seen in Table 5, the consolidation or full state funding policies have their largest impacts on families with high valuation of education. Previously, they tended to move to districts that had other high valuation families and that tended to vote higher taxes to support their schools.³³ The equalization inhibits expressing preferences for education through choice of higher taxes, and they end up in lower quality schools. Indeed, this looks like the results in California following the *Serrano v. Priest* court case. The state largely took over funding of all schools, the level of funding of schools fell (compared to other states), and schools dropped to near the bottom of state rankings on student achievement (Hanushek and Lindseth 2009).

Table 9 summarizes the welfare change of households resulting from the previous policy alternative. In Table 9 we provide a measure of the consumption change that is equivalent to the utility change with the introduction of the policy. The impact of constrained choices of consolidation on high valuation households (both skilled and unskilled) is the equivalent of a two percent consumption loss. But even the unskilled, low valuation households are hurt, because rents are driven up from the minimums previously available.³⁴

Although we have ideal conditions for governmental involvement – the presence of peer group effects and the redistributive motives for the government to reduce spending disparities – the welfare implications of the policies that are shown Table 9 are somewhat surprising. Due to distortions that could only be captured by a general equilibrium framework, crippling the Tiebout system by divorcing local property wealth (i.e. the price mechanism) from school spending results

Nonetheless, calculations of the new equilibrium under these policies yields the same qualitative answers as those shown: Overall school quality declines, and there are welfare losses for each of the subgroups.

³³Our previous analysis based on the classic monocentric employment model (Hanushek and Yilmaz 2007) also found the consolidation led to generalized welfare losses.

³⁴Table 9 provides an estimate of the consumption change required for constant utility. An alternative is to calculate the change in rent needed to hold household utility constant after the introduction of the policy. This calculation (not shown) yields the same qualitative conclusion

Type	Consolidation	Power Equalization
<i>Skilled High</i>	-0.52	-0.44
<i>Skilled Low</i>	-0.48	-0.34
<i>Unskilled High</i>	-0.58	-0.36
<i>Unskilled Low</i>	-0.44	-0.41

Table 9: Equivalent consumption changes as a result of governmental involvement (Minus implies losses).

Type/Commut. Cost	a=1.1, b=0.1	a=1.375, b=0.075	a=0.825, b=0.125	a=1.375, b=0.125
<i>Skilled High</i>	-0.52	-0.18	-0.35	-0.18
<i>Skilled Low</i>	-0.48	-0.17	-0.36	-0.16
<i>Unskilled High</i>	-0.58	-0.28	-0.33	-0.20
<i>Unskilled Low</i>	-0.44	-0.18	-0.29	-0.12

Table 10: Welfare change after consolidation by the cost of commuting. a is the monetary cost in dollars, and b is the time cost of commuting in hours.

in welfare losses for *all* households. The worst policy, in terms of welfare loss, is school district consolidation, but district power equalization does surprisingly bad.

5 Sensitivity Analysis

The specific equilibrium outcome reported is clearly based upon specific choices of the functional form and parameters for the key underlying utilities and costs. Without an analytical solution, we cannot get away from this parameter dependency. As a way of assessing the generalizability of the results, we consider varying two key sets of parameters: those related to the cost of commuting and those related to importance of the specific taste parameters.

We start out with the cost of commuting. We consider a range of many simulations in which we increase/decrease the time cost and pecuniary cost by 25%, and repeat school district consolidation and district power equalization exercises. Due to space consideration, we just report welfare change after school district consolidation from some simulations in Table 10. Note that the first column is the benchmark, and the reported ones are chosen to give a general picture of all simulations. In almost all simulations, everybody is worse off. Though not reported here, average school quality by income or taste groups reconfirms the previous findings. Everybody gets a lower quality school.

Type/Taste	$\alpha_L=0.17, \alpha_H=0.19$	$\alpha_L=0.15, \alpha_H=0.17$	$\alpha_L=0.15, \alpha_H=0.19$	$\alpha_L=0.17, \alpha_H=0.21$
<i>Skilled High</i>	-0.52	-0.02	-0.93	-0.84
<i>Skilled Low</i>	-0.48	-0.05	-0.91	-0.85
<i>Unskilled High</i>	-0.58	-0.15	-0.48	-0.31
<i>Unskilled Low</i>	-0.44	-0.04	-0.42	-0.30

Table 11: welfare change after consolidation by taste parameters.

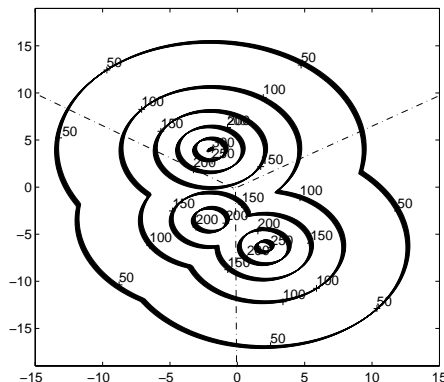


Figure 5: *Iso-rent curves for the pure urban model have been shown.*

As for power district equalization, we reconfirm our previous results. Everybody is worse off after the policy is implemented, and all household income groups or taste groups get a lower quality of education.

Table 11 shows welfare change after consolidation for different taste values. Recall that taste values determine the willingness to pay, the property tax rate. We have nothing new. Everybody is worse off, and the quality of education for income groups by income or tastes is lower. For district power equalization, the welfare change and average quality of education for household groups by income or tastes is less robust.

One interesting case is to compare our benchmark to a pure urban model by removing the Tiebout dimension to choice. We set property taxes to be zero and quality of educations as one. Figure 5 shows the iso-rent curves for the pure urban model. Again, rents go down as we move away from the employment centers. Around centers, we see the capitalization of wages. Wages are highest (lowest) at the CC (west) subcenter. So are rents. We also see a more balanced distribution

of population distribution. In the benchmark, we see more high valuation households in the school district that provides the a better education. Moreover, we also see the capitalization of better education. Though the east subcenter has higher wages than the west subcenter, the rents around the west subcenter are higher due to a higher quality of education. These behaviors and equilibrium outcomes are no longer present in the pure urban model. Finally, in the pure urban model, we do not see any jumps or drops in rents as we cross the district border because there is no education quality to be capitalized into housing prices.

6 Conclusions

This paper provides a unified treatment of the two separated streams of literature, urban location theory and Tiebout models of community choice. It also takes the model beyond the monocentric city model by introducing decentralized employment locations. The base locational outcomes are more consistent with empirical observation. As opposed to the prediction of Tiebout models, where there is stratification of households across communities by income, all communities are heterogenous and contain all household types. Compared to monocentric models, the model generates a realistic commuting pattern.

From an analytical viewpoint, it is clear that considering multiple jurisdictions with decentralized employment opportunities is important. The competition among suburban districts, a fact of today's locational patterns, can only be handled with models such as those outlined here. Additionally, when considering the kinds of significant changes in school finance policies that have occurred frequently for the past 50 years, it is extremely important to consider general equilibrium formulations, because households will adjust to the changed attractiveness of different locations in a metropolitan area.

The richness of this depiction of household locational choices does, however, come at a cost. We cannot be completely general but must rely upon fixed specifications and selected values for key parameters. There is no reason to believe that the results are artifacts of the specific choices,

and a sensitivity analysis involving alternative values of key parameters yields similar results as the baseline model. Nonetheless, conclusions about the generalizability of the policy simulations must await further analyses.

We use the model to assess the impact of two central types of reforms in the pursuit of equity in school finance on the quality of education and individual welfare. A significant finding of our paper is that households seem to be uniformly worse off as a result of governmental involvement. In the baseline, communities establish different levels of education and property taxes, and households "vote with their feet" to choose the optimal bundle. The property tax essentially becomes a fee for education and location.

When government policy intervenes, our baseline policy assessments suggest that households are not made better off. Simplistic assumptions based on perfect stratification and no reactions of households lead to policy proposals that are not welfare improving.

Moreover, with school district consolidation and power equalization programs, districts have the same expenditure per pupil, yet end up providing different levels of education. Improving the fiscal capacity of schools may be a necessary requirement to improve outcomes, but clearly it is not sufficient to achieve equity of educational opportunity. Operating on just the spending margin does not make the situation better, even when one has specific distributional objectives.

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Appendix

Some Equations used in the Algorithm:

The Bid-rent and lot size functions for a type i household with workplace l, school district j, and location (x,y) are given by:

$$\Psi(l, j, r, u_i, q_j, \tau_j) = \frac{k_i^{1/\eta_i}}{(1 + \tau_j)(w_i^l)^\delta/\eta_i} q_j^{\alpha_i/\eta_i} Y_i^l(r)^{\frac{\eta_i + \gamma + \delta}{\eta_i}} u_i^{-1/\eta_i} \quad (1)$$

$$s(l, j, r, u_i, q_j, \tau_j) = \frac{\eta_i}{(\eta_i + \gamma + \delta)(1 + \tau_j)} \frac{Y_i^l(r)}{\Psi(l, j, r, u_i, q_j, \tau_j)} \quad (2)$$

The indirect utility function for the household is given by :

$$V_i(\cdot) = \frac{k_i}{R(x, y)^{\eta_i} (1 + \tau_j)^{\eta_i} (w_i^l)^\delta} q_j^{\alpha_i} Y_i^l(r)^{\eta_i + \gamma + \delta} \quad (3)$$

Our model relies on the single-crossing property of bid-rent functions that needs to be confirmed. Consider two different households with workplace l, school district j, and bid-rent functions $\Psi_1(l, j, r, u_1, \cdot)$ and $\Psi_2(l, j, r, u_2, \cdot)$. Let r^* stand for an intersection point (i.e. $\Psi_1(r^*, u_1, \cdot) = \Psi_2(r^*, u_2, \cdot)$ for some (r^*, u_1, u_2)). We say $\Psi_1(\cdot)$ is steeper than $\Psi_2(\cdot)$ if the following condition holds: Whenever $\Psi_1(r^*, u_1, \cdot) = \Psi_2(r^*, u_2, \cdot) > 0$,

$$\frac{\partial \Psi_1(r^*, u_1, \cdot) / \partial r}{\partial \Psi_2(r^*, u_2, \cdot) / \partial r} = \frac{(\eta_1 + \gamma + \delta) \eta_2 Y_2^l(r^*) (a + b w_1^l)}{(\eta_2 + \gamma + \delta) \eta_1 Y_1^l(r^*) (a + b w_2^l)} > 1 \quad (4)$$

Since bid-rent functions are continuous and decreasing in r, if $\Psi_1(\cdot)$ is steeper than $\Psi_2(\cdot)$, then it must be the case that $\Psi_1(\cdot)$ and $\Psi_2(\cdot)$ intersects at most once. Moreover, the bid-rent function of household 1 dominates that of Household 2 to the left of r^* (i.e. $\Psi_1(r, u_1, \cdot) > \Psi_2(r, u_2, \cdot)$ for all $0 \leq r < r^*$) and the latter dominates to the right of r^* (i.e. $\Psi_2(r, u_2, \cdot) > \Psi_1(r, u_1, \cdot)$ for all $r > r^*$ and $\Psi_1(\cdot) > 0$). Note that the ratio has nothing to do with the quality of education and property tax rate in school district j. I.e. the spatial order for the rings around workplace l is valid in any school district.

The population constraint for a type i household:

$$\int_{(x,y)} \int \frac{1}{S(x, y)} I[type^*(x, y) = i] dx dy = \bar{N}_i \quad (5)$$

where $S(x,y)$ is lot size in equilibrium, $I[type^*(x, y) = i]$ is the indicator function that takes the value 1 if the occupant at (x,y) in equilibrium is type i household, 0 otherwise, and \bar{N}_i is the exogenously given population for type i household (closed city model).

ALGORITHM:

The following algorithm is used to solve the models in the paper. Major points are highlighted. Also see the picture at the end. The school district j could be any district. If we know the bid-rent curve of a type i household with workplace l and school district j , then we can draw the bid-rent for that type in all districts, because identical households obtain the same level of utility. In the same way, we can calculate the bid rent in all districts for type i households with a workplace in the other two employment places.

1. Define model parameters and discretize the space.
2. By using equation 4 in this appendix, check if single crossing property holds and determine the spatial order of households. The ratio in equation 4 is always larger than 1 for any r^* and the spatial order happens to be Unskilled High, Unskilled Low, Skilled High, and Skilled Low from the workplace l .
3. Randomize the initial tax rate/quality of education package in each district.
4.
 1. Initialize fringe distance, r_f^{*j} . Find u_{SL}^* by using equation 3 in the paper and the fact that at r_f^{*j} , the rent is r_a . Use this information to calculate bid-rent and lot sizes (equations 1&2) for Skilled Low valuation households with workplace l in all districts. Also calculate bid-rent and lot sizes in all districts for Skilled Low valuation households with a workplace in other employment centers.
 2. The land area where agricultural use outbids Skilled Low valuations is determined from the bidding process.
 3.
 1. Initialize r_{SHSL}^{*j} .
 2. Calculate the rent at r_{SHSL}^{*j} by using the bid-rent function (equation 1) of Skilled Low valuation households. Use this information to find u_{SH}^* by using equation 3. Calculate bid-rent and lot sizes (equations 1 & 2) for Skilled High valuation households with workplace l in all districts. Also calculate bid-rent and lot sizes in all districts for Skilled High valuation households with a workplace in other employment centers.
 3. Determine the land area that Skilled Low valuations outbid Skilled High valuations. By using lot sizes, calculate the population of Skilled Low valuations (equation 5).
 4. If it is larger (smaller) than the target value, go back to 4.3.1 and increase (lower) r_{SHSL}^{*j} . if it is close to the target value, move to step 4.4.
 4. The land occupied by Skilled Low valuations are determined from the bidding process.
 5.
 1. Initialize r_{ULSH}^{*j} .
 2. Calculate the rent at r_{ULSH}^{*j} by using the bid-rent function (equation 1) of Skilled High valuation households. Use this information to find u_{UL}^* by using equation 3. Calculate bid-rent and lot sizes (equations 1 & 2) for Unskilled Low valuation households with workplace l in all districts. Also calculate bid-rent and lot sizes in all districts for Unskilled Low valuation households with a workplace in other employment centers.
 3. Determine the land area that Skilled High valuations outbid Unskilled Low valuations. By using lot sizes, calculate the population of Skilled High valuations (equation 5).
 4. If it is larger (smaller) than the target value, go back to 4.5.1 and increase (lower) r_{ULSH}^{*j} . if it is close to the target value, move to step 4.6.
 6. Land occupied by Skilled High valuations are determined from the bidding process.

7. 1. Initialize r_{UHUL}^{*j} .
2. Calculate the rent at r_{UHUL}^{*j} by using the bid-rent function (equation 1) of Unskilled Low valuation households. Use this information to find u_{UH}^* by using equation 3. Calculate bid-rent and lot sizes (equations 1 & 2) for Unskilled High valuation households with workplace 1 in all districts. Also calculate bid-rent and lot sizes in all districts for Unskilled High valuation households with a workplace in other employment centers.
3. Determine the land area that Unskilled Low valuations outbid Unskilled High valuations. By using lot sizes, calculate the population of Unskilled Low valuations (equation 5).
4. If it is larger (smaller) than the target value, go back to 4.7.1 and increase (lower) r_{UHUL}^{*j} . If it is close to the target value, move to step 4.8.
8. Land occupied by Unskilled Low valuations are determined from the bidding process.
9. Determine the land area that Unskilled High valuations outbid Unskilled low valuations. By using lot sizes, calculate the population of Unskilled High valuations (equation 5).
5. If it is smaller is (larger) than the target value, go back to 4.1 and increase (lower) r_f^{*j} . If it is close to the target value, move to step 6.
6. Find majority winner property tax rates, tax bases, and quality of education in each school district.
7. Go back to step 3 and update tax rate/education package. Stop if the current period tax rate/education packages are close to those in the last period.

