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Characterizing Global Value Chains

Zhi Wang | Shang-Jin Wei | Xinding Yu | Kunfu Zhu

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Stanford
Center for International Development

John A. and Cynthia Fry Gunn Building | 366 Galvez Street, Stanford, CA 94305-6015

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Zhi Wang

University of International Business and Economics & George Mason University

Shang-Jin Wei, Asia Development bank

Xinding Yu and Kunfu Zhu

University of International Business and Economics, China

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Abstract

Since the extent of both outsourcing and offshoring varies by sector and country, we develop a set of country-sector level measures of global value chains (GVCs) in terms of average production length, intensity of participation, and relative upstream positions on a production network. We distinguish production activities that are inside a country, and that cross borders once or multiple times. Using these measures, we characterize cross-country production sharing patterns and their evolutions for 35 sectors and 40 countries over 17 years. While the production chain for the world as a whole has become longer, there are interesting variations in the length, participation, and positions across different country-sectors. The results contribute to a better understanding of the character of various global value chains and patterns of participation by individual country-sectors.

Key Words: Production length, Position and Participation in Global Value Chains

JEL Number: F1, F6

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1. Introduction

The emergence of global value chains (GVCs) has changed the pattern of international trade in recent decades. Different stages of production now are often conducted by multiple producers located in several countries, with parts and components crossing national borders multiple times. While the deficiency (i.e., due to trade in intermediates) of official trade statistics as a description of true trade patterns has been well recognized, measures of global value chains based on sequential production are still under development.

A "value chain" represents value added at various stages of production, which runs from the initial phase such as R&D and design to the delivery of the final product to consumers. A value chain can be national if all stages of production occur within a country, or regional or global if different stages take place in different countries. In practice, most products or services are produced by a regional or global value chain.

Production length, as a basic measure of GVCs, is defined as the number of stages in a value chain, reflecting the complexity of the production process. Such measures are necessary to assess specialization patterns of countries in relatively upstream versus downstream stages of global production processes (Antras et al., 2012). Based on the production length, the upstreamness and downstreamness indexes are proposed in the recent literature (see Antras et al., 2012; and Miller and Temurshoev, 2015) to measure a sector/country's position in a global production process.

The recent work in the production length measures for GVCs started with Fally (2012), who proposed two measures, "distance to final demand," or "upstreamness" i.e., the average number of stages between production and final consumption, and "the average number of production stages embodied in each product" or the "downstreamness" to quantify the length of production chains and a sector's position in the chain simultaneously. These two measures are further explored in Antras et al. (2012) and Antras and Chor (2013), respectively. Curiously, sector rankings by these production length or "upstreamness" and "downstreamness" measures do not coincide with each other. This implies certain inconsistency in the way that these measures are defined. As we will argue, a key source of the problem is that the existing measures start from a sector's gross output. We will propose a new production length measure that starts from a country-sector's value added or primary inputs. With such an approach, the "upstreamness" and "downstreamness" in our newly defined global value chain position index will be completely consistent with each other.

As argued by Erik (2005, 2007), a production chain starts from the sector's primary inputs (or value added) such as labor and capital, not its gross output.[1] By defining production length as the number of stages between primary inputs in one country/sector to final products in another country/sector, our new measure provides better internal consistency and easier economic interpretations. For example, in our framework, the average production length of a value chain is the average times that the value-added created by production factors employed in the sequential production process has been counted as gross output in the value chain; it equals the ratio of the accumulated gross outputs to the corresponding value-added that induces the output. In addition, following the gross trade accounting framework proposed by Koopman, Wang, and Wei (to be subsequently cited as KWW, 2014) and Wang, Wei, and Zhu (to be subsequently cited as WWZ, 2013), we further split the total production length into a pure domestic segment, a segment related to "traditional" trade, and a segment related to GVCs that involve production sharing activities crossing national borders. This allows us to define the GVC production length more clearly for the first time in the literature.

While "production length" counts the number of production stages, "production position" on a value chain is a relative concept. The relative distance of a particular production stage (country-sector) to the two ends of a global value chain constitutes a measure of production line position. They are two related but different measures. We also modify the measures of global value chain participation indices, originally proposed by KWW (2014), based on forward and backward industrial linkages for a

[1] It is important to bear in mind that gross outputs are endogenous variables, while primary inputs and final demand are exogenous variables in the standard Leontief model. Converting gross output (gross exports are part of it) into final demand is the key technical step to establishing their gross trade accounting framework in both Koopman, Wang, and Wei (2014) and Wang, Wei, and Zhu (2013).

country-sector. These newly defined or modified measures allow us to completely characterize the role, intensity and upstream/downstream positions of all country sectors in global production networks.

We apply these new measures to the recently available Inter-Country Input Output (ICIO) database and obtain some interesting results. We show that Fally's (2012) result that the production length is getting shorter (based on the US IO table) is not globally representative. Consequently, his main hypothesis that value-added has gradually shifted towards the downstream stage, closer to the final consumers, may only apply to very few high income countries such as Japan and the United States.

Our results differ from the existing literature in a number of ways. First, we show that emerging economies such as China experience a lengthening of the overall production chains over time, and the lengthening of production by these countries dominates shortening of production by others, so that for the world as a whole, the production line is getting longer over time. Second, we decompose changes in total production length into changes in the pure domestic segment, changes in the segment related to traditional trade, and changes in the segment related to global value chains. With such decomposition, we show that the ratio of international production length versus total production length of GVCs has increased for all countries. Third, we show that all countries in the world increased their GVC participation during 1995–2011. And finally, we analyze the role GVCs have played in transmitting economic shocks in the recent global financial crisis and find that a country/sector's GVC participation intensity has significant effects. The deeper and more intense is a country sector participating GVCs, the stronger the impact of the global economic shock. In addition, the effect of the financial crisis increases with the length of the international portion of the relevant global value chains.

This paper takes advantage of but also goes beyond the gross trade accounting framework developed in KWW and WWZ. Constructing various GVC indexes that characterizes global production network from different perspectives based on consistent decomposition and accounting exercises, will allow follow-up econometric studies of determinants and consequences of cross country production sharing as guided by economic theory. The GVC participation, length and position indexes defined in this paper are part of our efforts in this direction.

The rest of the paper is organized as follows: Section 2 formally defines the GVC participation, length and position indexes and discuss how these new GVC measures different from measures existing in current literature; Section 3 reports major empirical results based on WIOD; and Section 4 explores the implications of our findings and concludes.

2. Global Value Chain Participation, Length and Position Indexes

2.1 Value added and final goods production decomposition

Without loss generality, let's consider an Inter-Country Input-Output (ICIO) model for *G* countries and *N* sectors. Its structure can be described by Table 1:

Table 1 General Inter-Country Input-Output table

Outputs / Inputs		Intermediate Use				Final Demand				Total Output
		1	2	⋯	G	1	2	⋯	G	
Intermediate Inputs	1	Z^{11}	Z^{12}	⋯	Z^{1g}	Y^{11}	Y^{12}	⋯	Y^{1g}	X^1
	2	Z^{21}	Z^{22}	⋯	Z^{2g}	Y^{21}	Y^{22}	⋯	Y^{2g}	X^2
	⋮	⋮	⋮	⋱	⋮	⋮	⋮	⋱	⋮	⋮
	G	Z^{g1}	Z^{g2}	⋯	Z^{gg}	Y^{g1}	Y^{g2}	⋯	Y^{gg}	X^g
Value-added		Va^1	Va^2	⋯	Va^g					
Total input		$(X^1)'$	$(X^2)'$	⋯	$(X^g)'$					

where Z^{sr} is an $N \times N$ matrix of intermediate input flows that are produced in country *s* and used in country *r*; Y^{sr} is an $N \times 1$ vector giving final products produced in country *s* and consumed in country *r*; X^s is also an $N \times 1$ vector giving gross outputs in country *s*; and VA^s denotes an $N \times 1$ vector of direct value added in country *s*. In this ICIO model, the input coefficient matrix can be defined as $A = Z\hat{X}^{-1}$, where $\hat{X}$ denotes a diagonal matrix with the output vector *X* in its diagonal. The value added coefficient vector can be defined as $V = Va\hat{X}^{-1}$. Gross outputs X can be split into intermediate goods and final goods, $AX + Y = X$. Rearranging terms, we can reach the classical Leontief

(1936) equation, $X = BY$, where $B = (I - A)^{-1}$ is the well-known (global) Leontief inverse matrix.

2.1.1 Decomposition of value added production at the country-sector level

The gross output production and use balance, or the row balance condition of the ICIO table in Table 1 can be written as:

$$X^s = A^{ss}X^s + \sum_{r\neq s}^{G} A^{sr} X^r + Y^{ss} + \sum_{r\neq s}^{G} Y^{sr} = A^{ss}X^s + Y^{ss} + E^{s*} \quad (1)$$

where A^{ss} is an *N*×*N* domestic input coefficient matrix of country *s* (block diagonal), A^{sr} is an *N*×*N* import input coefficient matrix of country *r* (block off diagonal) , $E^{s*} = \sum_{s\neq r}^{G} E^{sr}$ is the *N×1* vector of total gross exports of country *s*, and E^{sr} is the *N×1* vector of gross exports from country *s* to country *r*.

Rearranging the equation (1) yields

$$X^s = (I - A^{ss})^{-1}Y^{ss} + (I - A^{ss})^{-1}E^{s*} = L^{ss}Y^{ss} + L^{ss}E^{s*} \quad (2)$$

where $L^{ss} = (I - A^{ss})^{-1}$ is defined as local Leontief inverse. With a further decomposition of gross exports into exports of intermediate/final products and their final destinations of absorption, it can be shown that

$$L^{ss}E^{s*} = L^{ss} \sum_{r\neq s}^{G} Y^{sr} + L^{ss} \sum_{r\neq s}^{G} A^{sr} \sum_{u}^{G} B^{ru} \sum_{t}^{G} Y^{ut} \quad (3)$$ [2]

where B^{ru}s are block matrices in the global Leontief inverse.

Inserting (3) into (2) and pre-multiplying with the direct value-added diagonal matrix $\hat{V}$, we can decompose value-added generated from each industry/country pair (GDP by industry) into different components:

$$(Va^s)' = \hat{V}^s X^s = \underbrace{\hat{V}^s L^{ss} Y^{ss}}_{(1)-V\_D} + \underbrace{\hat{V}^s L^{ss} \sum_{r\neq s}^{G} Y^{sr}}_{(2)-V\_RT} + \underbrace{\hat{V}^s L^{ss} \sum_{r\neq s}^{G} A^{sr} \sum_{u}^{G} B^{ru} \sum_{t}^{G} Y^{ut}}_{(3)-V\_GVC}$$

$$= \underbrace{\hat{V}^s L^{ss} Y^{ss}}_{(1)-V\_D} + \underbrace{\hat{V}^s L^{ss} \sum_{r\neq s}^{G} Y^{sr}}_{(2)-V\_RT} + \underbrace{\hat{V}^s L^{ss} \sum_{r\neq s}^{G} A^{sr} L^{rr} Y^{rr}}_{(3a)-V\_GVC\_R}$$

$$+ \underbrace{\hat{V}^s L^{ss} \sum_{r\neq s}^{G} A^{sr} \sum_{u}^{G} B^{ru} Y^{us}}_{(3b)-V\_GVC\_D} + \underbrace{\hat{V}^s L^{ss} \sum_{r\neq s}^{G} A^{sr} \left(\sum_{u}^{G} B^{ru} \sum_{t\neq s}^{G} Y^{ut} - L^{rr} Y^{rr}\right)}_{(3c)-V\_GVC\_F} \quad (4)$$

There are five terms in this decomposition, each representing domestic value-added generated by the industry in its production to satisfy different segments of the

[2] A detailed mathematical proof of equation (3) is provided in Appendix A.

global market. These domestic value-added or GDP in each country/sector pair is generated from the following three types of production activities:

(1) Production of domestically produced and consumed value-added ($\hat{V}^s L^{ss} Y^{ss}$). This is domestic value added to satisfy domestic final demand that is not related to international trade, and no cross country production sharing is involved. We label it as V_D for short.

(2) Production of value-added embodied in final product exports ($\hat{V}^s L^{ss} \sum_{r \neq s}^{G} Y^{sr}$). This is domestic value added to satisfy foreign final demand that does not involve any cross country production activities. It cross national border for consumption only so is very similar to traditional "Ricardian" type trade such as "French wine in exchange for England cloth", in the term used by Borin and Mancini (2015)[3]. We label it as V_RT for short.

(3) Production of value-added embodied in exports of intermediate goods and services. It is domestic value-added that relates to production activities outside the source country, and is the source country's contribution to global production. We label it as V_GVC for short. It measures the amount of domestic value added that is generated from the production of intermediate exports regardless of how and where these value-added are finally absorbed. It can be further split into three categories:

3a. Directly absorbed by partner country *r* without further border crossing. Value-added embodied in intermediate exports that is used by trading partner to produce its domestic final products and consumed in the direct importing country *r*. DVA crosses national border only once, with no indirect exports via third countries or re-exports activities involved. We label it as V_GVC_R for short.

3b. Returned (re-imported) to exporting country *s* and finally consumed domestically. Value-added embodied in intermediate exports that are used by the importing country to produce either intermediate or final goods and services and

[3] In Ricard time, exports are 100% domestically produced value-added, while today, even in final product exports from a country, there always embodied foreign value-added, domestically produced value-added become only a part of the exports. However, use our decomposition method, we are still able to compute the portion of "Ricardian trade" analytically.

shipped back to the source country (possibly via third countries in the production chain) as imports and consumed there (i.e., domestic value-added to satisfy domestic final demand that is related to international trade, production sharing between home and foreign countries); we denote it as V_GVC_D[4];

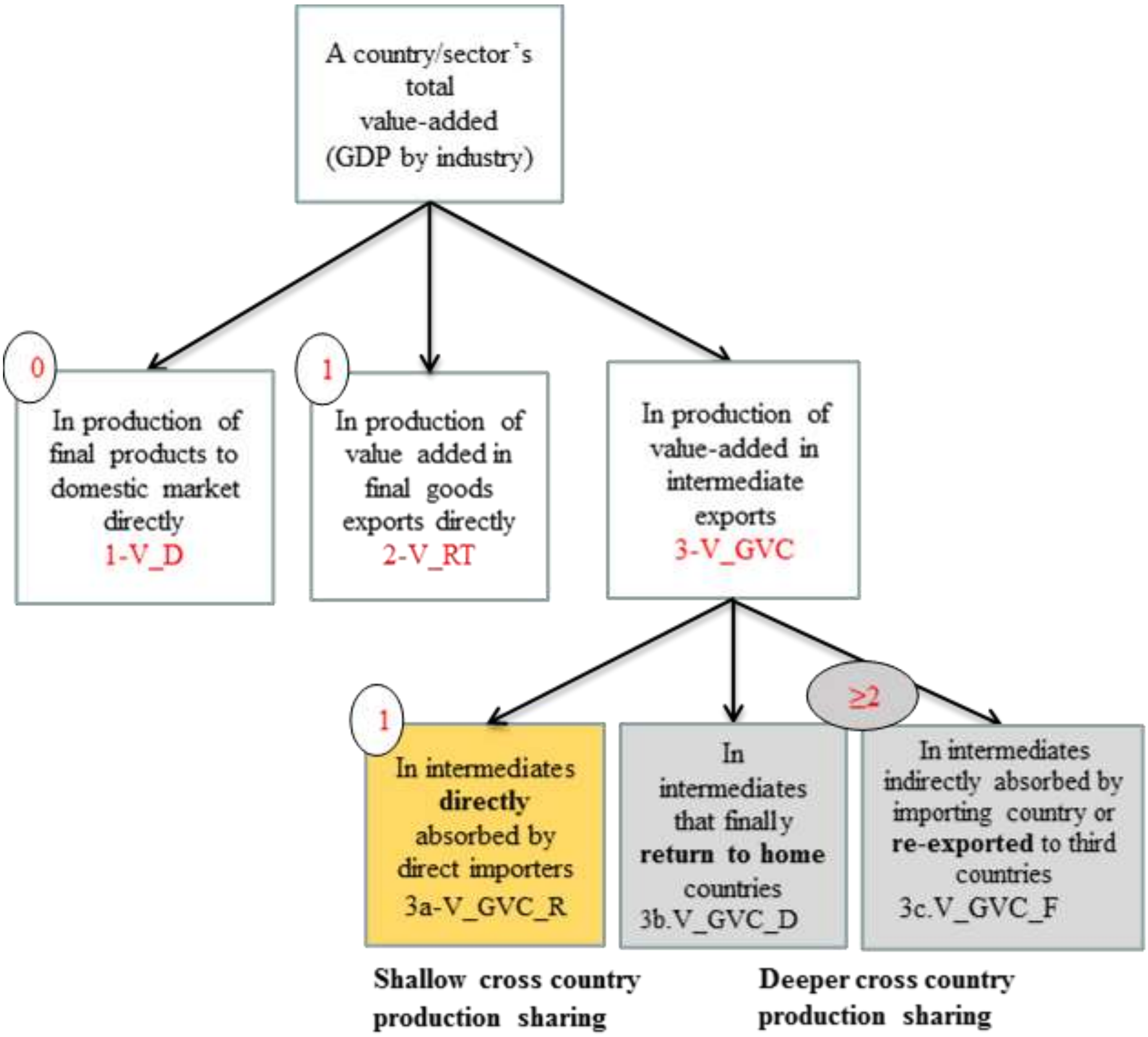
3c. Indirectly absorbed by partner country *r or* re-exported to a third country *t*. Domestic value-added embodied in intermediate exports that is used by partner country *r* to produce exports of final products or intermediate inputs for other countries' production of final goods and services that are eventually re-exported and consumed abroad (i.e., domestic value added to satisfy other country's final demand, production sharing among at least three countries), we label it as V_GVC_F for short.

Note that we use the term GVC related trade here refer to domestic value added in intermediate products that contributes to production activities outside the source country. It can be further divided into shallow and deeper cross country production sharing activities based on number of border crossing of source country's domestic value-added. Term 3a,value added in intermediate exports that crosses borders only once represents relatively shallow cross country production sharing activities, while terms 3b and 3c are "Deeper GVCs", measuring domestic value added that crosses national borders at least twice, represent relative deeper cross country production sharing activities. The summation in the last four terms indicates that the domestic value-added generated by export production can be further split at the bilateral level into each trading partner's market. The sum of terms 2, 3a, and 3c gives the amount of value-added exports as defined by Johnson and Negara (2012), which is the total (direct and indirect) domestic value added to satisfy foreign final demand, while the sum of terms 1 and 3b is the total (directly and indirectly) domestic value-added to satisfy domestic final demand. Finally, the sum of (2) and (3) gives the measure of domestic value-added (GDP) in gross exports proposed by KWW (2014).

The decomposition is also illustrated in Figure 1

[4] It can be further divided by returned routes such as via trading partner country r or via third country t.

Figure 1 Decomposition of GDP by industry
— Which types of production and trade activities belong to Global Value Chains?



2.1.2 Decomposition of final product production by country/sector pairs

Similar to the decomposition of country-sector value-added production into five major components based on whether and how they are involved in cross country production sharing activities, final goods production at each country/sector pair also can be decomposed into five different parts as follows:

$$Y^s = \sum_r^G Y^{sr} = \underbrace{V^s L^{ss} \hat{Y}^{ss}}_{(1)-Y\_D} + \underbrace{V^s L^{ss} \sum_{s \neq r}^G \hat{Y}^{sr}}_{(2)-Y\_RT}$$
$$+ \underbrace{\sum_{r \neq s}^G V^r\, L^{rr} A^{rs} L^{ss} \hat{Y}^{ss}}_{(3a)-Y\_GVC\_R} + \underbrace{V^s \sum_{s \neq r}^G B^{sr}\, A^{rs} L^{ss} \sum_t^G Y^{st}}_{(3b)-Y\_GVC\_D}$$

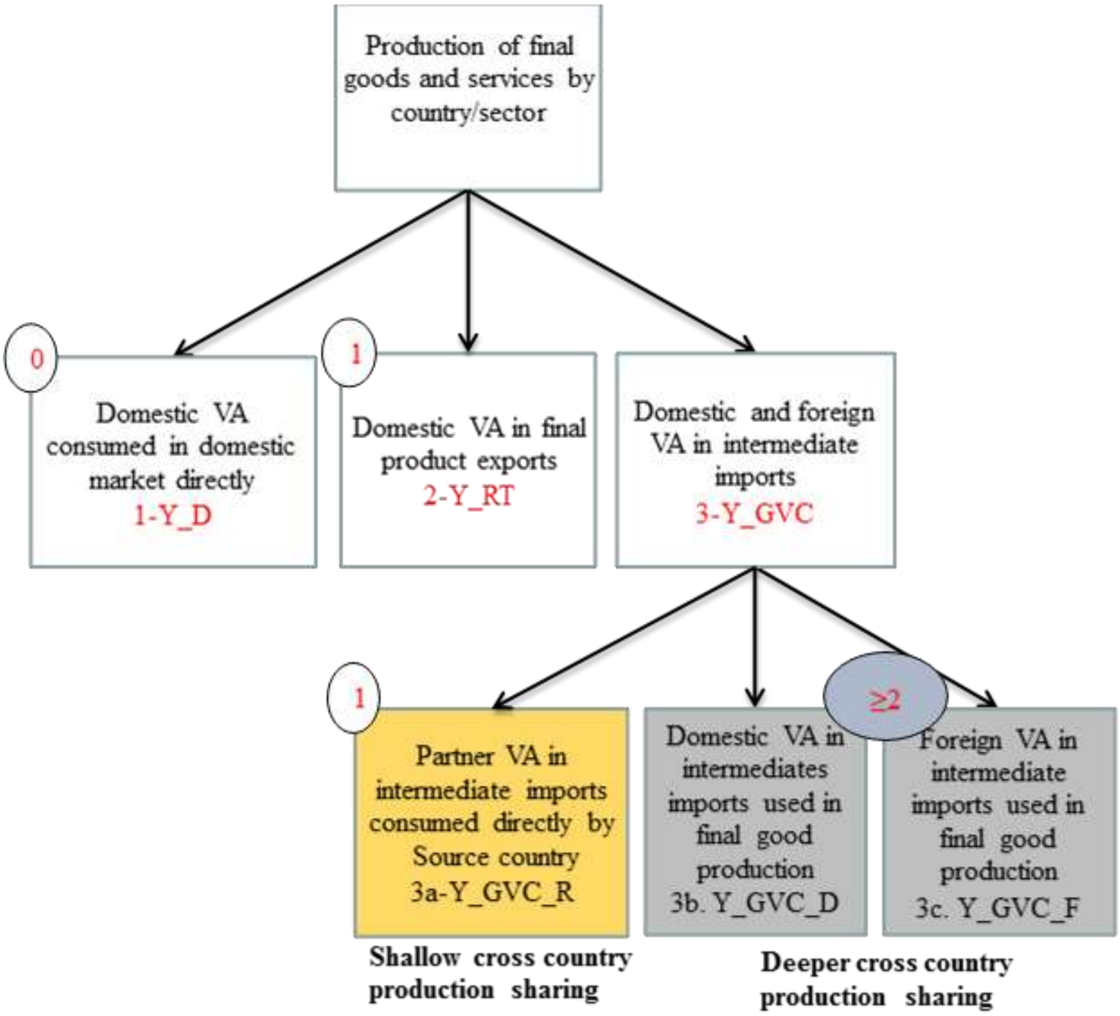
$$+\underbrace{\left(\sum_{r\neq s}^{G} V^{r} \sum_{u\neq r}^{G} B^{ru} A^{us} L^{ss} \sum_{t}^{G} \hat{Y}^{st} - \sum_{r\neq s}^{G} V^{r} L^{rr} A^{rs} L^{ss} \hat{Y}^{ss}\right)}_{(3c)-Y\_GVC\_F} \qquad (5)$$ [5]

where Y^s is a vector, representing final products produced in country *s*, which is the sum of country *s*'s final products for domestic use and exports. The first term in equation (5) is domestic value-added in domestically produced final products that satisfy domestic final demand (DFD) without involving any cross border trade and production activities; we label it as pure domestic product for domestic market (Y_D). The second term is domestic value-added directly embodied in final product exports, there is no cross board production activities involved, we label it as pure domestic product for tradition trade (Y_RT). Conceptually, these two terms are exactly the same as the first two terms in equation (4), but numerically they only equal each other at country aggregate. The third term (3a) is value added of partner country *r* embodied in the intermediate imports of source country s from *r* and used in its production of domestically consumed final products, involve production activities in both country r and s, but only cross border once, we label it as Y_GVC_R. The fourth term (3b) is domestic value-added embodied in intermediate exports first exported but return to the source country through intermediate imports and used in final goods production to satisfy domestic demand or re-exports, we label it as Y_GVC_D. It includes returned domestic value-added (RDV), but also includes returned value-added in final product exports to other countries. The fifth term (3c) is foreign value-added embodied in intermediate imports used by country *s* to produce its final products (for domestic use and exports), we label it as Y_GVC_F. The sum of the first two terms and term 3b equals domestic value-added in country s produced final products, the sum of the terms 3a and 3c equals foreign value-added in country s produced final products. This backward-linkage based decomposition is also depicted in Figure 2.

[5] A detailed mathematical proof of equation (5) is provided in Appendix B.

Figure 2 Decompose final goods production by country/sector

--Which part of final goods production and trade belong to GVCs?



Decomposition of value-added and final goods production into GVC related and unrelated activities based on forward or backward inter-industry linkage is the foundation of the GVC index system will be defined in this paper. Both way to decompose production activities in a country/sector pair include five parts: value-added in Parts 1 and 2 involve no cross country production activities, satisfy domestic and foreign demand respectively. Value-added in Part 2 cross board once, but only for consumption activities, all its intermediate inputs come from domestic sources, so it is the Ricardo trade in value-added (French wine change for England cloth). Value added in Part 3 is embodied in trade in intermediate products and can be decomposed further into three portions: 3a is value-added embed in intermediate goods absorbed by direct importer, there is cross board production activities, but only within the direct importing country without further board crossing. 3b and 3c are value-added cross board at least twice to satisfy domestic and foreign final demand respectively. The last three parts measure GVC related value-added production activities. It excludes domestic value-added measured by the first two terms in equations (4) and (5) because those production activities are accomplished completely within the national boundaries, so both of them can be treated as pure domestic production activities.

2.2 Global Value-Chain participation indexes

The amount of Vertical Specialization (measured by both VS and VS1 as proposed by Hummels et al., 2001) as percent of gross exports have been used widely in the literature as the index to quantify the extent of a country's participation in global value chains (Koopman et al., 2010, 2014; OECD, 2013). However, it excludes production to satisfy domestic final demand (which includes both pure domestic and international trade related production activities), and by only considering export activities, may not cover all the possible ways a country could contribute its domestic value-added into the global production network.

Firms in a country/industry may participate in international production chains in three ways:

1. Exporting its domestic value-added in intermediate exports used by other countries to produce other countries' domestically consumed final products that shows up as foreign value-added in other countries' domestic final products used domestically ;
2. Exporting its domestic value-added in intermediate exports used by other countries to produce exports directly or indirectly; it is the source country's value-added that shows up as foreign value-added in other countries' gross exports;

3. Using other countries' value-added to produce its gross exports directly or indirectly; it is the other countries' value-added that shows up as foreign value-added in the source countries' gross exports.
4. Using other countries' value-added to produce its gross output for domestic use directly or indirectly; it is the other countries' value-added that shows up as foreign value-added in the source countries' gross output used domestically.

The global value chain participation indexes used in the literature, such as the VS and VS1 as percent of gross exports, only take channels 2 and 3 into consideration, even if the first and last channels may be quite substantial especially for large economies as both sources and destinations.

Using the downstream decomposition of value-added generated from each industry/country pair (GDP by industry statistics) expressed in equation (4), and the upstream decomposition of final goods production expressed in equation (5), we can fully identify all the four possible ways a country can participant in the global production network and construct indexes that helps us to measure the full extent to which production factors are employed in a particular country-sector involved in the global production process. Such a GVC participation index based on forward industrial linkage can be defined mathematically as follows:

$$GVCPt\_f^s = \frac{V\_GVC^s}{\hat{V}^s X^s} = \frac{V\_GVC\_R^s}{\hat{V}^s X^s} + \frac{V\_GVC\_D^s}{\hat{V}^s X^s} + \frac{V\_GVC\_F^s}{\hat{V}^s X^s} \tag{6}$$

The denominator of equation (6) is the value-added generated in production from a country/sector pair; the numerator of equation (6) is domestic value added of country *s* embodied in its intermediate exports to the world. So equation (6) gives domestic value-added generated from GVC related production activities as a share of total sector value added. It differs from the forward industrial linkage based GVC participation index defined in previous literature (VS1 as percent of gross exports) in two ways: (a) it is based on the value-added concept while both VS1 and gross exports are based on the gross concept; (b) it is a production concept, not only trade. It includes domestic value-added embodied in intermediate inputs from the exporting country that is directly and indirectly absorbed by its direct trading partners. Therefore, it completely reflects the degree of participation of domestic production factors employed in a particular country/sector in cross border production sharing activities.

Based on the upstream decomposition of final goods production we can define

$$GVCPt\_b^s = \frac{Y\_GVC^s}{Y^s} = \frac{Y\_GVC\_R^s}{Y^s} + \frac{Y\_GVC\_D^s}{Y^s} + \frac{Y\_GVC\_F^s}{Y^s} \quad (7)$$

where $Y^s = \sum_r^G Y^{sr}$ is the total final goods production of sector i at Country s; The second term in (7) gives the portion of domestic value-added embodied in home country's intermediate imports used to produce final products consumed domestically or re-exported to other countries as share of final goods produced in country *s*. The first and last term in (7) give the share of foreign value-added in the total value of final products produced in country *s*. The global sum of its numerator equals the global sum of the numerator in equation (6).[6] Therefore, at the global level, the forward and backward industrial linkage based GVC participation indexes equal each other, a similar property of VS and VS1 based GVC participation indexes. However, it also differs from the backward industrial linkage based GVC participation index defined in previous literature (VS as percent of gross exports) in two ways: (a) it is based on a net concept while both VS and gross exports are based on a gross concept; (b) it is a production concept, not only trade. It not only includes foreign value-added embodied in intermediate imports that is direct or indirectly absorbed by the importing country (production sharing activities with the source or third countries), so completely reflects the degree of foreign production factors' participation in the home country/sectors' production of final products, and measures international production sharing activities from another perspective: how a country's final goods production relies on other countries' production factors' contribution (the first and last term), but also reveal the role of domestic factor has played in deep cross country production sharing arrangement (the second term).

Note that the sum of the first and last term equals foreign value-added in the exporting country's final goods production, including its domestically consumed final

[6] The mathematical proof is provided in Appendix C.

products, divided by the total value of final products produced by the home country, which equals the FVAS measure defined by Los, Timmer and Vries (2015). This means this new backward linkage based GVC participation index is broader than FVAS, it not only consider foreign value-added, but also domestic value-added return home then re-export again and consumed by other countries, thus better reflect the percentage of a country's final goods production is related (contributed) by cross country production sharing activities.

2.3 The length of production chain

We define the length of production as the average number of production stages between the primary inputs in a country-sector to final products in another country/sector, it is the average number of times that value-added created by the prime factors employed in the country/sector pair has been counted as gross output in the production process until its embodied in a final products.

2.3.1 Length of total production

Based on the Leontief insight (Leontief, 1936), value added and final products in the global ICIO model specified in Table 1 are linked by the following equation: $Va' = \hat{V}X = \hat{V}BY$. It is obvious that primary inputs (value added) of sector *i* only can be directly embodied in final products of sector *j* if sector *i* and sector *j* are the same. Therefore, in the first stage of any production process, the value added of sector *i* embodied in final products of sector *j* can be quantified as $\delta_{ij} v_i y_j$, where δ_{ij} is a dummy variable. If *i* and *j* are the same, δ_{ij} equals 1, otherwise it equals 0. At this stage, the length of the production chain is 1, and the output in this production chain (induced by this production chain) is $\delta_{ij} v_i y_j$.

In the second stage, the value added of sector *i* directly embodied in its gross output that is used as intermediates to produce final products of sector *j* can be measured as $v_i a_{ij} y_j$, which is the value added of sector *i* indirectly(first round) embodied in final products of sector *j*. Up to this stage, the length of the production chain is 2, and the output induced by this production chain is $2v_i a_{ij} y_j$, which account value-added $v_i a_{ij} y_j$ as output twice, once for sector i, once for sector j .

In the third stage, indirect value added from sector *i* can be embodied in intermediate goods from any sector, which are used as intermediates to produce sector j final products. Domestic value added from sector i in this stage can be measured as $v_i \sum_k^n a_{ik} a_{kj} y_j$. This is the second round indirect value-added from sector *i* embodied in intermediate goods used by any sector k and absorbed by final goods of sector *j*. At this stage, the length of the production chain is 3, and the output induced by this production chain is $3v_i \sum_k^n a_{ik} a_{kj} y_j$. The same value-added originally produced from sector i is counted as output three times, once in sector i, once in sector k, and once in sector j.

The same goes on for the succeeding stages.

Generalizing the above process to include all rounds of value-added in sector *i* directly and indirectly embodied in final goods of sector *j*, we obtain the following:

$$\delta_{ij} v_i y_j + v_i a_{ij} y_j + v_i \sum_k^n a_{ik} a_{kj} y_j + \cdots = v_i b_{ij} y_j \qquad \delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad \text{(8a)}$$

Expressing (8a) in matrix notation

$$\hat{V}\hat{Y} + \hat{V}A\hat{Y} + \hat{V}AA\hat{Y} + \cdots = \hat{V}(I + A + AA + \cdots)\hat{Y}$$

$$= \hat{V}(I - A)^{-1}\hat{Y} = \hat{V}B\hat{Y} = \begin{bmatrix} v_1 b_{11} y_1 & \cdots & v_1 b_{1n} y_n \\ \vdots & \ddots & \vdots \\ v_n b_{n1} y_1 & \cdots & v_n b_{nn} y_n \end{bmatrix} \quad \text{(8b)}$$

The matrix in last right side of equation (8b) gives the estimates of value added in final goods production by sector source. Each element in the matrix represents the value added from a source sector directly or indirectly used in the production of final goods in the destination sector. The element of row *i* and column *j* in the matrix, $v_i b_{ij} y_j$, is the total value added of sector *i* embodied in the final goods of sector *j*. Looking at the matrix along the row yields the distribution of value added created from one sector absorbed by final goods of all sectors. Looking at the matrix along the column yields the contribution of value added from all source sectors embodied in final goods produced by a particular sector.

Using the length of each stage as weights and summing across all production stages, we obtain the following equation that gives the total output in (induced by) a particular production chain (sector *i* to sector *j*):

$$\delta_{ij} v_i y_j + 2 v_i a_{ij} y_j + 3 v_i \sum_k^n a_{ik}\, a_{kj} y_j + \cdots$$
$$= v_i \sum_k^n b_{ik}\, b_{kj} y_j \qquad \delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \tag{9a}$$

It captures the footprint of sector value added in each production stage. Expressing in matrix notation

$$\hat{V}\hat{Y} + 2\hat{V}A\hat{Y} + 3\hat{V}AA\hat{Y} + \cdots = \hat{V}(I + 2A + 3AA + \cdots)\hat{Y}$$
$$= \hat{V}(B + AB + AAB + \cdots)\hat{Y} = \hat{V}BB\hat{Y} \tag{9b}$$

The element of row *i* and column *j* in the matrix at the right side of equation (9b), $v_i \sum_k^n b_{ik}\, b_{kj} y_j$, is the total output induced by the production chain from sector i's value added and finally absorbed by sector j's final products. Dividing by $v_i b_{ij} y_j$, the total value-added of sector i embodied in final product of sector j , the average production length of value added from sector *i* to final products of sector *j* can be computed as:

$$plvy_{ij} = \frac{v_i \sum_k^n b_{ik} b_{kj} y_j}{v_i b_{ij} y_j} \tag{10}$$

The denominator is total value added from sector i contribute to final product in sector j, and the numerator is total output accumulated along the production chain induced by the value-added. When value added is used as input in a production stage, either as primary input or embodied in intermediate inputs, it will be count as output where it is used. Therefore, the length of a production chain is the times of value added counted as output in the production chain, from the first time it is used as primary input until it absorbed by a final product, thus exist the production process.

Aggregating equation (10) over all products *j*, we obtain the total average production length of value added generated in sector *i*, i.e., the average production length measure based on forward industrial linkage:

$$plv_i = \sum_j^n \left(\frac{v_i b_{ij} y_j}{v_i \sum_k^n b_{ik} y_k} \times \frac{v_i \sum_k^n b_{ik} b_{kj} y_j}{v_i b_{ij} y_j} \right)$$
$$= \frac{v_i \sum_k^n b_{ik} \sum_j^n b_{kj} y_j}{v_i \sum_k^n b_{ik} y_k} = x_i^{-1} \sum_k^n b_{ik}\, x_k = \sum_k^n g_{ik} \tag{11a}$$

where $\sum_k^n b_{ik}\, y_k = x_i$and $\sum_j^n b_{kj} y_j = x_k$. Expressing in matrix notation gives:

$$PLv = \frac{\hat{V}BB\hat{Y}\mu}{\hat{V}B\hat{Y}\mu} = \frac{\hat{V}BBY}{\hat{V}BY} = \hat{X}^{-1}BX = \hat{X}^{-1}B\hat{X}\mu' = G\mu' \quad (11b)$$

where μ is a 1×N unit vector with all its elements equal to 1, and G is the Ghosh inverse matrix[7].

It is the sum along the rows of the Ghosh inverse matrix, which equals the total value of gross outputs that are related to one unit of value added created by primary inputs from a particular sector. Therefore, equation (11) measures total gross outputs induced by one unit of value added at the sector level, which are the footprints of each sector's value added in the economy as a whole. The longer the production chain, the greater the number of downstream production stages a sector's value added is counted as gross output in the economy.

To better understand this point, let us use the diagonal matrix of sectoral value added multiply with PLv, obtaining:

$$Xv = \widehat{Va}PLv = \widehat{Va}\hat{X}^{-1}B\hat{X}\mu' = \hat{V}BBY = \hat{V}BX$$

$$= \hat{V}X + \hat{V}AX + \hat{V}AAX + \hat{V}AAAX + \cdots \quad (12a)$$

Its ith element equals

$$Xv_i = va_i plv_i = va_i x_i^{-1} \sum_k^n b_{ik}\, x_k = v_i \sum_k^n b_{ik}\, x_k \quad (12b)$$

$$= v_i x_i + v_i \sum_k^n a_{ik}\, x_k + v_i \sum_j^n a_{ij} \sum_k^n a_{jk}\, x_k$$

where Xv is the gross output induced by sector value added. On the right side of equation (12a), the first term is the value added directly embodied in its own sector's output, and we may name it as the first footprint of the sector value added in its own sector gross output; the second term is the value added embodied in its own sector's gross output used by all sectors as intermediates to produce outputs, and we may name it as the second footprint of the sector value added directly and indirectly embodied in total gross outputs of this second stage production process. Summing up all terms on

[7] The definition of Ghosh model and the linkage with Leontief model can be expressed in Appendix D.

the right hand side of (12a), we obtain all footprints of sector value added in the whole economy, which equals the total value of gross outputs that relates to the sector value added created by primary inputs from a particular sector. Therefore, the average production length of sector *i* based on forward industrial linkages equals the ratio of sector value added induced total gross output in the whole economy and the sector value-added.

Using the shares of sectoral value added in GDP as weights to aggregate equation (12) over all sectors, we obtain:

$$PLvw = \left(Va\hat{X}^{-1}B\hat{X}\mu'\right)/(\mu Va) = (VB\mathrm{X})/GDP = (\mu X)/GDP \tag{13}$$

where $Va\hat{X}^{-1} = V$, $\hat{X}\mu' = X$ and $VB = \mu$. Equation (13) indicates that the average length of the production chain in the world economy equals the ratio of total gross outputs to GDP,[8] which can be regarded as an index of complexity of the production process in the economy, i.e., the higher this ratio, the more complex the production process in the economy.

Aggregating equation (10) over value-added from all sectors *i* that have contributed to the final goods and services produced by sector *j*, we obtain the production length measure based on backward industrial linkages as:

$$ply_j = \sum_i^n \left(\frac{v_i b_{ij} y_j}{\sum_k^n v_k b_{kj} y_j} \times \frac{v_i \sum_k^n b_{ik} b_{kj} y_j}{v_i b_{ij} y_j}\right) = \frac{\sum_i^n v_i \sum_k^n b_{ik} b_{kj} y_j}{\sum_k^n v_k b_{kj} y_j} = \sum_k^n b_{kj} \tag{14a}$$

where $\sum_i^n v_i b_{ik} = \sum_k^n v_k b_{kj} = 1$. Expressing in matrix notation

$$PLy = \frac{\mu\hat{V}BB\hat{Y}}{\mu\hat{V}B\hat{Y}} = \frac{VBB\hat{Y}}{VB\hat{Y}} = \mu B \tag{14b}$$

It is the sum along the column of the Leontief inverse matrix, which equals the total value of inputs induced by a unit of final product produced in a particular sector. Therefore, equation (14) measures total intermediate inputs induced by a unit value of a particular final product throughout all upstream sectors in the economy, which is called the footprints of final goods and services in the literature. The longer the production chain, the greater the number of upstream production stages a particular

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[8] This is also recognized by Fally (2012).

final product has in the economy. Using the sectoral ratio of final goods to GDP as weight to aggregate equation (14) over all sectors, we obtain:

$$PLyw = \left(\mu B\hat{Y}\mu'\right)/(\mu Y) = (\mu BY)/GDP = (\mu X)/GDP \qquad (15)$$

which gives the same gross output to GDP ratio as equation (13) and therefore has the same economic interpretation.

It is worth noting that the length of a production chain based on forward industrial linkages as expressed in equation (11) is mathematically equivalent to the upstreamness index defined by Fally (2012a, 2012b, 2013) and Antras et al. (2012, 2013);[9] On the other hand, the length of a production chain based on backward industrial linkages expressed in equation (14) is mathematically equivalent to the downstreamness index defined by Antras and Chor (2013). However, there are two notable differences. First, similar to Miller and Temurshoev (2013), our indexes are obtained by the sum of the rows/columns of the Ghosh/Leontief inverse matrices respectively, which are simpler in mathematics and are part of the classic input-output literature; Second, we measure a production chain length from primary inputs in sector *i* to final products of sector *j*, starting from primary inputs (value added), not gross outputs (as Fally and Antras did), and provide very clear economic interpretations for both the numerator and denominator in the production length indexes discussed above. Most important, such concepts of production length allow us to decompose the total production length in the world economy into different segments thus accurately define the length measure of Global Value Chain first time in the literature based on decomposition of value-added and final goods production activities.

2.3.2 Length of pure domestic production

Let us first consider the segment of domestic value added that is generated and absorbed by production activities entirely within the country at each stage of production.

We know from equation (4), in an infinite production process, domestic value added of country *s* embodied in its final products that satisfy its domestic final demand

[9] The proof is provided in Appendix E.

equals $\hat{V}^s L^{ss} Y^{ss}$ (DVA_D^s). Following a similar logic as equation (9), using the length of each production stage as weights and summing up all production stages, we obtain an equation that gives the gross output induced by value-added $\hat{V}^s L^{ss} Y^{sr}$ as follows:

$$Xv\_D^s = \hat{V}^s Y^{ss} + 2\hat{V}^s A^{ss} Y^{ss} + 3\hat{V}^s A^{ss} A^{ss} Y^{ss} + \cdots$$
$$= \hat{V}^s (I - A^{ss})^{-1} (I - A^{ss})^{-1} Y^{ss} = \hat{V}^s L^{ss} L^{ss} Y^{ss} \quad (16)^{10}$$

where $I + A^{ss} + A^{ss} A^{ss} + \cdots = (I - A^{ss})^{-1} = L^{ss}$

Because production activities that generate this part of domestic value-added have no relation with cross border trade, we define its production length as that of pure domestic production. It equals the portion of gross output of country *s* generated by the production of the country's GDP without any cross-border production and trade activities (how many times $\hat{V}^s L^{ss} Y^{ss}$ has been counted as gross output in the economy). Therefore, the average pure domestic production length of country *s* equals the ratio of this portion of gross output to the corresponding domestic value added, and can be expressed as[11]

$$PLv\_D^s = \frac{Xv\_D^s}{DVA\_D^s} = \frac{\hat{V}^s L^{ss} L^{ss} Y^{ss}}{\hat{V}^s L^{ss} Y^{ss}} \quad (17)$$

Similarly, production of value-added in "traditional trade" is also entirely take place domestically, the gross output it induced can be expressed as

$$Xv\_RT^s = \hat{V}^s \sum_{r \neq s}^{G} Y^{sr} + 2\hat{V}^s A^{ss} \sum_{r \neq s}^{G} Y^{sr} + 3\hat{V}^s A^{ss} A^{ss} \sum_{r \neq s}^{G} Y^{sr} + \cdots$$
$$= \hat{V}^s (I - A^{ss})^{-1} (I - A^{ss})^{-1} \sum_{r \neq s}^{G} Y^{sr} = \hat{V}^s L^{ss} L^{ss} \sum_{r \neq s}^{G} Y^{sr} \quad (18)$$

And its production length equals the average times $\hat{V}^s L^{ss} \sum_{s \neq r}^{G} Y^{sr}$ has been counted as gross output in the economy:

$$PLv\_RT^s = \frac{Xv\_RT^s}{DVA\_RT^s} = \frac{\hat{V}^s L^{ss} L^{ss} \sum_{r \neq s}^{G} Y^{sr}}{\hat{V}^s L^{ss} \sum_{r \neq s}^{G} Y^{sr}} \quad (19)$$

2.3.3 Length of Global Value Chain production[12]

The production process of GVC related trade is more complicated than the previous two segments. To better understand such a process, let us start from

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[10] A detailed mathematical proof of equations (16) and (17) is provided in Appendix F.

[11] A division symbol below denotes elements-wide divisions.

[12] A detailed mathematical proof of equations (20), (22), (25) and (29) is provided in Appendix G.

considering the segment of domestic value added that is generated by production activities related to a country's bilateral intermediate exports at each stage of production.

Obviously, intermediate exports only occur in cross country production process that has at least two stages. In such a two stage production process, domestic value added generated from country *s* will be first embodied in its gross output that is used as intermediate exports to other countries and used by these countries to produce final products consumed there or export. It can be measured as $\hat{V}^s A^{sr} \sum_t^G Y^{rt}$. Both its domestic and international production length equal 1. The output induced by this production chain is$\hat{V}^s A^{sr} \sum_t^G Y^{rt}$, which account value-added $\hat{V}^s A^{sr} \sum_t^G Y^{rt}$ as output twice, once in country s, once in country r.

In a three stage production process, the domestic value added generated from country *s* will be embodied in the final products produced from the third stage and consumed in all possible destination counties. It can be measured as$\hat{V}^s A^{ss} A^{sr} \sum_t^G Y^{rt} + \hat{V}^s A^{sr} \sum_u^G A^{ru} \sum_t^G Y^{ut}$ and can be decomposed into two parts: $\hat{V}^s A^{ss} A^{sr} \sum_t^G Y^{rt}$, and $\hat{V}^s A^{sr} \sum_u^G A^{ru} \sum_t^G Y^{ut}$. Their domestic production lengths equal 2, and 1, respectively, and their international production lengths equal 1, and 2, respectively. The output induced by this production chain is $2\hat{V}^s A^{ss} A^{sr} \sum_t^G Y^{rt} + \hat{V}^s A^{sr} \sum_u^G A^{ru} \sum_t^G Y^{ut}$ and $\hat{V}^s A^{ss} A^{sr} \sum_t^G Y^{rt} + 2\hat{V}^s A^{sr} \sum_u^G A^{ru} \sum_t^G Y^{ut}$, respectively. The same value-added originally produced from country s is counted as output three times, either twice in country s, once in country r, or once in country s, once in country r, and once in country u.

The same goes on for an *n*-stage production process.

Summing all over production stages in an infinite stage production process, we have

$$V\_GVC^{sr} = \hat{V}^s A^{sr} \sum_t^G Y^{rt} + \hat{V}^s A^{ss} A^{sr} \sum_t^G Y^{rt} + \hat{V}^s A^{sr} \sum_u^G A^{ru} \sum_t^G Y^{ut}$$
$$+\hat{V}^s A^{ss} A^{ss} A^{sr} \sum_t^G Y^{rt} + \hat{V}^s A^{ss} A^{sr} \sum_u^G A^{ru} \sum_t^G Y^{ut} + \cdots$$
$$= \hat{V}^s L^{ss} A^{sr} \sum_t^G Y^{rt} + \hat{V}^s L^{ss} A^{sr} \sum_u^G A^{ru} \sum_t^G Y^{ut} + \cdots$$
$$= \hat{V}^s L^{ss} A^{sr} \sum_u^G B^{ru} \sum_t^G Y^{ut} = \hat{V}^s L^{ss} A^{sr} X^r \quad (20)$$

where $\sum_u^G B^{ru}$ is the limit of the series $I + \sum_u^G A^{ru} + \sum_k^G A^{rk} \sum_u^G A^{ku}$ .... It measures the amount of domestic value added that can be generated from the production of gross intermediate exports $A^{sr}X^r$ in country *s*, regardless of whether these exports are finally absorbed in importing country *r* or not. Summing equation (20) over all trading partner countries (i.e., over *r*), we obtain the last 3 terms in equation (4), which are the domestic value-added of country *s* generated from all production activities that are needed in the production of its gross intermediate exports to the world.

The source country's domestic value-added embodied in its intermediate exports can be further decomposed into three parts according to equation (4) as follows:

$$V\_GVC^s = \underbrace{\hat{V}^s L^{ss} \sum_{r\neq s}^G A^{sr} L^{rr} Y^{rr}}_{(3a)-V\_GVC\_R} + \underbrace{\hat{V}^s L^{ss} \sum_{r\neq s}^G A^{sr} \sum_u^G B^{ru} Y^{us}}_{(3b)-V\_GVC\_D}$$
$$+ \underbrace{\hat{V}^s L^{ss} \sum_{r\neq s}^G A^{sr} \left(\sum_u^G B^{ru} \sum_{t\neq s}^G Y^{ut} - L^{rr} Y^{rr}\right)}_{(3c)-V\_GVC\_F} \qquad (21)$$

They are the source country's DVA in bilateral intermediate exports directly absorbed in importing country *r* (V_GVC_R), returned (re-imported) and absorbed at the source country (V_GVC_D), and used by importing country *r* to produce final or intermediate goods that finally consumed abroad (indirect absorbed by direct importing country or re-exported to third countries, V_GVC_F), respectively. All of them are involved in production activities abroad, so we label them together as GVC related DVA production activities.

Following the same logic to derive equations (9), i.e., using the domestic or international production length of each stage of intermediate exports production discussed earlier as weights and summing across all production stages, we can obtain the global gross output generated by GVC related trade as well as its 3 components in any particular bilateral route.

$$Xvd\_GVC^{sr} = \hat{V}^s A^{sr} \sum_t^G Y^{rt} + 2\hat{V}^s A^{ss} A^{sr} \sum_t^G Y^{rt} + \hat{V}^s A^{sr} \sum_u^G A^{ru} \sum_t^G Y^{ut} + \cdots$$
$$= \hat{V}^s L^{ss} L^{ss} A^{sr} \sum_u^G B^{ru} \sum_t^G Y^{ut} \qquad (22)$$

For instance, the portion of gross output in country *s* (labeled as Xvd_GVC^{sr}) induced by the production of country *s*'s domestic value-added embodied in its GVC related exports equals:

$$Xvd\_GVC^s = \underbrace{\hat{V}^s L^{ss} L^{ss} \sum_{r \neq s}^{G} A^{sr} L^{rr} Y^{rr}}_{(3a)-Xvd\_GVC\_R} + \underbrace{\hat{V}^s L^{ss} L^{ss} \sum_{r \neq s}^{G} A^{sr} \sum_{u}^{G} B^{ru} Y^{us}}_{(3b)-Xvd\_GVC\_D}$$
$$+ \underbrace{\hat{V}^s L^{ss} L^{ss} \sum_{r \neq s}^{G} A^{sr} \left(\sum_{u}^{G} B^{ru} \sum_{t \neq s}^{G} Y^{ut} - L^{rr} Y^{rr} \right)}_{(3c)-Xvd\_GVC\_F} \quad (23)$$

Term 3a is domestic gross outputs generated by country s's domestic value added in intermediate exports directly consumed by its trading partners, we label it as $Xvd\_GVC\_R^s$. Term 3b is country s's gross outputs induced by country s's domestic value added in GVC related exports returned and finally consumed at home, we label it as $Xvd\_GVC\_D^s$. Term 3c is country s's gross outputs induced by country s's value added in GVC related exports that are embodied in intermediate exports and finally consumed abroad, we label it as $Xvd\_GVC\_F^s$. All of these different parts of gross outputs are associated with domestic value-added in GVC related exports before it leaves the country through forward domestic inter-industrial linkage.

Therefore, the average domestic production length of GVC exports can be computed as the weighted sum of the ratio of the portion of gross output to its corresponding domestic value-added of its 3 components in equations (23) and (21) respectively:

$$PLvd\_GVC^s = \frac{Xvd\_GVC^s}{V\_GVC^s} = \frac{\sum_{k}^{M} V\_GVC\_k^s * PLvd\_GVC\_k^s}{V\_GVC^s} \qquad M = (R, D, F) \quad (24)$$

The average domestic production length of the three components are labeled as $PLvd\_GVC\_R^s$, $PLvd\_GVC\_D^s$, and $PLvd\_GVC\_F^s$ respectively.

Similarly, the total international (foreign) gross outputs induced by domestic value-added of country s embodied in its GVC related intermediate exports can be expressed as:

$$Xvi\_GVC^{sr} = \hat{V}^s A^{sr} \sum_{t}^{G} Y^{rt} + \hat{V}^s A^{ss} A^{sr} \sum_{t}^{G} Y^{rt} + 2\hat{V}^s A^{sr} \sum_{u}^{G} A^{ru} \sum_{t}^{G} Y^{ut} + \cdots$$
$$= \hat{V}^s L^{ss} A^{sr} \sum_{v}^{G} B^{rv} \sum_{u}^{G} B^{vu} \sum_{t}^{G} Y^{ut} \quad (25)$$

It also can be decomposed into 3 components:

$$Xvi\_GVC^s = \underbrace{\hat{V}^s L^{ss} \sum_{r \neq s}^{G} A^{sr} L^{rr} L^{rr} Y^{rr}}_{(3a)-Xvi\_GVC\_R} + \underbrace{\hat{V}^s L^{ss} \sum_{r \neq s}^{G} A^{sr} \sum_{v}^{G} B^{rv} \sum_{u}^{G} B^{vu} Y^{us}}_{(3b)-Xvi\_GVC\_D}$$
$$+ \underbrace{\hat{V}^s L^{ss} \sum_{r \neq s}^{G} A^{sr} \left(\sum_{v}^{G} B^{rv} \sum_{u}^{G} B^{vu} \sum_{t \neq s}^{G} Y^{ut} - L^{rr} L^{rr} Y^{rr} \right)}_{(3c)-Xvi\_GVC\_F} \quad (26)$$

Term 3a represents international gross outputs generated in the process between domestic value-added of country s embodied in its intermediate exports arriving at country *r* and the value-added directly absorbed by final products consumed in country *r*, we label it as $Xvi\_GVC\_R^s$. Term 3b represents international gross outputs generated in the process between domestic value-added of country *s* embodied in its GVC exports arriving at country *r* and the value-added shipped back after further processing and absorbed by final products that are consumed at home, we label it as $Xvi\_GVC\_D^s$. Term 3c represents international gross outputs generated in the process between domestic value-added of country *s* embodied in its intermediate exports arriving at country *r* and the value-added finally absorbed by final products consumed by other countries(including r), we label it as $Xvi\_GVC\_F^s$. All of these different parts of gross outputs are associated with domestic value-added in GVC exports of country *s* after it leaves the country through forward inter-industrial inter-country linkages. Therefore, the average international production length of country *s*'s GVC exports can be computed as the weighted sum of the ratio of the portion of gross output to its corresponding domestic value-added of its 3 components in equations (26) and (21), respectively:

$$PLvi\_GVC^s = \frac{Xvi\_GVC^s}{V\_GVC^s} = \frac{\sum_k^M V\_GVC\_k^{sr} * PLvi\_GVC\_k^{sr}}{V\_GVC^{sr}} \qquad M = (R, D, F) \qquad (27)$$

The average international production length of the three components are labeled as $PLvi\_GVC\_R^s$, $PLvi\_GVC\_D^s$, and $PLvi\_GVC\_F^s$ respectively.

Summing equations (24) and (27), we obtain the total average production length of domestic value-added of country *s* embodied in its bilateral intermediate exports as follows:

$$PLv\_GVC^s = PLvd\_GVC^s + PLvi\_GVC^s$$
$$= \frac{Xvd\_GVC^s}{V\_GVC^s} + \frac{Xvi\_GVC^s}{V\_GVC^s} = \frac{Xv\_GVC^s}{V\_GVC^s} \qquad (28)$$

Obviously, Xv_GVC^s measures total world gross outputs generated by domestic value-added of country *s* embodied in its total intermediate exports. The weighted sum of PLv_GVC^s defines the average production length of domestic value-added embodied in bilateral intermediate exports. Intermediate exports used by direct importers in their production of domestically consumed final products are involved in the production process only within the direct importing country, therefore, the international production length of the source countries' domestic value-added embodied in such intermediate exports equals their production length in the direct importing country r. The international production length of remaining two parts of gross intermediate exports can be very different from their domestic production length due to these embodied domestic value-added cross national borders at least twice, so they represent deeper cross country production sharing arrangements.

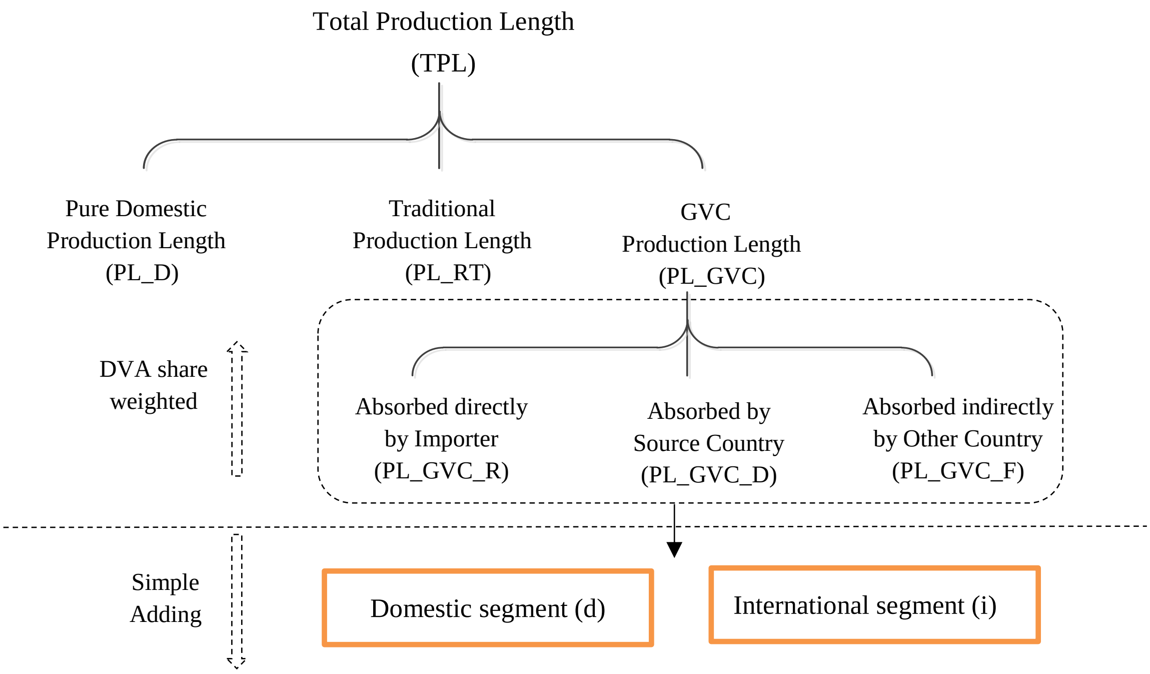
There is a nice symmetry among the terms in equations (22)–(28): all of them are based on the measurement and decomposition of both domestic value-added in intermediate exports and global gross outputs. It is consistent with the gross trade accounting framework proposed in KWW (2014). Using corresponding components of domestic value-added in GVC related trade in equation (21) as the denominators to divide equations (22) and (23) (i.e., the corresponding part of value-added induced gross outputs as numerators), we can obtain the average length of production of each segment and their weighted average in a particular global value chain (equations (24) and (27)). This measures the amount of global gross output that can be generated by one unit of domestic value-added in country s and its total subsequent utilization in the global production network.

Summing the numerator of equations (16), (18), and Xv_GVC^s defined in equation (28) over all trading partner countries r, we obtain

$$\begin{aligned}&Xv\_D^s + Xv\_RT^s + Xv\_GVC^s\\&= \hat{V}^s L^{ss} L^{ss} Y^{ss} + \hat{V}^s L^{ss} L^{ss} \sum_{r\neq s}^{G} Y^{sr} + \hat{V}^s \sum_{s}^{G} B^{sr} \sum_{u}^{G} B^{ru} \sum_{t}^{G} Y^{ut} - \hat{V}^s L^{ss} L^{ss} \sum_{r}^{G} Y^{sr}\\&= \hat{V}^s \sum_{s}^{G} B^{sr} \sum_{u}^{G} B^{ru} \sum_{t}^{G} Y^{ut} = \hat{V}^s \sum_{s}^{G} B^{sr} X^r = Xv^s \qquad (29)\end{aligned}$$

Equation (29) shows clearly that the sum of value-added induced gross output of traditional and GVC exports (equals global total output induced by domestic value-added in gross exports of country s to the world) defined in equations (18) and (28) plus the gross output induced by pure domestic production defined in equation (16) equals $\hat{V}BBY = \hat{V}BX$, the total gross output induced by sector value added in the whole world economy as defined in equation (12). The structure and internal linkage of our production length index system can be represented as a tree diagram, as shown in Figure 3.

Figure 3 An Index System for Production Length



2.3.4 Production length based on backward inter-industry cross country linkage

Based on the decomposition of final goods and services production at each country/sector pair in equation 5, following the same logic of Sections 2.3.2 and 2.3.3, we can compute domestic and international gross outputs drive by different parts of final product production as follows.

$$Xy\_D^s = V^s L^{ss} L^{ss} \hat{Y}^{ss} \tag{30a}$$

$$Xy\_RT^s = V^s L^{ss} L^{ss} \sum_{s\neq r}^G \hat{Y}^{sr} \tag{30b}$$

$$\begin{aligned} Xyd\_GVC^s &= \sum_r^G V^r \sum_{u\neq s}^G B^{ru} A^{us} L^{ss} L^{ss} \sum_v^G \hat{Y}^{sv} \\ &= \underbrace{\sum_{r\neq s}^G V^r L^{rr} A^{rs} L^{ss} L^{ss} \hat{Y}^{ss}}_{Xyd\_GVC\_R} + \underbrace{V^s \sum_{u\neq s}^G B^{su} A^{us} L^{ss} L^{ss} \sum_v^G \hat{Y}^{sv}}_{Xyd\_GVC\_D} \\ &+ \underbrace{\sum_{r\neq s}^G V^r \left[\sum_{u\neq s}^G B^{ru} A^{us} L^{ss} L^{ss} \sum_v^G \hat{Y}^{sv} - L^{rr} A^{rs} L^{ss} L^{ss} \hat{Y}^{ss}\right]}_{Xyd\_GVC\_F} \end{aligned} \tag{30c}$$

$$\begin{aligned} Xyi\_GVC^s &= \sum_r^G V^r \sum_u^G B^{ru} \sum_{t\neq s}^G B^{ut} A^{ts} L^{ss} \sum_v^G \hat{Y}^{sv} \\ &= \underbrace{\sum_{r\neq s}^G V^r L^{rr} L^{rr} A^{rs} L^{ss} \hat{Y}^{ss}}_{Xyi\_GVC\_R} + \underbrace{V^s \sum_v^G B^{sv} \sum_{u\neq s}^G B^{vu} A^{us} L^{ss} \sum_v^G \hat{Y}^{sv}}_{Xyi\_GVC\_D} \\ &+ \underbrace{\sum_{r\neq s}^G V^r \left[\sum_v^G B^{rv} \sum_{u\neq s}^G B^{vu} A^{us} L^{ss} \sum_v^G \hat{Y}^{sv} - L^{rr} L^{rr} A^{rs} L^{ss} \hat{Y}^{ss}\right]}_{Xyi\_GVC\_F} \end{aligned} \tag{30d}$$

$$Xy\_GVC^s = Xyd\_GVC^s + Xyi\_GVC^s \tag{30e}$$

$$Xy^s = Xy\_D^s + Xy\_RT^s + Xy\_GVC^s = \sum_r^G V^r \sum_v^G B^{rv} \sum_u^G B^{vu} \sum_t^G \hat{Y}^{ut} \tag{31}$$

Detailed derivations can be found in Appendix I.

Therefore, the ratio of these gross outputs to the value of final products produced in country s,Y^s, is the average domestic and international production length based on backward inter-industry and cross-country linkage.

$$PLyd\_GVC^s = \frac{Xyd\_GVC^s}{Y\_GVC^s} = \frac{\sum_k^M Y\_GVC\_k^s * PLyd\_GVC\_k^s}{Y\_GVC^s} \qquad M = (R, D, F) \quad (32a)$$

$$PLyi\_GVC^s = \frac{Xyf\_GVC^s}{Y\_GVC^s} = \frac{\sum_k^M Y\_GVC\_k^s * PLyf\_GVC\_k^s}{Y\_GVC^s} \qquad M = (R, D, F) \quad (32b)$$

$$PLy\_GVC^s = PLyf\_GVC^s + PLyi\_GVC^s = \frac{Xy\_GVC^s}{Y\_GVC^s} \quad (32c)$$

2.3.5 Number of border crossing in the GVC production length index system

International production length specified above can be further decomposed into number of border crossing of intermediate trade flows (border crossing for production, which is different with border crossing for consumption) and domestic production length in all countries involved in the global value chain after intermediate exports leaving the source country.

It can be shown that Equation (25) can be further decomposed into 3 terms:

$$Xvi\_GVC^{sr} = \hat{V}^s L^{ss} A^{sr} \sum_v^G B^{rv} \sum_u^G B^{vu} \sum_t^G Y^{ut} = \hat{V}^s L^{ss} A^{sr} \sum_v^G B^{rv} X^v$$
$$= \hat{V}^s L^{ss} A^{sr} \sum_v^G B^{rv} \sum_t^G Y^{vt} + \hat{V}^s L^{ss} A^{sr} \sum_v^G B^{rv} \sum_{t \neq v}^G A^{vt} X^t$$
$$+ \hat{V}^s L^{ss} A^{sr} \sum_v^G B^{rv} A^{vv} X^v \quad (33)$$

The first term is that V_GVC^{sr} accounts as intermediate exports when it cross country from s to r for production at the first time, equals domestic value-added of source country s embodied in its intermediate exports to partner Country r to produce final products for domestic consumption or exports; the second term is that V_GVC^{sr} accounts as intermediate exports the second time embodied in country r's intermediate exports to all countries; the last term is that V_GVC^{sr} account as intermediate inputs in all countries' domestic inter-sectorial production. Therefore, the sum of the first and second terms equals the total amount of V_GVC^{sr} has been accounted as cross border intermediate exports and can be expressed as

$$Ev\_GVC^{sr} = \hat{V}^s L^{ss} A^{sr} \sum_v^G B^{rv} \sum_t^G Y^{vt} + \hat{V}^s L^{ss} A^{sr} \sum_v^G B^{rv} \sum_{t \neq v}^G A^{vt} X^t$$

$= \hat{V}^s L^{ss} A^{sr} X^r + \hat{V}^s L^{ss} A^{sr} \sum_v^G B^{rv} \sum_{t \neq v}^G A^{vt} X^t$ (34a)

Aggregate equation (34a) over all trading partner r, we obtain

$Ev\_GVC^s = \hat{V}^s L^{ss} \sum_{r \neq s}^G A^{sr} X^r + \hat{V}^s L^{ss} \sum_{r \neq s}^G A^{sr} \sum_v^G B^{rv} \sum_{t \neq v}^G A^{vt} X^t$

$= \hat{V}^s L^{ss} \sum_{r \neq s}^G A^{sr} X^r + \hat{V}^s \sum_v^G B^{sv} \sum_{t \neq v}^G A^{vt} X^t - \hat{V}^s L^{ss} \sum_{r \neq s}^G A^{sr} X^r$

$= \hat{V}^s \sum_v^G B^{sv} \sum_{t \neq v}^G A^{vt} X^t$ (34b)

The last term in the right hand of equation (34b) is total intermediate exports that induced by domestic value added in intermediate exports of country s. Further aggregate over all source country s, we obtain total global intermediate exports.

Ev_GVC can be further decomposed into gross exports induced by shallow V_GVC and deep V_GVC production activities in country s as follows:

$Ev\_GVC\_shallow^s = \hat{V}^s L^{ss} \sum_{r \neq s}^G A^{sr} L^{rr} Y^{rr}$ (35a)

$Ev\_GVC\_deep^s = \hat{V}^s \sum_v^G B^{sv} \sum_{t \neq v}^G A^{vt} X^t - \hat{V}^s L^{ss} \sum_{r \neq s}^G A^{sr} L^{rr} Y^{rr}$

$= \hat{V}^s \sum_v^G B^{sv} \sum_{t \neq v}^G A^{vt} \sum_u^G B^{tu} \sum_r^G Y^{ur} - \hat{V}^s L^{ss} \sum_{r \neq s}^G A^{sr} L^{rr} Y^{rr}$ (35b)

Similarly, aggregate the last term in equation (33) over all trading partners, we obtain the total amount of intermediate inputs that V_GVC^{sr}has been accounted for after it cross national borders and used in domestic production within all countries involved in the global value chain:

$Xvf\_GVC^s = \hat{V}^s L^{ss} \sum_{r \neq s}^G A^{sr} \sum_v^G B^{rv} A^{vv} X^v$

$= \hat{V}^s \sum_v^G B^{sv} A^{vv} X^v - \hat{V}^s L^{ss} A^{ss} X^s$ (36)

Xvf_GVC^s can also be further decomposed into gross outputs induced by shallow V_GVC and deep V_GVC of country s.

$Xvf\_GVC\_shallow^s = \hat{V}^s L^{ss} \sum_{r \neq s}^G A^{sr} (L^{rr} - I) L^{rr} Y^{rr}$ (36a)

$Xvf\_GVC\_deep^s = \hat{V}^s L^{ss} \sum_{r \neq s}^G A^{sr} [\sum_v^G B^{rv} A^{vv} X^v - (L^{rr} - I) L^{rr} Y^{rr}]$

$= \hat{V}^s L^{ss} \sum_{r \neq s}^G A^{sr} [\sum_v^G B^{rv} A^{vv} \sum_u^G B^{vu} \sum_t^G Y^{ut} - (L^{rr} - I) L^{rr} Y^{rr}]$ (36b)

Divide equations (34b) and (36) by V_GVC^s, we decompose international production length of global value chain into 2 part: (1) the average number of border crossing for production activities; (2) the average domestic production length of V_GVC^s within all countries involved in the GVCs after V_GVC^s leaving the source country. Adding up the average domestic production length of V_GVC, equation (24), we decompose the total average production length of V_GVC into three portions:

$$PLv\_GVC^s = PLvd\_GVC^s + PLvi\_GVC^s$$

$$= PLvd\_GVC^s + CBv\_GVC^s + PLvf\_GVC^s$$

$$= \frac{Xvd\_GVC^s}{V\_GVC^s} + \frac{Ev\_GVC^s}{V\_GVC^s} + \frac{Xvf\_GVC^s}{V\_GVC^s} \quad (37)$$

Similarly, following the same logic of equations (33) and (34), we can decompose Xyi_GVC^s into two parts: (1) intermediate imports induced by final goods and services production in country s, Ey_GVC^s; and (2) intermediate inputs used within all countries involved in GVCs that induced by final goods and services production in country s, Xyf_GVC^s. In mathematical notations:

$$Ey\_GVC^s = \sum_{t \neq s}^{G} A^{ts} L^{ss} \sum_{v}^{G} \hat{Y}^{sv} + \mu \sum_{r}^{G} \sum_{u \neq r}^{G} A^{ru} \sum_{t \neq s}^{G} B^{ut} A^{ts} L^{ss} \sum_{v}^{G} \hat{Y}^{sv}$$

$$= \sum_{t}^{G} V^t \sum_{u \neq r}^{G} B^{tr} A^{ru} B^{us} \sum_{v}^{G} \hat{Y}^{sv} \quad (38)$$

$$Xyf\_GVC^s = \mu \sum_{u}^{G} A^{uu} \sum_{t \neq s}^{G} B^{ut} A^{ts} L^{ss} \sum_{v}^{G} \hat{Y}^{sv} \quad (39)$$

Both of them can be further decomposed according to shallow Y_GVC and deep Y_GVC of country s.

$$Ey\_GVC\_shallow^s = \sum_{r \neq s}^{G} V^r L^{rr} A^{rs} L^{ss} \sum_{v}^{G} \hat{Y}^{sv} \quad (38a)$$

$$Ey\_GVC\_deep^s = [\sum_{t}^{G} V^t \sum_{u \neq r}^{G} B^{tr} A^{ru} B^{us} - \sum_{r \neq s}^{G} V^r L^{rr} A^{rs} L^{ss}] \sum_{v}^{G} \hat{Y}^{sv} \quad (38b)$$

$$Xyf\_GVC\_shallow^s = \sum_{r \neq s}^{G} V^r L^{rr} A^{rs} (L^{ss} - I) L^{ss} \sum_{v}^{G} \hat{Y}^{sv} \quad (39a)$$

$$Xyf\_GVC\_deep^s = \mu \sum_{u}^{G} A^{uu} \sum_{t \neq s}^{G} B^{ut} A^{ts} L^{ss} \sum_{v}^{G} \hat{Y}^{sv}$$

$$- \sum_{r \neq s}^{G} V^r L^{rr} A^{rs} (L^{ss} - I) L^{ss} \sum_{v}^{G} \hat{Y}^{sv} \quad (39b)$$

Divide equations (38) and (39) by Y_GVC^s, we can obtain (1) the average number of border crossing of intermediate imports used in country s's final product production activities; (2) the average domestic production length of Y_GVC^s within countries involved in the GVCs before entering the importing country. Adding up the average domestic production length of Y_GVC, we decompose the total average production length of Y_GVC into three portions

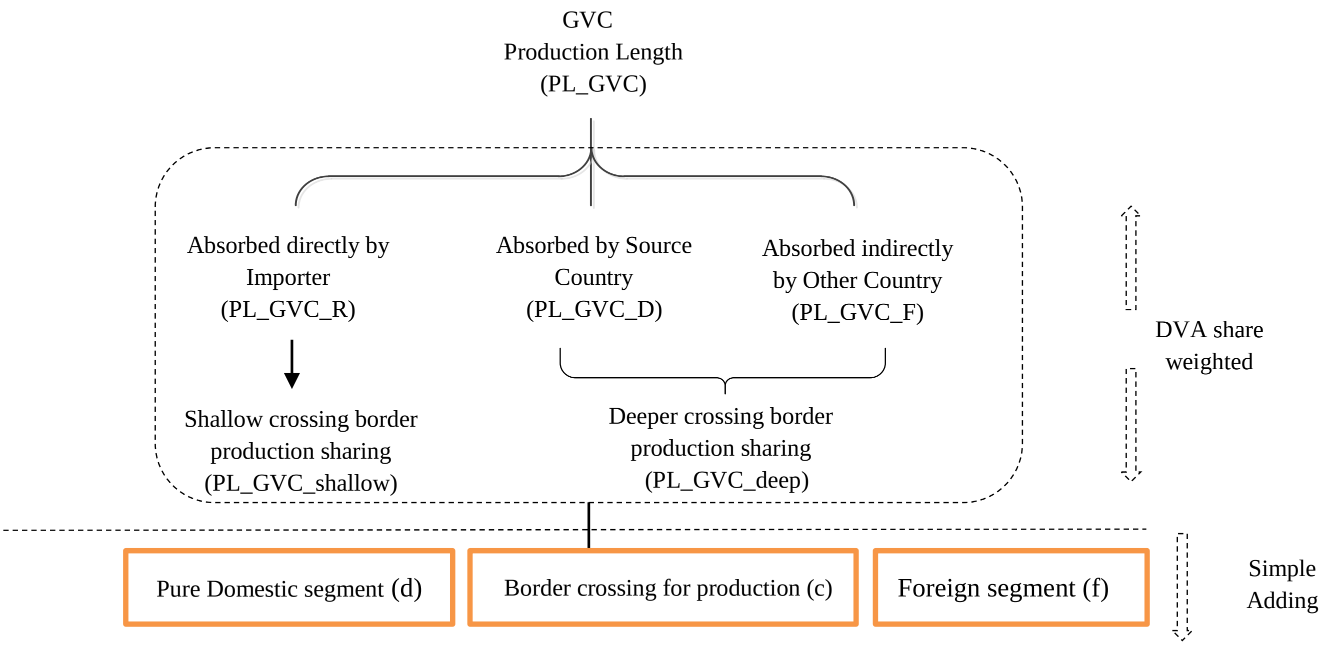
$$PLy\_GVC^s = PLyd\_GVC^s + PLyi\_GVC^s$$

$$= PLyd\_GVC^s + CBy\_GVC^s + PLyf\_GVC^s \quad (40)$$

$$= \frac{Xyd\_GVC^s}{Y\_GVC^s} + \frac{Ey\_GVC^s}{Y\_GVC^s} + \frac{Xyf\_GVC^s}{Y\_GVC^s}$$

The number of border crossing for production in the GVC production length index system expressed by equations (37) and (40), can also be represented as a tree diagram, as shown in Figure 4.

Figure 4 Number of border crossing and the GVC production length index system



Muradov (2016) has proposed a measure of the average number of border crossing. Different from the number of border cross measure defined in this paper, his measure includes not only border crossing for production, but also includes border crossing for consumption (it also accounts border crossing of final goods trade). Both measures are useful and can be used in different settings. Detailed derivation of equations (34) to (40) and the relationship between border crossing measure of Muradov (2016) and what is defined in this paper is provided in appendix H for interested readers.

Because global final demand always sums to global value-added, the forward and backward based production lengths are equal to each other at the global level. However, they may not be equal to each other at the country or country/sector level due to international trade and cross border production activities. This naturally raises the question: What is the relation between production length measure and production line position? Whether we can use production length measure directly infer upstreamness or downstreamness of a country or a country/sector pair? Current literature is not clear on such important questions and often uses production length measures to infer production line position directly. This is the topic we will address in the next section.

2.4 From production length to production line positions

As we have defined GVC related production and trade activities earlier, it is easy to see that a GVC production line not only has a starting and an ending stage, it also involves at least one and often many additional middle stages because value-added in global production chains needs to have production activities cross national borders. Therefore, GVC position index is a relative measure. If a country/sector pair participant GVC in a particular production stage, the less production stages occurring before the stage it engages, the relative more upstream the country/sector pair's position in the particular GVC. In the other hand, the less production stages occurring after the stage it engages, the relative more downstream the country/sector pair's position in the particular GVC. This indicates that a meaningful production line position index needs be able to measure the production stage concerned to both end of a particular global value chain.

Let us consider sector *i* of country *s* as such a middle stage in a global value chain. Based on equation (28), we can obtain the average production length forward as the ratio of GVC related domestic value-added and its induced gross output:

$$PLv\_GVC^s = \frac{Xv\_GVC^s}{V\_GVC^s} \tag{41}$$

It measures the average production length of domestic value-added embodied in intermediate exports from its first use as primary inputs until it finally absorbed in final goods and services thus exist the production process.

Based on equation (32), we can obtain the average production length backward as the ratio of GVC related foreign value-added and its induced gross output：

$$PLy\_GVC^s = \frac{Xy\_GVC^s}{Y\_GVC^s} \tag{42}$$

It measures the average production length of foreign value-added embodied in intermediate imports from their first use as primary inputs until their absorption in Country s's production of final products (for domestic use and exports).

As a special production node in the global production network, the longer that sector *i* of Country *s'* s forward linkage based production length is, the upstream the country/sector is located; the longer that sector *i* of Country *s's* backward linkage based production linkage is, the downstream the country/sector is located. In other words, the upstream the country/sector located, the longer its forward linkage based production length is, and the shorter its backward linkage based production is. Therefore, its average production line position in the global value chain can be defined as the ratio of the two production length:

$$GVCPs^s = \frac{PLv\_GVC^s}{[PLy\_GVC^s]'} \tag{43}$$

The larger the index, the more upstream is the country/sector pair. Equation (43) indicates that the production line positon index is closely related to the measure of production length, but the production length measure may not directly imply production line position. Only through aggregation, considering both forward and backward linkage based production length measures of a particular country/sector pair, by first determining its "distance" to both the starting and ending stages of all related production lines, the relative "upstreamness" or "downstreamness" in global production for a particular country/sector pair can be correctly determined. Most importantly, under definition of (43), the upstreamness and downstreamness of a given country/sector pair are really the same, thus overcoming the inconsistency of the production position indexes widely used in current literature, such as the N* and D* indexes proposed by Fally (2012) and the Down measure proposed by Atras and Chor (2013). In addition, such a GVC position index has a nice numerical property: it is distributed around one because at global aggregation, the forward and backward linkage based GVC production lengths are the same so at global level, this index equals to one.

The inconsistence of using forward and backward linkage based production length measures to infer production line position also recognized by others in recent literature.

For example, Antras et al. (2016) has defined a “upstreamness” index between any two industry pair based on “average propagation lengths" (APL) measure proposed by Dietzenbacher et al. (2005,2008), which is also invariant to whether one adopts a forward or backward linkage perspective when computing the average number of stages between a pair of industries. Escaith and Inomata (2016) have proposed similar ideas that use the ratio of forward and backward linkage based APLs to identify the relative position of economies within regional and global supply chains, and applied such measure to study the changes in relative positions of East Asian economies between 1985 and 2005. However, the production length index system we defined in this paper are different from APL measure defined in the literature in several important ways as we discuss below.

APL is formulated as the average number of production stages that it takes an exogenous change in one sector to affect the value of production in another sector, using the share of impact at each stage as weight. Based on Erik et.al (2005, 2008), APL can be defined as

$$APL = \frac{G(G-I)}{G-I} = \frac{B(B-I)}{B-I} \tag{44}$$

And the APL from sector i to sector j can be expressed as

$$apl_{ij} = \frac{\sum_k^n g_{ik}g_{kj}-g_{ij}}{g_{ij}-\delta_{ij}} = \frac{\sum_k^n b_{ik}b_{kj}-b_{ij}}{b_{ij}-\delta_{ij}} \tag{45}$$

Compare equations (45) with equation (10), we can see clear difference between APL and average production length we defined in this paper. First, the diagonal elements of Goash/Leontief inverse are subtracted for APL in order to take out initial cost shock/demand injection, because such exogenous changes do not depend on the economy’s industrial linkage and hence is not relevant to how long the “distance” between two industries. While the diagonal elements of Goash/Leontief inverse need to be kept for average production length, because the direct value-added created by primary factor inputs in the first stage are matters for average production length. Without take it into account, the production line is not complete. Second，the economic interpretation of the two measures are different. Production length measures the average times of value-added created from primary factors from a particular industry are counted as gross output along a production chain until it is embodied in final products, thus exit the production process. It is the footprint of value-added created from a particular country/sector pair in the whole economy, needs measure both the sector value-added and its induced gross output. APL is defined as the average number of stages that an exogenous impulse starting in one industry has to go through before it has impacts on another industry, measuring the average distance of inter-industrial linkages between two industries. It focuses on propagation transmission of gross output among industries only and has no relation with the size of value-added in the economy. Finally, they are computed differently. Production length is the ratio of gross output to related value-added or final products. Its denominator is value-added or final products generated from a value chain, its nominator is gross output of the value chain induced (driven) by the corresponding value-added (final products). While APL can be computed by Ghosh or Leontief inverse alone without involve sector value-added. Both measures are useful depend on the research question at hand. However, the numerical results of production length are relative robust. For example, the total production length will not change as number of sector classification increase as long as the total gross output and GDP keep constant, while the numerical estimates of APL will change as number of sector classification changes.[13] In addition, production length can be further decomposed into different segments according to different cross country production sharing arrangement based on the decomposition of sector GDP or final goods production, thus allow us define GVC position index rather than total production line position first time in the literature. More details difference between production length and APL in mathematical and their aggregation property are provided in Appendix J.

3. Estimation Results

Applying the GVC participation, length and position measures developed in the previous section to WIOD data, a set of indexes can be computed and used to

[13] In Appendix I，we also provide a numerical example to show such difference between Production length and APL.

quantitatively describe the multi-dimensional structures and the evolving trend of various GVCs for 41 countries and 34 industries over 1995–2011. Since all the indexes can be computed at both the most aggregated "world" and the more disaggregated "bilateral-sector" levels, we obtain a large amount of numerical results. To illustrate the computation outcomes in a manageable manner, we first report a series of examples at various disaggregated levels to highlight the stylized facts based on our new GVC index system and demonstrate their advantages compared to the existing indexes in the literature, we then conduct econometric analysis on the role of GVCs in the economic shocks brought by the recent global financial crisis as a more comprehensive application of these newly developed GVC measures.

3.1 Decomposition of production activities

As discussed in section 2.1, a country-sector's production activities (value added generation or final goods and service production) can be decomposed into four parts: Pure domestic, Ricardian trade, shallow and deep GVC related cross country production sharing. The last part (Deep GVC) can be further divided into two components according to the value-added is finally consumed in home (GVC_D) or abroad (GVC_F). Three stylized facts in global production activities can be observed in our decomposition results at the global level:

First, the pure domestic activities still account for the largest portion of production activities, but its relative importance is decreasing over time (Figure 5); Second, among the 3 parts related to international trade, the relative importance of traditional Ricardian type trade is increased more slowly than GVC related activities, although such general trends have been temporarily interrupt by the 2008-09 global financial crisis (Figure 5); Finally, among GVC related production activities, the percentage of factor content embodied in intermediate trade flows cross national border only once (shallow GVCs) is higher than that of Deep GVC activities, but its relative importance is diminishing over time before the year 2009, while domestic factor content exported via deep production sharing activities has been increased dramatically, although such a trend was also interrupted temporarily by the Global Financial Crisis (Figure 6). The three facts also can be find at country level decomposition results as reported in tables 2a (forward linkage based) and 2b (backward linkage based). For example, as shown in column (8) in both tables, the pure domestic production activities were declining overtime, especially for Emerging economies. In GVC related production activities, the share of shallow GVC(column 9) was increasing in most economies，although the increase is slower compare to Deep GVC activities(column 10), which has increased in all countries during the sample period, indicating all economies have deepen their integration with global value chains.

Figure 5 Decomposition at the Global Level

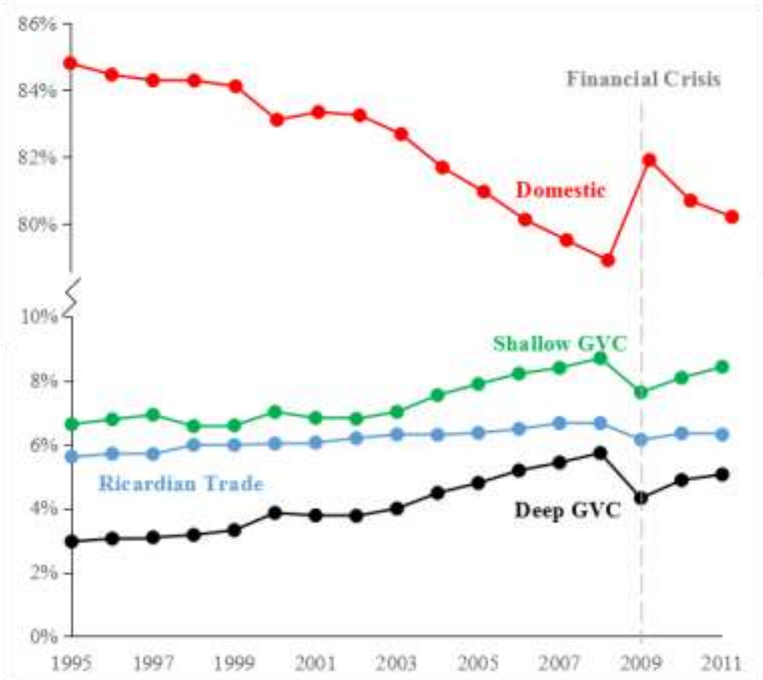


Figure 6 The percentage of Shallow GVC part in cross country production sharing activities

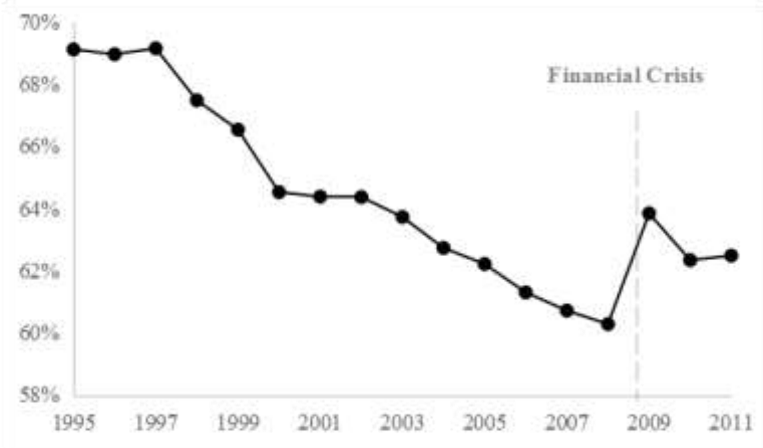


Table 2a Decomposition of GDP Growth at country level

Economies	GDP（VA）			Change of VA in 1995-2011 due to Change in				Change of Percentage Share in VA, 2011 over 1995			
	1995	2011	2011 over 1995	Domestic	Ricardian Trade	Shallow GVC	Deep GVC	Domestic	Ricardian Trade	Shallow GVC	Deep GVC
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Mature	**22776.2**	**41305.4**	**18529.1**	**14262.0**	**1239.0**	**1759.8**	**1268.3**	**-3.7%**	**0.5%**	**1.5%**	**1.8%**
USA	7449.8	15161.2	7711.4	6844.2	243.7	372.6	250.9	-1.0%	0.1%	0.3%	0.6%
JPN	5252.4	5896.2	643.8	355.7	81.8	103.0	103.3	-3.9%	1.0%	1.3%	1.6%
DEU	2390.3	3488.7	1098.4	427.7	244.2	240.6	185.9	-13.0%	4.4%	4.6%	4.0%
FRA	1468.0	2676.7	1208.7	985.6	87.9	72.7	62.6	-0.1%	-0.2%	-0.5%	0.7%
GBR	1104.5	2335.6	1231.1	926.8	75.3	121.8	107.2	-1.4%	-1.2%	0.4%	2.2%
ITA	1073.9	2098.9	1025.0	805.6	89.6	67.5	62.3	-0.7%	-0.5%	-0.1%	1.4%
CAN	569.5	1665.5	1096.0	842.8	58.2	136.4	58.6	3.1%	-2.3%	-1.8%	1.0%
ESP	573.5	1447.4	873.9	689.9	67.2	69.2	47.6	-3.6%	0.3%	1.6%	1.7%
AUS	369.1	1439.6	1070.5	853.9	23.1	124.2	69.3	-2.0%	-1.5%	1.2%	2.3%
KOR	514.6	1084.8	570.2	318.4	77.3	105.7	68.8	-11.7%	2.9%	4.7%	4.1%
NLD	402.2	814.2	412.0	237.8	50.9	62.7	60.7	-2.7%	-1.2%	0.4%	3.5%
SWE	238.4	509.5	271.1	173.4	25.1	43.6	29.0	-3.2%	-0.9%	1.9%	2.2%
BEL	272.6	497.0	224.4	133.0	23.4	37.0	31.0	-0.5%	-1.9%	0.6%	1.9%
Emerging	**3282.3**	**17273.2**	**13991.0**	**10929.8**	**1096.9**	**1176.5**	**787.8**	**-4.8%**	**1.5%**	**1.0%**	**2.3%**
CHN	734.4	7386.9	6652.6	5178.5	659.4	502.3	312.3	-2.6%	-0.5%	1.1%	2.0%
BRA	724.8	2251.6	1526.8	1319.0	57.3	93.9	56.5	-4.4%	1.4%	1.5%	1.5%
IND	371.1	1885.4	1514.3	1287.6	103.9	79.5	43.3	-3.9%	2.1%	0.5%	1.2%
RUS	324.1	1701.8	1377.7	999.0	28.0	185.0	165.8	-3.3%	-1.2%	0.0%	4.5%
MEX	318.7	1146.8	828.1	648.2	59.9	82.0	37.9	-2.2%	0.6%	0.3%	1.3%
IDN	243.7	855.8	612.0	471.2	20.8	76.8	43.2	-3.0%	-1.3%	1.6%	2.8%
TUR	223.7	733.6	509.9	412.3	36.0	35.6	26.0	-5.7%	0.5%	2.7%	2.5%

Note: Twenty Largest Economies ranked on GDP are included. Following the classification used in Timmer et al.(2012), Mature economies include Australia, Canada, Japan, South Korea, Taiwan, US, and 15 countries that joined the EU before 2004. Emerging economies include Brazil, China, Russia, India, Indonesia, Mexico and Turkey and 12 countries that joined the EU in 2004.

Table 2b Decomposition of Final goods and services production growth at country level

Economy	Final Goods Output（Y）			Change of Y in 1995-2011 due to Change in				Change of Percentage Share in Y, 2011 over 1995			
	1995	2011	2011 over 1995	Domestic	Ricardian Trade	Shallow GVC	Deep GVC	Domestic	Ricardian Trade	Shallow GVC	Deep GVC
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Mature	**22549.6**	**41177.4**	**18627.8**	**14262.0**	**1239.0**	**1836.8**	**1290.0**	**-4.3%**	**0.4%**	**2.0%**	**1.9%**
USA	7389.6	15334.2	7944.5	6844.2	243.7	597.3	259.4	-2.8%	0.1%	1.7%	1.0%
JPN	5174.6	5898.3	723.7	355.7	81.8	206.5	79.7	-5.4%	1.0%	3.1%	1.2%
DEU	2339.1	3328.8	989.6	427.7	244.2	110.6	207.1	-11.5%	4.8%	1.5%	5.2%
FRA	1456.7	2731.7	1275.0	985.6	87.9	94.5	107.1	-2.4%	-0.4%	0.5%	2.2%
GBR	1088.8	2278.8	1190.0	926.8	75.3	115.9	72.0	-0.6%	-1.1%	0.9%	0.9%
ITA	1087.5	2189.8	1102.3	805.6	89.6	126.7	80.4	-3.0%	-0.8%	2.1%	1.7%
CAN	540.0	1584.8	1044.9	842.8	58.2	100.2	43.7	3.0%	-2.4%	0.7%	-1.3%
ESP	594.1	1498.0	903.9	689.9	67.2	82.7	64.2	-3.4%	0.3%	1.0%	2.0%
AUS	356.7	1332.0	975.3	853.9	23.1	71.2	27.2	1.7%	-1.4%	-0.6%	0.3%
KOR	520.8	1102.3	581.5	318.4	77.3	105.2	80.6	-11.9%	2.8%	4.0%	5.1%
NLD	392.7	791.7	399.0	237.8	50.9	46.7	63.5	-2.5%	-1.2%	0.5%	3.2%
BEL	267.2	497.1	229.8	133.0	23.4	30.4	43.1	-1.8%	-2.2%	1.4%	2.5%
Emerging	**3287.5**	**17437.6**	**14150.1**	**10929.8**	**1096.9**	**1292.7**	**830.7**	**-5.4%**	**1.4%**	**1.7%**	**2.3%**
CHN	757.3	7725.0	6967.8	5178.5	659.4	713.7	416.1	-3.6%	-0.6%	2.3%	1.9%
BRA	721.1	2235.8	1514.7	1319.0	57.3	99.5	38.9	-4.3%	1.4%	1.8%	1.1%
IND	378.2	1977.4	1599.2	1287.6	103.9	129.3	78.4	-6.1%	1.9%	1.7%	2.5%
RUS	281.1	1397.2	1116.1	999.0	28.0	61.7	27.5	1.0%	-1.3%	-0.2%	0.5%
MEX	322.2	1160.5	838.3	648.2	59.9	65.1	65.0	-2.2%	0.6%	-0.4%	2.0%
IDN	243.7	828.2	584.5	471.2	20.8	62.1	30.3	-0.4%	-1.2%	0.3%	1.4%
TUR	235.3	755.3	520.0	412.3	36.0	41.9	29.7	-3.7%	0.6%	0.9%	2.2%
POL	130.0	505.4	375.4	233.9	47.9	48.2	45.4	-13.7%	3.3%	4.0%	6.4%

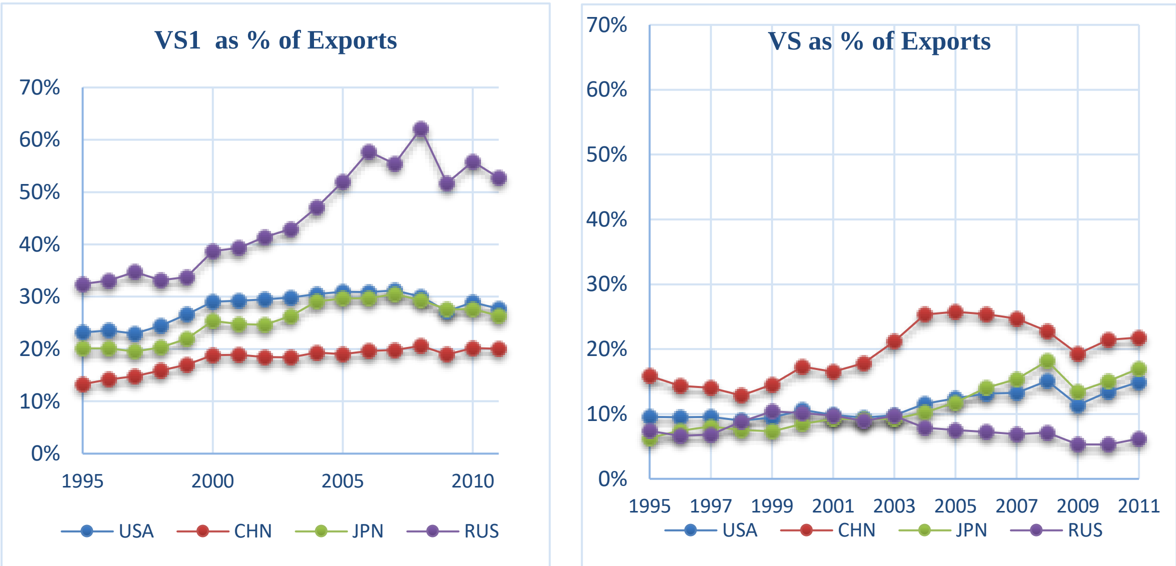
Note: Twenty Largest Economies ranked on final goods and service production are included. Following the classification used in Timmer et al.(2012), Mature economies include Australia, Canada, Japan, South Korea, Taiwan, US, and 15 countries that joined the EU before 2004. Emerging economies include Brazil, China, Russia, India, Indonesia, Mexico and Turkey and 12 countries that joined the EU in 2004.

3.2 Participation index

3.2.1 Traditional indexes

Hummels et al. (2001)'s Vertical Specification indexes, the share of VS and VS1 in gross exports, are widely used in the literature to measure the extent of GVC participation since they were first proposed by Koopman et al. (2010). Take the top 3 countries in terms of GDP (United Statas, China and Japan) and a typical energy-exporting country (Russia) as examples, the VS and VS1 ratios shown in Figure 7 can provide us with useful information of GVC participation from at least two aspects: (1) Generally speaking, the degree of participation of all the four countries increase over the 17 year period; (2) The upward trend of Vertical Specification has been temporarily interrupted by the global financial crisis.

Figure 7 VS and VS1 ratios, 1995 to 2011



However, there are major shortcomings in those traditional participation indexes:

1) Using gross exports as the denominator. The share might be very high just because some sectors may have very little direct exports (e.g., Mining and Service). In such a case, the index value might become very large. In many empirical cases as we will show later, we may not be able to determine whether the index becoming larger is due to the large numerator or the small denominator (in math terms, the index goes to

infinity when the denominator goes to zero) and whether such index actually overestimates GVC participation for a country/sector pair.

2) The fundamental characters of GVCs is cross country production sharing activities, VS and VS1 only consider export related activities, exclude a large portion of production activities that satisfies domestic final demand through international production sharing.

3) Not able to distinguish shallow and deep GVC participation. The former only involve production sharing activities between the exporting and importing country, while the later measures more complex sequential production sharing activities across countries.

The GVC participation index proposed in this paper has overcome the above-mentioned shortcomings and is able to better measure the degree of GVC participation as the share of total value-added/final goods production for any country/sector pair and can be further decomposed into shallow and deep parts based on number of border crossing. Such detailed GVC participation measure provides better indexes that are needed to conduct GVC related empirical analysis.

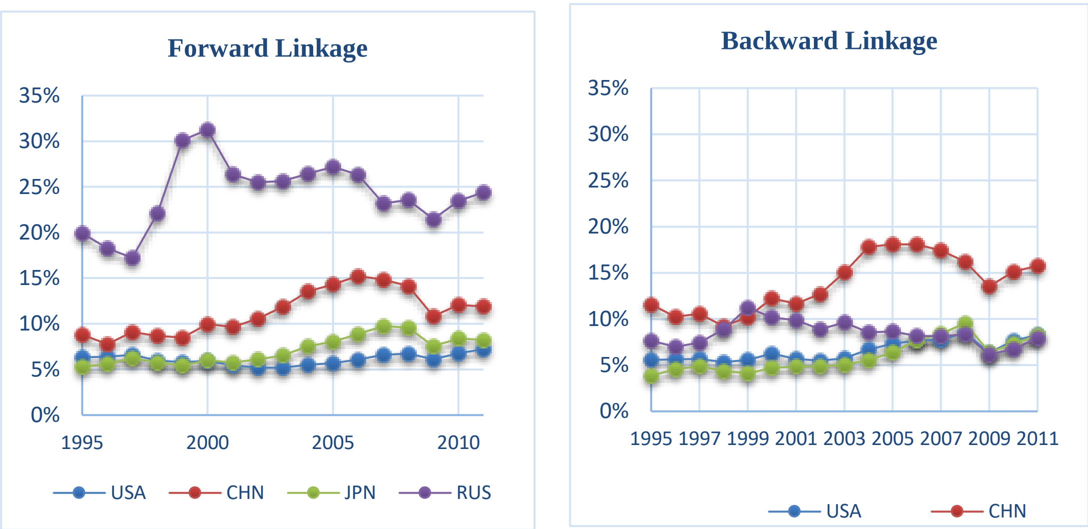
3.2.2 New indexes

The forward linkage based participation index proposed in this paper can be understood as “What is the percentage of production factors employed in a country-sector pair has been involved in cross country production sharing activities?” while the backward linkage based participation index can be understood as “What is the percentage of final products produced by a country-sector pair that comes from GVC related production and trade activities?”

(1) Country level

Continue using the US, China, Japan and Russia as examples, Figure 8 plots out the time trend of both forward/backward industrial linkage based participation indexes. The general patterns revealed are similar in both the traditional and the new indexes. For example, an upward trend GVC participation intensity for the four economies, and the negative impact of the global financial crisis on such trends can been clearly observed in both index. At country level, both indexes show that Russia as an important energy supply country, its forward industrial linkage based participation index is significantly higher than its backward industrial linkage based participation index, indicating Russian participant GVCs mainly from upstream. While China is opposite. As the "world factory", China's backward industrial linkage based index is significantly higher than its forward industrial linkage based index, indicating China participant GVCs relatively more from the downstream.

Figure 8 Forward/Backward Participation Indexes, 1995 to 2011



However, there are also clear difference between the new and old indexes. For instance, the traditional indexes show that there is an inconsistent time trend for Russia's GVC participation: there is an increasing trend for its forward participation but a decline trend for its backward participation. While our new indexes indicate there is a declined trend for Russia's GVC participation from both directions since 2000. Another interest difference is that the U.S. and Japan have a much higher forward participation intensity than that of China based on the traditional index, while our new index indicate the opposite. This is largely due to Chinese economy is more depend on trade than the US and Japan (China's exports to GDP ratio is much higher than the US and Japan), therefore, using gross exports as denominator, traditional index will overestimate GVC participation intensity for the US and Japan.

(2) Sectoral level

The intensity of GVC participation varies by sector. Table 3a and 3b reports both forward and backward linkage based GVC participation indexes by four major industrial groups (Agriculture, Mining, Manufacturing and services) and their changes over 17 years. We also report the top five sectors within manufacturing and services industrial group with fastest GVC participation rate growth. Among the four major industrial groups, Mining has the highest forward linkage based and lowest backward linkage based GVC participation rate (48% over 9%), consisting the industry's upstream position in global production network. Manufacturing has the highest backward linkage based participation rate (26.3%) and second highest forward linkage based index (25.7%), consist with the fact that the industry has been most deeply integrated into the global production network. As expected, services has the lowest GVC participation intensity, but its participation rate has grown faster than agriculture in recent years.

Table 3a GVC Participation Index at sectoral level (Forward Linkage)

Sector	GVCPt_f			Shallow			Deep		
	1995	2011	2011 over 1995	1995	2011	2011 Over 1995	1995	2011	2011 over 1995
Agriculture	**9.3%**	**10.8%**	**1.6%**	**7.2%**	**7.8%**	**0.6%**	**2.1%**	**3.0%**	**1.0%**
Mining	**39.3%**	**47.9%**	**8.6%**	**29.0%**	**30.1%**	**1.2%**	**10.3%**	**17.8%**	**7.5%**
Manufacturing	**19.5%**	**25.7%**	**6.1%**	**12.8%**	**15.2%**	**2.4%**	**6.7%**	**10.4%**	**3.7%**
Refined Petroleum	15.9%	27.8%	11.9%	11.6%	17.6%	6.1%	4.4%	10.1%	5.8%
Recycling	10.2%	19.8%	9.6%	7.3%	12.2%	4.9%	3.0%	7.6%	4.7%
Rubber and Plastics	24.0%	33.5%	9.5%	16.0%	20.2%	4.2%	8.0%	13.4%	5.3%
Basic Metals	29.2%	38.0%	8.8%	18.1%	21.1%	3.0%	11.1%	16.8%	5.7%
Electrical Equipment	26.5%	34.8%	8.3%	16.2%	19.1%	3.0%	10.4%	15.7%	5.3%
Service	**6.0%**	**8.3%**	**2.2%**	**4.3%**	**5.3%**	**1.0%**	**1.8%**	**3.0%**	**1.2%**
Electricity, Gas and Water	8.7%	13.5%	4.8%	5.9%	8.2%	2.3%	2.8%	5.3%	2.5%
Inland Transport	12.1%	16.3%	4.2%	8.7%	10.2%	1.5%	3.4%	6.1%	2.7%
Business Activities	13.4%	17.6%	4.2%	9.4%	11.3%	1.9%	4.0%	6.3%	2.3%
Air Transport	22.3%	25.8%	3.5%	17.1%	18.8%	1.7%	5.2%	7.0%	1.8%
Wholesale Trade	11.5%	14.6%	3.1%	8.1%	9.0%	0.9%	3.4%	5.6%	2.2%

Further distinguish “deep” and “shallow” participation, and analyze the time trend, we find that the increase of GVC participation intensity is mainly drive by the “deep” cross country production sharing activities. However, it is interesting to note that the backward “shallow” participation for service industry is grow faster than that of “deep “ participation, indicating that although on global average service production rely more foreign factor content than 17 years ago, increasingly more of these foreign factor content is directly coming from the direct partner country than that come from third countries.

Table 3b GVC Participation Index at sectoral level (Backward Linkage)

Sector	GVCPt_b			Shallow			Deep		
	1995	2011	2011 over 1995	1995	2011	2011 over 1995	1995	2011	2011 Over 1995
Agriculture	8.8%	10.0%	1.2%	6.7%	6.9%	0.2%	2.2%	3.1%	0.9%
Mining	8.9%	8.9%	0.0%	7.0%	6.8%	-0.2%	1.8%	2.1%	0.2%
Manufacturing	18.8%	26.3%	7.5%	10.3%	11.8%	1.5%	8.5%	14.5%	6.0%
Refined Petroleum	29.5%	45.9%	16.4%	22.0%	27.5%	5.4%	7.5%	18.4%	11.0%
Basic Metals	19.7%	29.6%	9.8%	12.8%	17.3%	4.5%	6.9%	12.3%	5.4%
Electrical Equipment	21.5%	30.8%	9.2%	8.1%	9.6%	1.5%	13.4%	21.1%	7.7%
Machinery	17.1%	26.1%	9.0%	8.1%	10.9%	2.8%	9.0%	15.1%	6.2%
Transport Equipment	24.3%	33.1%	8.8%	12.6%	13.7%	1.1%	11.7%	19.4%	7.6%
Service	**6.7%**	**10.0%**	**3.3%**	**5.4%**	**7.5%**	**2.1%**	**1.2%**	**2.5%**	**1.2%**
Electricity, Gas and Water	10.7%	20.3%	9.6%	9.1%	16.8%	7.7%	1.6%	3.6%	2.0%
Air Transport	12.4%	22.1%	9.6%	8.4%	12.8%	4.5%	4.1%	9.2%	5.2%
Inland Transport	9.1%	14.6%	5.5%	7.5%	10.6%	3.2%	1.7%	4.0%	2.3%
Construction	12.5%	18.0%	5.5%	10.2%	13.6%	3.4%	2.2%	4.3%	2.1%
Water Transport	18.0%	23.1%	5.1%	7.8%	9.2%	1.3%	10.2%	14.0%	3.8%

(3) Country-Sector level

Table 4 lists the forward and backward linkage based participation indexes in year 2011 for 3 sectors in 8 countries, which implies the characteristics of different countries/sector pair when participating in GVC production.

For example, in the agriculture sector of Finland and Brazil, the forward linkage based participation rate is significantly higher than that in other countries. This numerical result is in line with the facts that forestry is the dominant industry in Finland

and animal husbandry and agriculture are pillar industries in Brazil. Similarly, since Russia is the giant in energy, its mining sector's forward linkage based participation rate is as high as 75.6%. In contrast, due to the energy shortage, Japan's mining sector has the highest backward linkage based participation rate (50.5%).

Regarding the typical manufacturing industry, "transportation equipment", Germany is the global manufacturing power, so its forward and backward linkage based participation rates are both higher than that of other countries. With a high forward linkage based participation rate, a high proportion of production factors employed by German transportation equipment sector has engaged into the network of Global Value Chains directly or indirectly. With a higher backward linkage based participation rate, a high proportion of components and parts in the final products produced by Germany are produced by other countries in GVCs. In contrast, the way United States participating global production is quite different from Germany with a much higher backward linkage based participation rate (57.9 over 9), indicating US transportation equipment sector use more intermediate inputs than components and parts it exports.

(4) Dynamic Pattern: the case of transportation equipment sectors (WIOD sector 13)

Transportation equipment industries (Auto is one of its major sector) is a typical industry of global value chains. Due to complexity of its production process, input demand and cost structure varies widely in different production stages and locations. To minimize cost, Auto firms need constantly optimize its production process and cost structure based on comparative advantage of different geographic locations. Therefore, the intensity of GVC participation at each country/sector pair will change over time as comparative advantage of each production location and production cost changes.

Table 4 Sectoral Level Participation Index, Forward/Backward Linkage

	Forward Linkage Based Participation Index (GVCPt_f)		
	Agriculture	**Mining**	**Transport Equipment**
BRA	**23.2%**	40.6%	8.1%
CHN	6.6%	17.1%	14.0%
DEU	18.0%	58.8%	**30.1%**
FIN	**33.5%**	47.9%	28.9%
IND	6.4%	26.1%	12.6%
JPN	2.6%	27.2%	25.4%
RUS	5.4%	**75.6%**	11.1%
USA	11.4%	50.4%	**9.0%**
	Backward Linkage Based Participation Index (GVCPt_b)		
	Agriculture	**Mining**	**Transport Equipment**
BRA	8.6%	11.9%	19.7%
CHN	7.5%	14.9%	22.9%
DEU	19.9%	18.6%	**36.8%**
FIN	17.5%	24.6%	33.7%
IND	2.9%	4.9%	19.9%
JPN	10.7%	**50.5%**	16.0%
RUS	10.3%	4.3%	33.3%
USA	9.9%	6.7%	**57.9%**

BRA=Brazil; CHN=China; DEU=Germany; FIN=Finland; IDN=Indonesia; IND=India; RUS=Russia; USA=United States

We selected 21 economies from the 40 WIOD covered countries according to the rank of their gross export value of transportation equipment and share of value-added in GDP (highest 15.in each criteria mix together). We separate the 21 economies into 4 groups in table 5 based on their forward or backward GVC participation, and increase or decrease of their participation intensity. Such grouping reflects the different pattern of different economy engage in global transportation equipment production network. There are four observations from table 5:

First, six European countries (Germany, French、UK, Poland, Check and Hungry) and two East Asian countries(Japan, Korea) have increased their participation in global auto production chains through both forward and backward linkage (they are both suppliers and demanders of parts and components), cross country production activities are booming in these economies; second, the United States, Austria and Slovak increased their backward linkage based GVC participation, but their GVC participation based on backward linkage declined, indicating these economies increased their offshoring of parts and components (the US and Austria), or assembling activities (Slovak); Third, China, Mexico and Indonesia, etc. increased their GVC participation from forward industrial linkage, aim to become global suppliers of auto parts, but reduced their activities in importing and assembling; finally, compare to all other economies, the auto GVC participation by Canada, Sweden and Brazil is in an decline trend.

Table 5 The Dynamics of GVC participation in transportation equipment industry

	GVCPt_f ↑	GVCPt_f ↓
GVCPt_b ↑	DEU FRA GBR POL CZE HUN KOR JPN	USA AUT SVK
GVCPt_b↓	CHN MEX IDN BEL ITA ESP ROU	CAN SWE BRA

3.2.3 Why do we need the new "GVC Participation Index"?

(1) Eliminate the sectoral level bias in traditional indexes

As mentioned previously, using gross exports as the denominator may lead to overvalue bias at the bilateral/sectoral level.

For comparison, we use both gross exports and sector GDP as the denominator to compute the forward linkage based participation index, VS1 as share of gross exports and GVC_Pr_f, respectively. As shown in Table 6, The VS1 share measure for 6 sectors (marked with gray background color) are substantially larger than 100%. These sectors have one thing in common: a great proportion of their value added is exported indirectly, which is embodied in other sectors' exports.

The overvaluation problem is more pronounced for utility and service sectors, as a large proportion of their value added is exported indirectly. We choose three typical sectors to illustrate this point. Two of them belong to the utility and service industries ("Electricity, Gas and Water" and "Retail Trade"), while the third one, "Leather and Footwear," is a typical "direct" exporting sector. Table 7 lists 15 largest countries ranking by GDP to show the comparison between traditional VS1 ratio and our forward linkage based GVC participation index. As we have expected, the overvaluation problem is more serious in the utility and service industries.

Table 6 Forward Linkage Participation Index for US sectors, 2011
Comparison between Traditional and New Measures

Sector	VS1	GVCPt_f	Sector	VS1	GVCPt_f
Agriculture	19.5%	11.4%	Sale of Vehicles and Fuel	224.7%	5.7%
Mining	47.0%	50.4%	Wholesale Trade	60.2%	9.3%
Food	4.0%	7.8%	Retail Trade	194.9%	5.0%
Textiles Products	0.8%	7.4%	Hotels and Restaurants	27.9%	6.4%
Leather and Footwear	4.3%	11.9%	Inland Transport	48.6%	12.5%
Wood Products	17.8%	27.0%	Water Transport	3536.1%	13.6%
Paper and Printing	42.3%	16.7%	Air Transport	51.9%	8.4%
Refined Petroleum	9.4%	47.1%	Other Transport	416.3%	5.8%
Chemical Products	13.6%	58.9%	Post and Telecommunications	67.6%	7.3%
Rubber and Plastics	35.3%	22.8%	Financial Intermediation	64.0%	10.5%
Other Non-Metal	16.3%	13.3%	Real Estate	31.6%	5.9%
Basic Metals	24.2%	40.7%	Business Activities	38.6%	18.2%
Machinery	8.0%	23.2%	Public Admin	60.7%	0.9%
Electrical Equipment	13.9%	34.2%	Education	50.5%	0.7%
Transport Equipment	9.5%	9.0%	Health and Social Work	21.6%	0.9%
Recycling	7.6%	34.4%	Other Services	41.5%	8.1%
Electricity, Gas and Water	152.2%	17.9%	Private Households	107.5%	2.1%
Construction	34.4%	0.9%			

Table 7 Comparison between Traditional and New Participation Indexes for Three Typical Sectors

	Electricity, Gas and Water		Retail Trade		Leather and Footwear	
	VS1	**GVCPt_f**	**VS1**	**GVCPt_f**	**VS1**	**GVCPt_f**
USA	701.1%	4.4%	3647.0%	0.7%	4.7%	5.0%
CHN	797.1%	14.5%	34.7%	10.5%	3.1%	10.2%
JPN	816.3%	8.2%	70.9%	2.2%	26.1%	7.2%
DEU	65.6%	21.4%	985.0%	14.4%	6.1%	24.6%
FRA	90.4%	11.9%	2.8×10^7%	9.9%	2.2%	10.2%
GBR	368.4%	9.2%	448.8%	8.5%	6.9%	19.9%
BRA	152.6%	7.7%	282.8%	6.0%	16.3%	17.3%
ITA	401.6%	11.4%	50.6%	10.5%	4.7%	16.8%
IND	1.3×10^5%	8.4%	1184.5%	5.8%	8.0%	11.8%
RUS	363.2%	26.9%	48.2%	10.4%	49.3%	10.6%
CAN	69.1%	20.2%	151.3%	14.1%	3.3%	23.8%
ESP	253.0%	13.6%	318.2%	8.8%	3.5%	13.5%
AUS	935.5%	10.5%	83.9%	8.0%	11.3%	12.9%
MEX	444.4%	9.6%	52.6%	14.0%	10.9%	14.1%
KOR	2268.8%	21.9%	74.2%	8.6%	15.9%	25.0%

USA=United States; CHN=China; JPN=Japan; DEU=Germany; FRA=France; GBR=United Kingdom; BRA=Brazil; ITA=Italy; IND=India; RUS=Russia; CAN=Canada; ESP=Spain; AUS=Australia; MEX=Mexico; KOR=Korea;

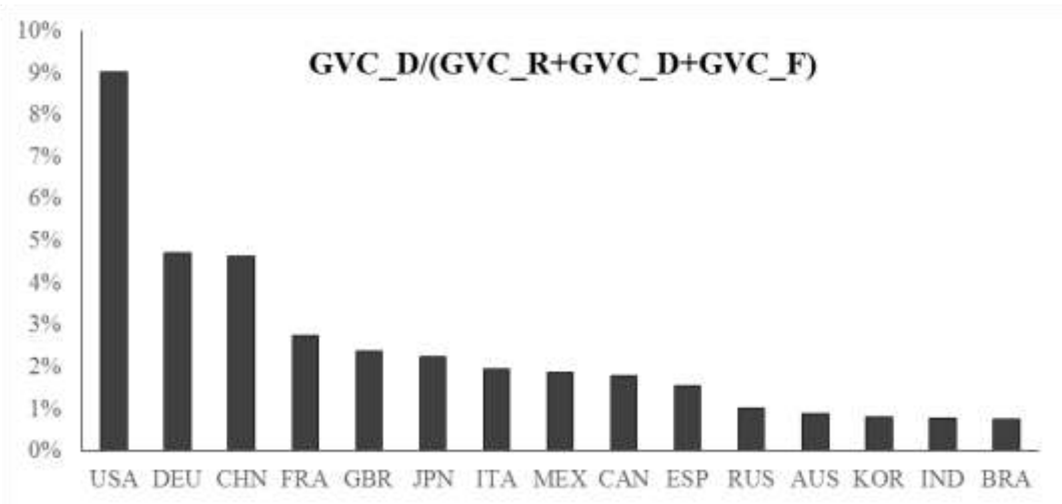
(2) Differentiate "shallow" and "deep" GVC participation

As shown in our decomposition framework, the domestic value added in gross intermediate exports of a country can be decomposed into two major parts: DVA crossing the national border only once – GVC_R, representing the type of cross border specialization that is relatively shallow; DVA cross border two or more times – GVC_D and GVC_F, representing the type of cross border specialization that is deeper. In our newly defined participation indexes, both way a country/sector pair to participant GVC can be identified and quantified.

We have already show in section 3.1 that the "shallow" and "deep" parts of GVC participation are different in size and the trend of change. The shallow part takes a relatively large proportion, but its relative importance is diminishing over time for almost all countries in the sample. Instead, the domestic value added exported via deep production sharing activities is increasing dramatically.

Besides that, the relative sizes of GVC_R, GVC_D, and GVC_F may reflect the differences of roles in the GVCs for different countries. Taking 10 countries with largest GDP as examples, as shown in Figure 9, GVC_D, "re-imported and absorbed domestically," accounts for a substantially larger proportion in the US, followed by Germany and China, as the US and Germany are controlling both ends (design and sales) of the value chain, and China serves as the "world's factory" and the world's largest consumption market.

Figure 9 The share of Returned Value Added (GVC_D)



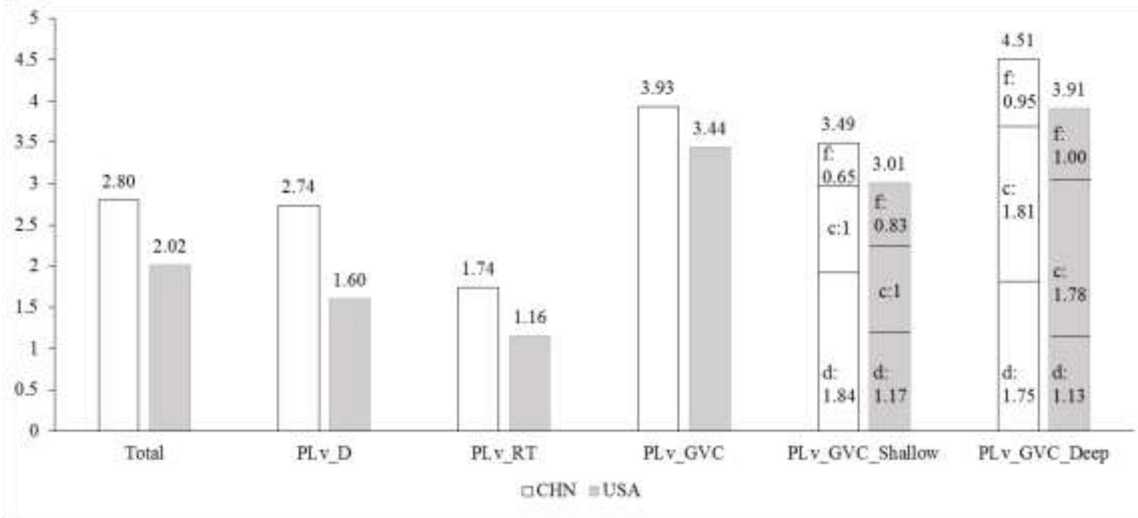
3.3 Production length index

3.3.1 Calculation results

To show the production length index structure proposed in this paper, we take the Electrical and Optical Equipment Sector as an example. Figure 10 reports the basic results for China and the US, at the "Country-Sector" level for 2011.

Figure 10 Production Length of Electrical and Optical Equipment Sector, 2011

Forward Linkage



Backward Linkage

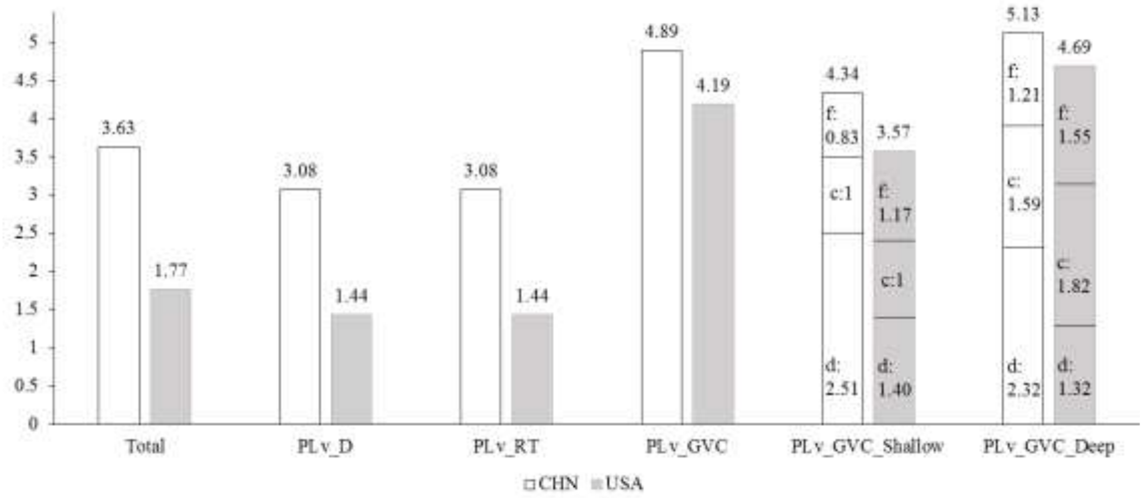


Figure 10 provide us with the following observations：

(1) The values of production length index are always higher for China than that for the US, which means the value added created by China (forward linkage) has to go through more steps before reaching its final uses, or China produced final products (backward linkage) has more stages in its upstream production process.

(2) Compared with the pure domestic and the traditional "Ricardian" type trade, value added created along the GVCs has the longest production length (PL_GVC). This

result is intuitively reasonable as more participants and production steps are involved in the GVC production process. When further divide GVC production into shallow and deep cross country production activities, the latter (PLv_GVC_Deep) has the longest production length. In such case, value added flows back to the global production network from the direct importing country, may further going through several production stages in third countries or return to the source country before they finally embodied in final products.

(3) The international portion of GVC production length (c + f) is always longer than the domestic portion (d), since it includes the average number of border crossing for production c and average production stages within other countries (f). This finding reflects the global increase in vertical specialization: the more fragmented is the production process, the more participants are involved, and the less is the portion of the whole production process allocated to each stage.

(4) Compare the three portions of length between shallow and deep GVCs, there is only one time border crossing for production in sallow GVC activities by definition, but also less than twice in deep GVC activities for both US and China, this indicates a large amount of intermediate imports used by the importing country is absorbed by exports of final products produced from the importing country, when these final products cross national border again to third countries, it is no longer a border crossing for production, thus reduce the size of average number of border cross for production. Compare the three portions of GVC length between US and China, China seems have a longer domestic portion than that of the US, indicating that China engage in more domestic production stages while the US tends to offshoring its production activities in the global electric and optical equipment production network.

Table 8a and 8b reports forward/backward linkage based production length in four aggregate industries: agriculture, mining, manufacture and services for 5 largest economies in terms of the highest GDP/final products output by industries.

Compare among industries, Agriculture, especially Mining, tend to have longer forward linkage based, shorter backward linkage based total and GVC production length, consistent to most products from these two sectors usually with longer distance to final consumers. Manufacture, as the industries dominate by various GVCs, its forward and backward linkage based production length often close to each other.

Compare among countries, China has the longest production length across most industries in both forward and backward linkage based measures. As the world factory and second largest world economy, more production stages of GVCs are take place within China, as measured by domestic portion of both shallow and deep GVC production activities listed in table 8a and 8b. Generally speaking, emerging economies have longer length in both total and GVC production than that of advance economies due to longer domestic portion of their production chain.

Compare different portion of the production length, we see a similar pattern as what shown in Figure 10: GVC production is significantly longer than pure domestic and traditional trade production in all country and industries, and deep GVC production activities is significantly longer that shallow GVC production activities.

Table 8a Forward-Linkage Production Length, Country-Sector Level, 2011

Sector	Country	Total	D	RT	GVC	Shallow_GVC	=	Domestic Portion	+	Border Crossing	+	Foreign Portion	Deep_GVC	=	Domestic Portion	+	Border Crossing	+	Foreign Portion
Agriculture	CHN	2.81	2.51	3.41	5.74	5.31		3.67		1		0.64	6.55		3.97		1.66		0.91
	USA	2.58	2.45	2.04	3.52	3.26		1.50		1		0.76	4.46		1.66		1.56		1.24
	BRA	2.30	1.92	2.12	3.48	3.29		1.49		1		0.79	4.06		1.50		1.49		1.07
	IDN	2.17	1.91	2.41	3.99	3.74		2.04		1		0.69	4.82		2.09		1.62		1.11
	IND	1.63	1.47	2.13	3.63	3.36		1.57		1		0.80	4.45		1.69		1.62		1.14
Manufacturing	CHN	3.07	2.86	2.39	4.53	4.10		2.45		1		0.66	5.19		2.43		1.77		0.99
	JPN	2.65	2.23	1.97	4.23	3.78		1.88		1		0.90	4.96		1.85		1.82		1.29
	DEU	2.31	1.60	1.30	3.50	3.03		1.28		1		0.76	4.08		1.27		1.83		0.98
	USA	2.16	1.88	1.39	3.65	3.23		1.44		1		0.80	4.25		1.39		1.78		1.08
	BRA	2.05	1.78	1.53	3.74	3.33		1.56		1		0.77	4.49		1.55		1.80		1.14
Mining	CHN	4.48	4.13	4.48	5.97	5.46		3.75		1		0.71	6.79		3.80		1.84		1.15
	AUS	3.99	2.53	2.88	4.47	4.00		1.22		1		1.77	5.33		1.22		1.95		2.16
	RUS	3.87	3.45	2.67	4.02	3.57		1.33		1		1.24	4.58		1.32		1.93		1.33
	IDN	3.68	2.59	2.72	4.44	3.89		1.41		1		1.48	5.39		1.40		1.98		2.02
	USA	2.49	2.14	2.58	4.39	3.93		2.00		1		0.93	5.17		1.91		1.92		1.34
Service	CHN	2.34	2.00	3.42	4.91	4.39		2.71		1		0.68	5.84		2.91		1.83		1.11
	DEU	1.96	1.51	2.53	4.43	3.91		2.19		1		0.72	5.29		2.36		1.85		1.08
	FRA	1.78	1.50	2.71	4.46	3.92		2.22		1		0.70	5.39		2.49		1.83		1.07
	JPN	1.70	1.53	2.52	4.68	4.30		2.54		1		0.76	5.32		2.54		1.70		1.08
	USA	1.69	1.55	2.30	4.08	3.62		1.86		1		0.76	4.94		1.91		1.83		1.20

Table 8b Backward-Linkage Production Length, Country-Sector Level, 2011

Sector	**Country**	**Total**	**D**	**RT**	**GVC**	**Shallow_GVC**	=	Domestic Portion	+	Border Crossing	+	Foreign Portion	**Deep_GVC**	=	Domestic Portion	+	Border Crossing	+	Foreign Portion
Agriculture	USA	2.22	1.93	1.93	4.16	3.77		2.01		1		0.76	5.01		1.91		1.85		1.25
	RUS	2.05	1.83	1.83	3.95	3.41		1.58		1		0.83	5.60		1.73		2.23		1.64
	CHN	2.04	1.81	1.81	4.92	4.46		2.70		1		0.76	6.47		2.69		2.20		1.58
	BRA	1.78	1.57	1.57	4.03	3.59		1.71		1		0.88	5.16		1.59		2.16		1.41
	IND	1.40	1.30	1.30	4.63	4.21		2.29		1		0.92	6.01		2.13		2.25		1.63
Manufacturing	CHN	3.33	2.84	2.96	5.01	4.52		2.72		1		0.80	5.46		2.56		1.65		1.25
	JPN	2.63	2.21	2.45	4.30	3.75		1.94		1		0.81	5.32		2.17		1.71		1.44
	IND	2.60	2.20	2.11	4.12	3.73		1.79		1		0.95	4.55		1.51		1.66		1.38
	DEU	2.50	1.89	1.82	3.97	3.32		1.45		1		0.88	4.24		1.41		1.62		1.20
	USA	2.39	2.01	1.90	3.92	3.42		1.57		1		0.85	4.70		1.52		1.79		1.40
Mining	CHN	2.49	2.10	2.10	4.72	4.24		2.51		1		0.74	6.00		2.54		2.01		1.46
	USA	1.83	1.57	1.57	3.61	3.19		1.48		1		0.72	5.48		1.58		2.26		1.64
	AUS	1.77	1.54	1.54	3.73	3.33		1.54		1		0.79	5.62		1.67		2.28		1.67
	IND	1.47	1.32	1.32	4.57	4.11		2.17		1		0.94	6.02		2.03		2.30		1.69
	MEX	1.33	1.18	1.18	3.76	3.29		1.34		1		0.95	5.06		1.33		2.28		1.45
Service	CHN	2.67	2.28	2.13	5.15	4.69		2.92		1		0.77	6.63		2.75		2.26		1.63
	JPN	1.72	1.55	1.59	4.35	3.98		2.18		1		0.79	5.97		2.09		2.20		1.68
	DEU	1.71	1.46	1.67	3.99	3.43		1.60		1		0.83	5.33		1.59		2.26		1.47
	USA	1.71	1.55	1.59	4.14	3.68		1.84		1		0.85	5.65		1.73		2.27		1.65
	FRA	1.66	1.45	1.58	4.19	3.66		1.85		1		0.80	5.54		1.83		2.29		1.42

3.2.2 Has the length of Global Value Chains become longer or shorter over time?

One important question addressed in the recent GVC related literature is: Has the global production chain become less or more fragmented?

Most studies conclude that global production has become more fragmented today than decades ago. As shown in Feenstra and Hanson (1996), the imported intermediate inputs in the US have increased from 5.3% to 11.6% between 1972 and 1990. Similarly, Hummels et al. (2001) find that the world VS (Vertical specialization) share of exports has grown almost 30% between 1970 and 1990, which accounts for more than 30% of overall export growth.[14]

Our numerical results clearly show that the Global Value Chain is getting longer, which reflects the increasing fragmentation of GVC related production and trade activities. Moreover, the distinction between different types of production and trade activities enable us to further investigate the major drivers behind the lengthening of GVCs.

As shown in Figure 11, the world average "Total Production Length" has a clearly upward trend, especially after year 2002 (this trend was temporarily interrupted by the global financial crisis during 2008 to 2009). Furthermore, the average production length of GVCs (PLv_GVC) has increased by 0.28 from 2002 to 2011, which is much faster than traditional exports (PLv_RT) and pure domestic production length (PLv_D). The lengthening of GVC is reflected in both shallow and deep GVC production activities, but lengthening of deep GVC is more dramatic, for example, from 2002 to 2011, the length of deep GVC has increased 0.38, doubled the growth of the length of shallow GVCs.

[14] Fally (2011) indicates that the production chain (or the distance to final demand) in the US appears to have shortened over time and concludes that such a trend is also a global phenomenon. Consistent with Fally, our calculation also shows that the production length of the US is getting shorter. But this finding is reversed at the global level. In Appendix K, we show that the strong assumption "The same industries have the same production length across countries" is the main factor that leads to the puzzling finding by Fally.

In Figure 12, we focus on GVC production activities to investigate the changes of its domestic and international portions. We find that the increasing length of GVCs is primarily driven by two factors: 1).The increasing number of border crossing for production； 2). The lengthening of GVC production within foreign countries. Because the number of border crossing for production is constant in Shallow GVC activities, the lengthening of deep GVC activities is the major driven force.

Figure 11 The Upward Trend of Production Length, World Average

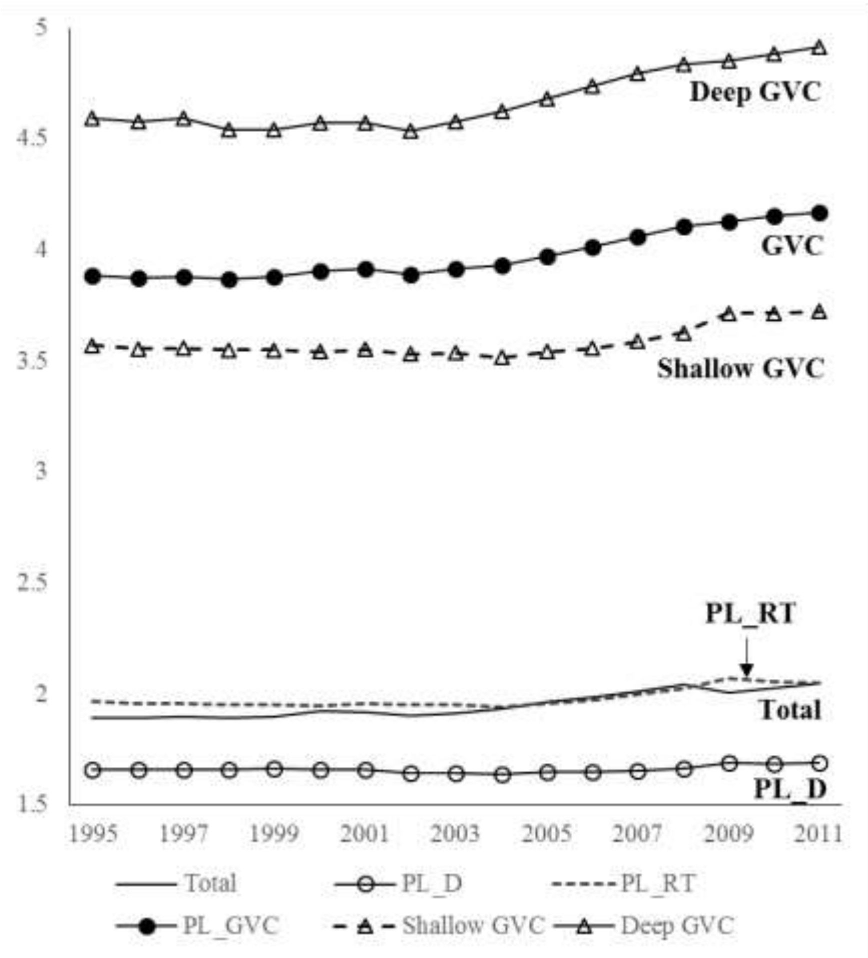
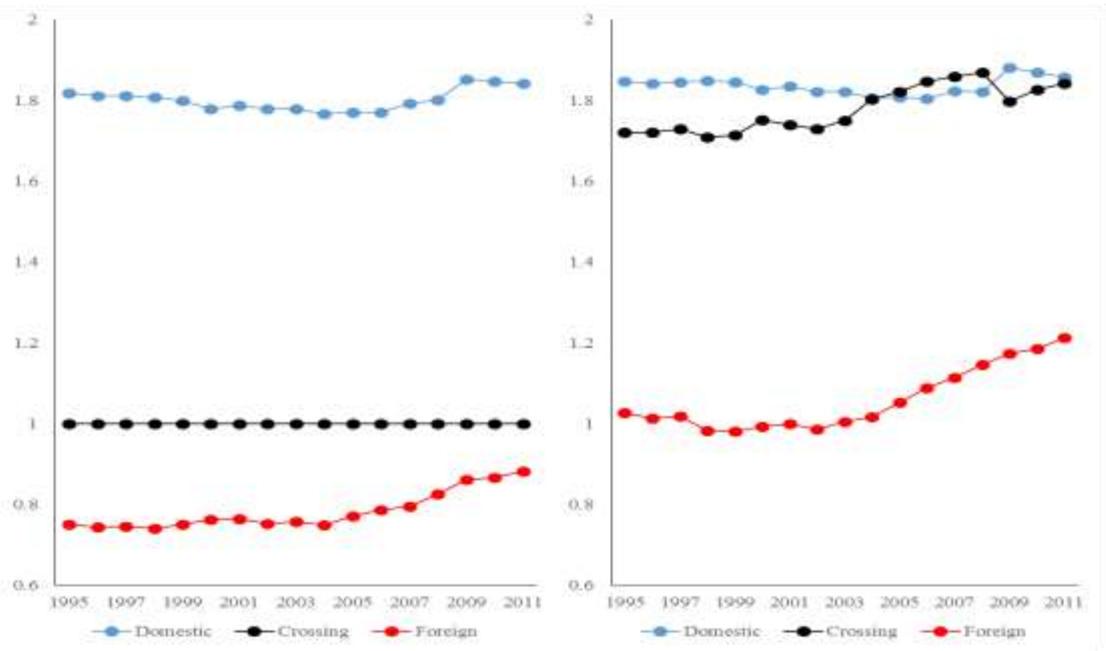


Figure 12 Changes of GVC Production Length: border crossing for production, domestic and foreign portion, World Average, 1995 to 2011

Shallow **Deep**

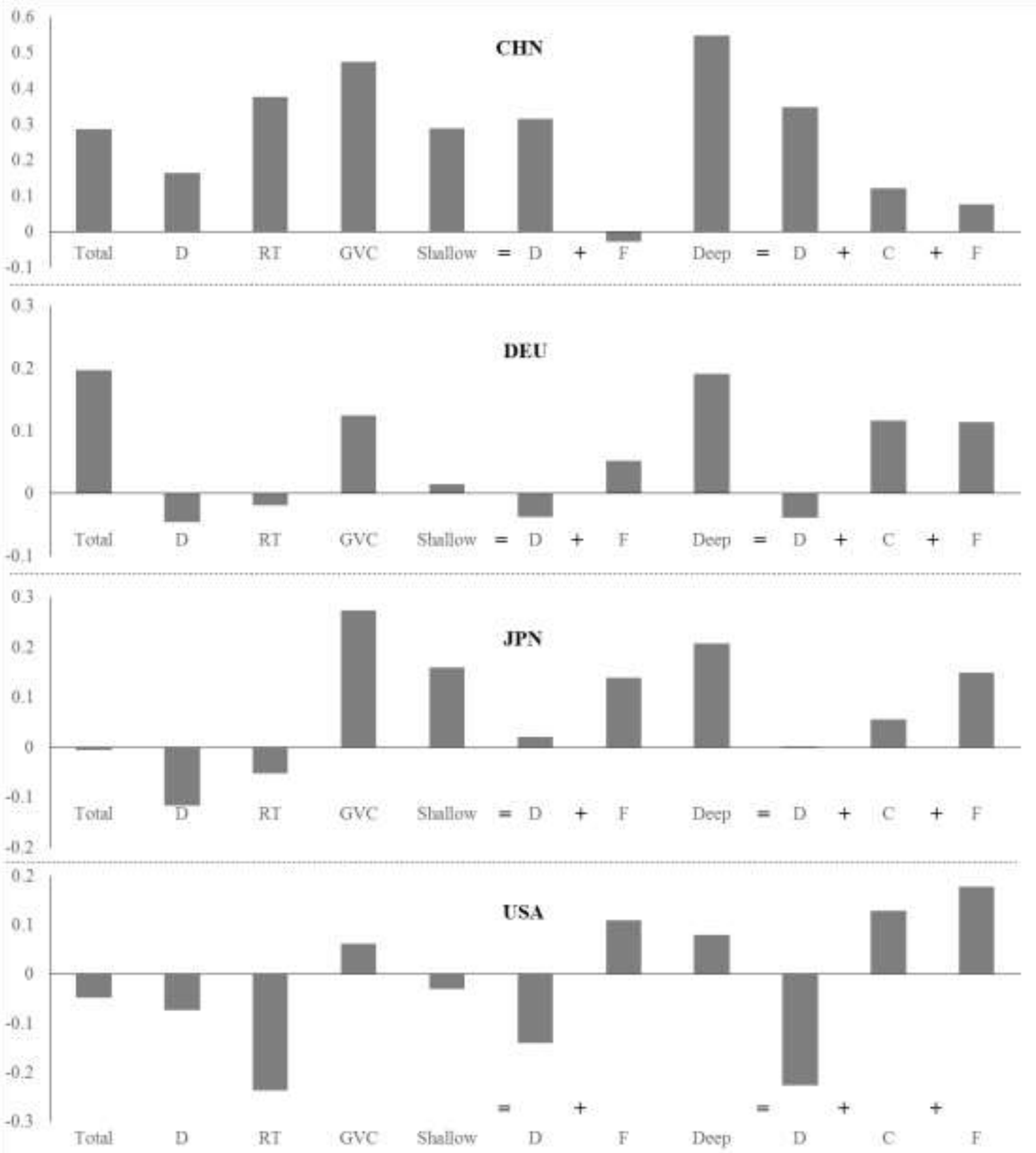


To ensure robustness of results, we further investigate the changes of production length at the country and industry level. In Figure 13, we select four largest countries ranking by GDP, the US, China, Japan and Germany, to compare the changes of major portions of forward linkage based production length.

For China, the total average production length, as well as all of its portions, except for the foreign portion in shallow GVC activities, is longer in 2011 than in 1995. For Germany, Japan, and the US, the production length for pure domestic (D) and traditional exports (RT) have decreased during the sample period. But the average GVC production length, especially the deep GVC part, has increased considerably for all countries over this period, even when the total average production length became shorter for Japan and the US.[15]

[15] This may reflect the phenomenon of "offshoring" production activities abroad in these developed economies. When more production activities go abroad, the international portion of GVCs gets longer while its domestic portion becomes shorter.

Figure 13 Production Length for Major Economies, Forward-Linkage based.

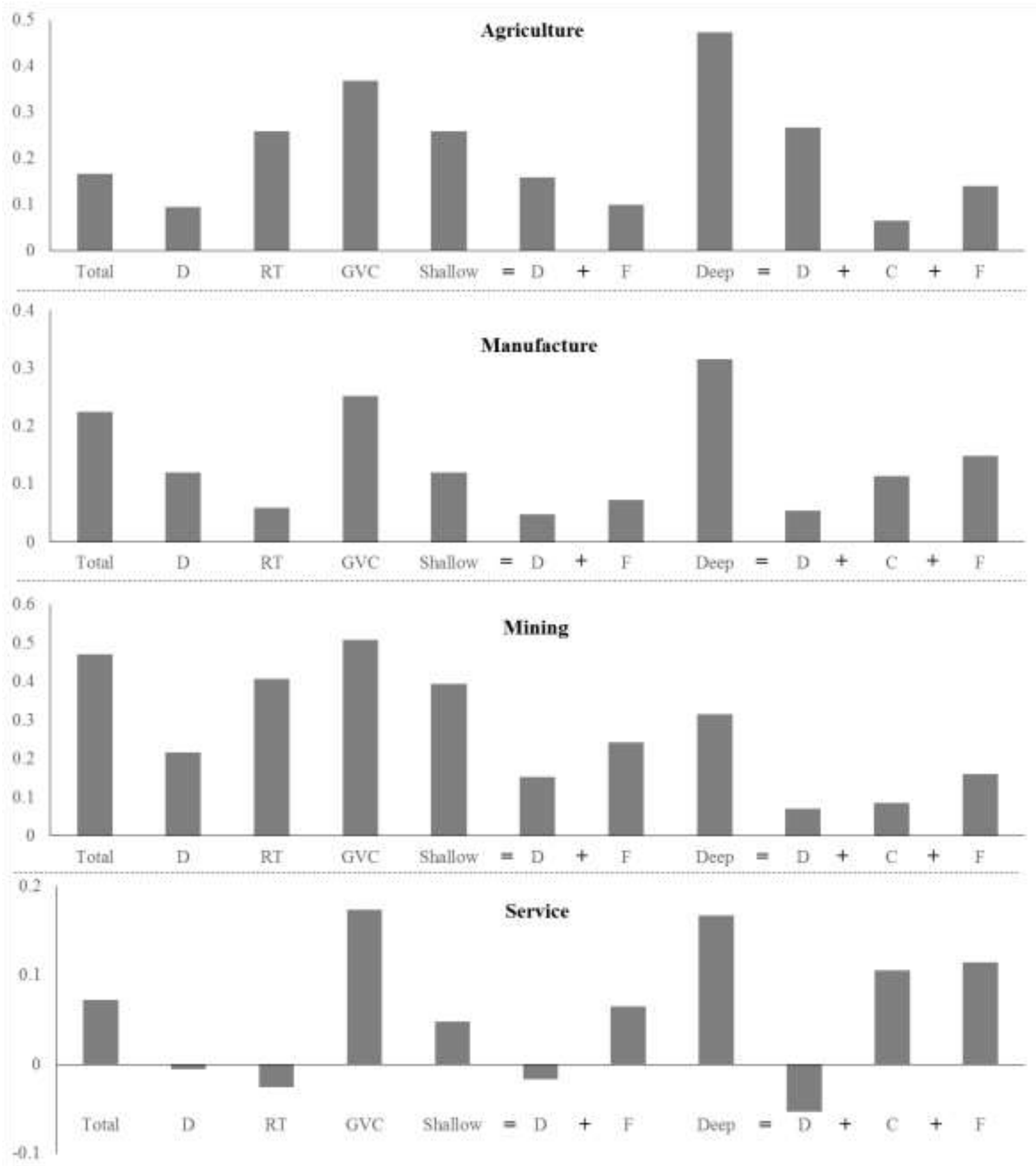


Although the length of GVC production, especially the deep GVC activities, has become longer during 1995 to 2011, their driven forces are quite different for the four selected large economies. For China the lengthening of domestic portion is dominate, representing China gains deeper and finer division of labor through attract more production stages move to its domestic economic landscape. For the three advanced industrial economies, the situation is opposite, the lengthening of their GVC production line is driven by the increase of number of border crossing and production stages with other countries, domestic production length even become short in Germany and the

United States. This is consistent with the fact that advanced countries rely heavily on offshoring to organize their global production network.

In Figure 14, we report the changes of production length of the four aggregate industries during 1995 to 2011.

Figure 14 Change of Production Length at Sectoral Level, 1995-2011, Forward-Linkage based.



Although GVC production length in all of the four aggregate industries have become longer during this period, the underlying driving force are different. In Agriculture and Manufacturing sectors, the length of Deep GVC segment increase much faster than that of the Shallow GVC segment, but the major driving factor in agriculture is the lengthening of domestic segment of production chain from finer division of labor, while the dominate force in manufacture is the increase of number of border crossing and production stages in foreign countries. In mining industries, the length of shallow GVC segment increases faster than its deep GVC segment and mainly driven by the increase of production stages in foreign countries.

Different from other three aggregate industries, the increase of total production length in services sectors is totally driven by the lengthening of GVC production, domestic portion of production length in all segments of the production chain actually decreased during this period. The increase of the length of deep GVC segment dominate the lengthening, which is driven by the growth of number of border crossing and number of production stages in foreign countries.

In conclusion, using the production length indexes newly defined in this paper, we have observed the increasing trend of fragmentation in production, especially in Global Value Chain related production activities.

3.3.3 A more robust measure of production length than APL

APL has been used in the literature to measure length in production (Dietzenbacher, and Romero,2007). .A major shortcoming is the measure changes as the aggregation level of industrial classification for the ICIO table changes. As we discussed in section 2, production length defined in this paper equals the times of value-added has been accounted as gross output since the first time it is used as primary factor until it is embodied in final products thus existing the production process. Because total value-added and gross output in a given statistical year are fixed at global level, they will not changes as the aggregation level changes, therefore the production length is constant at global level regardless aggregation of industrial classifications. This is an advantage of the production length defined in this paper over APL. To show this, we compute both the production length and APL based on original WIOD data, aggregate the original table from 35 sector into 10 and 3 sectors respectively and report our computation results of the average global production length in Figure 15 The results show clearly that the size of APL index increases as the aggregation level increase, while our production length measure keep constant at 2.05 across the 3 different sector aggregations.

At country and sector level, the new production length measure is also more robust than APL under different sector aggregations. Using the 2011 WIOD table at 10 and 35 sector as example, Figures 16 show that there is a systemic upward bias for the APL measure as sector classification become more aggregate, while the changes of newly defined production length measure due to aggregation are much smaller and around zero due to the use of industry group average value-added weights after aggregation. (Individual sector value-added share at disaggregate level is replaced by group average value-added share in the aggregation, which lead aggregation bias). Such aggregation bias will cancel each other when aggregate sector production length to global level. Similar situation can be find at the country and country/sector level, we report them in Appendix for interested readers.

Figure 15 Global Average APL, Calculated from IO Tables at Different Aggregation Levels

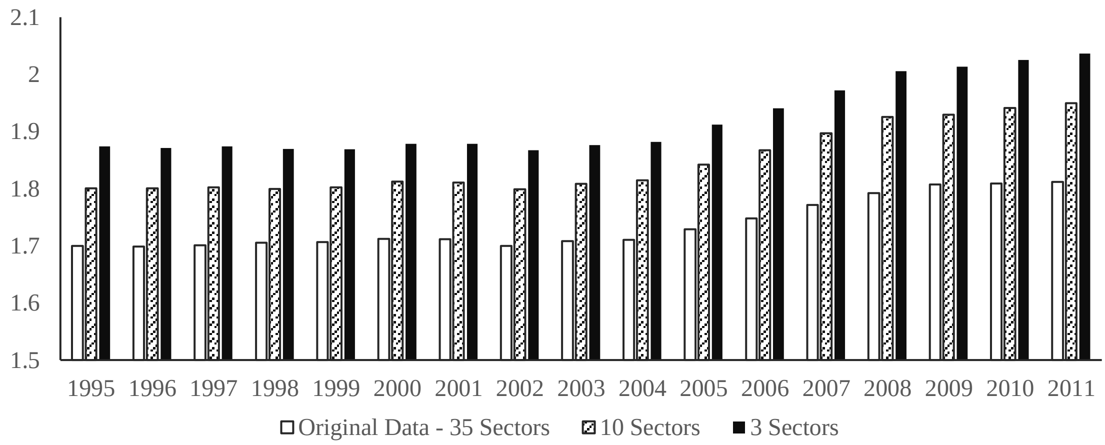
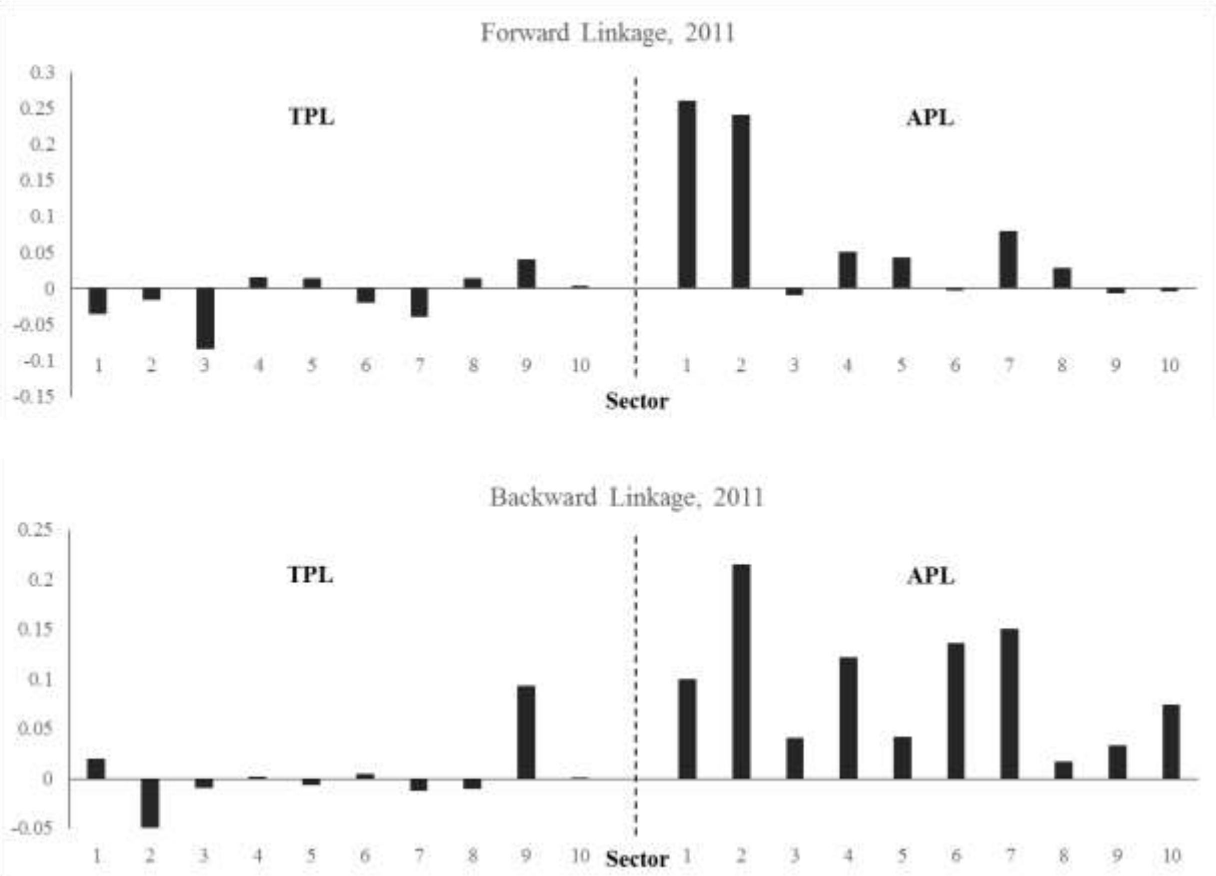


Figure 16 The Changes of TPL and APL at Sectoral Level



3.4 GVC Position Index

In section 2.5, we have discussed why the ratio of forward and backward industrial linkage based production length can been use as measure of the production line position in global value chains, now we report numerical results of such GVC position indexes in this subsection.

3.4.1 Country level

We compute the ratios of both forward and backward linkage based total and GVC production length in 2011, obtaining two type production line position indexes and report them in Table 9. Both type indexes indicate that China and India were located relatively closer to the bottom of GVCs among the 20 economies reported. However, the two type position indexes give very different ranking for upstream countries. For instance, Russia is ranked most upstream in the total production length based position index， while its ranking move down to the middle by the GVC production length based position index, while the ranking of Japan, Brazil, Italy and Belgium moved up. It is also interesting to note the ranking of Mexico’s GVC length based position is lower than both China and India.

Such difference may come from the structure difference between an economy’s total production and its GVC related production. The position index based on total production length measure a country’s production activities as a whole, including its pure domestic production and production of tradition trade that unrelated to cross country production activities (this part is often dominate in many economies),while the position index based on GVC production length only focus on the position of a country in cross country production sharing activities, so we define it in this paper as the “GVC Position Index”.

It is worth to point out, our numerical results of countries’ production line position seems contradictory to Miller et al. (2015). Their results show that, compared with other countries, China is the most upstream country in the world, far away from the final consumption end; but in fact, our results are not actually contradictory with Miller’s findings if carefully look through the numerical results. The reasons for the inconsistency are as follows:

When the “Upstreamness” (OU) and “Downstreamness” (ID) indexes of a country/sector pair computed by Miller et al. are high, it means that the distance between the country/sector pair to the factor input/final consumption end is longer. However, as we pointed out earlier, using backward or forward linkage based production lengths alone cannot tell the country/sector pair’s relative position in a production line because the country/sector pair as a middle stage of production process, its forward and backward length to each end of the production line could be relatively shorter or longer. Just as Table 9 shows, both the forward and backward total and GVC production length of China are significantly longer than that of other countries report here, which means that China would be placed at the upstream side if we use either the forward or backward linkage based total or GVC production length directly infer Upstreamness or Downstreamness as what Miller et al did in their 2015 paper.

Table 9 Country Level Position Index, 2011

Country	Production Position Index (Forward / Backward)	TPL Forward	TPL Backward	Country	GVC Position Index (Forward / Backward)	PL_GVC Forward	PL_GVC Backward
RUS	1.21	2.41	1.99	AUS	1.07	4.58	4.27
AUS	1.11	2.25	2.03	JPN	1.03	4.45	4.34
SWE	1.06	2.12	2.00	ITA	1.03	4.12	4.02
DEU	1.04	2.04	1.96	BEL	1.02	3.84	3.76
CAN	1.04	2.03	1.95	BRA	1.02	4.13	4.04
IDN	1.04	2.13	2.06	SWE	1.02	3.95	3.87
NLD	1.03	2.06	2.00	IDN	1.02	4.09	4.01
GBR	1.02	1.98	1.95	NLD	1.02	3.70	3.64
BRA	1.01	1.82	1.80	FRA	1.01	4.11	4.06
BEL	1.01	2.14	2.12	RUS	1.00	4.35	4.34
JPN	1.00	1.90	1.89	KOR	0.99	4.43	4.47
KOR	0.98	2.35	2.39	DEU	0.99	3.94	3.98
USA	0.98	1.77	1.81	GBR	0.99	3.78	3.83
FRA	0.98	1.85	1.89	TUR	0.99	4.09	4.15
MEX	0.97	1.74	1.81	ESP	0.98	4.01	4.08
TUR	0.96	1.90	1.97	USA	0.97	3.94	4.05
ITA	0.96	1.93	2.01	CAN	0.96	3.86	4.01
ESP	0.96	1.91	1.99	IND	0.95	4.00	4.19
CHN	0.95	2.72	2.85	CHN	0.95	4.84	5.08
IND	0.94	1.83	1.95	MEX	0.93	3.65	3.92

3.4.2 Sector level results

Table 10 reports average production line position for global industries in 2011. Similar to Table 9， we compute the ratios of both forward and backward linkage based total and GVC production length to obtain the two type production line position indexes.

There are differences in rank global industries by the two type position indexes. As we discussed, position index based on total production length measure a country's production as a whole, which pure domestic production activities are dominate, while GVC position index only concern cross country production activities in the global production network, as we showed earlier, the two type production activities are quite different even within the same industry. Taking construction sector as an example, when considering pure domestic production and cross country production as a whole, construction often located at the bottom of industrial production chain because it use large amount of intermediate inputs from other sectors and its products after completion will enter the consumer market immediately, therefore has a very short distance to final demand. However, when only cross border production activities is considered, products from construction sector may be difficulty to export directly due to the limitation of cross border factor mobility. Its factor content often embodied in other sectors' exports involving international production sharing indirectly. As a consequence, its position in GVC production network will move to relative upstream. Similar phenomena exist in many services sectors such as transportation and public services.

In summary, by excluding pure domestic production/consumption activities, our GVC position index ranks traditional non-tradeable sector such as utility and servicers upstream in the value chain, and most manufacturing sector such as leather and appeals, electronics and machinery downstream in the value chain. This is consistent with our economic intuitions.

Table 10 Sectoral Level Positions Index, World Average, 2011

Sector	Production Position Index	Sector	GVC Position Index
Mining	1.99	Electricity, Gas and Water	1.43
Business Activities	1.45	Refined Petroleum	1.31
Financial Intermediation	1.36	Private Households	1.22
Electricity, Gas and Water	1.29	Sale of Vehicles and Fuel	1.14
Water Transport	1.26	Retail Trade	1.14
Other Transport	1.24	Real Estate	1.13
Paper and Printing	1.24	Financial Intermediation	1.11
Wholesale Trade	1.22	Construction	1.10
Basic Metals	1.20	Public Admin	1.08
Agriculture	1.19	Post and Telecommunications	1.08
…		…	
Machinery	0.77	Rubber and Plastics	0.93
Hotels and Restaurants	0.77	Other Non-Metal	0.92
Education	0.74	Food	0.91
Recycling	0.72	Air Transport	0.88
Food	0.69	Textiles Products	0.88
Transport Equipment	0.67	Machinery	0.85
Leather and Footwear	0.65	Electrical Equipment	0.84
Public Admin	0.64	Recycling	0.83
Health and Social Work	0.60	Leather and Footwear	0.80
Construction	0.52	Transport Equipment	0.80

3.4.3 Country-sector level

Our calculation results show that the GVC position for a sector may vary considerably across countries, which reflects the differences in location by each country along a particular production network. Three typical sectors in 20 largest countries in terms of GDP are shown in Table 11.

Table 11 Sectoral Level: A comparison of GVC positions across Countries, 2011

Electrical Equipment		Business Service		Textiles Products	
Country	Position	Country	Position	Country	Position
RUS	1.083	AUS	1.258	RUS	1.209
AUS	0.954	RUS	1.186	JPN	0.901
NLD	0.923	MEX	1.169	TUR	0.848
SWE	0.892	TUR	1.154	AUS	0.840
BRA	0.891	DEU	1.113	USA	0.831
IDN	0.862	BEL	1.097	KOR	0.828
DEU	0.848	JPN	1.094	MEX	0.826
KOR	0.846	BRA	1.093	BRA	0.819
ITA	0.841	ITA	1.079	SWE	0.815
CAN	0.840	GBR	1.075	FRA	0.798
JPN	0.838	FRA	1.075	CAN	0.786
GBR	0.833	NLD	1.063	BEL	0.782
BEL	0.828	KOR	1.029	GBR	0.779
USA	0.822	USA	1.022	ITA	0.775
FRA	0.821	SWE	1.020	NLD	0.774
ESP	0.812	CAN	1.017	DEU	0.762
TUR	0.808	ESP	0.956	CHN	0.760
CHN	0.803	CHN	0.851	IND	0.750
MEX	0.770	IND	0.850	IDN	0.734
IND	0.763	IDN	0.789	ESP	0.733

In the “Electrical and Optical Equipment” sector, countries that specialize in assembling and processing activities, such as China, India and Mexico, are located on the most downstream end, as they are placed at the final stage of the production chain. In contrast, two natural-resources-abundant countries, Australia and Russia, are positioned in the most upstream end to provide energy and mining needs for the whole value chain. Germany and Korea are also located on the upstream end of the production chain, as they participate in GVCs as providers of design and core components.

In textile sector，China, India and Indonesia are located in the final product end of the value chain, while Russia and Australia positioned more upstream by providing natural resource based intermediate inputs. Japan’s position is also more upstream by providing more manufactured intermediate inputs into the production chain. In business services sector, China, India and Indonesia are also located at the end of the value chain by providing direct services such as calling center and clinic record keeping. Countries located upstream are those countries where business services are important intermediate input for their manufacturing industry such as Germany, and natural resource providing countries such as Australia and Russia.

3.4.4 Time Trend: selected industries

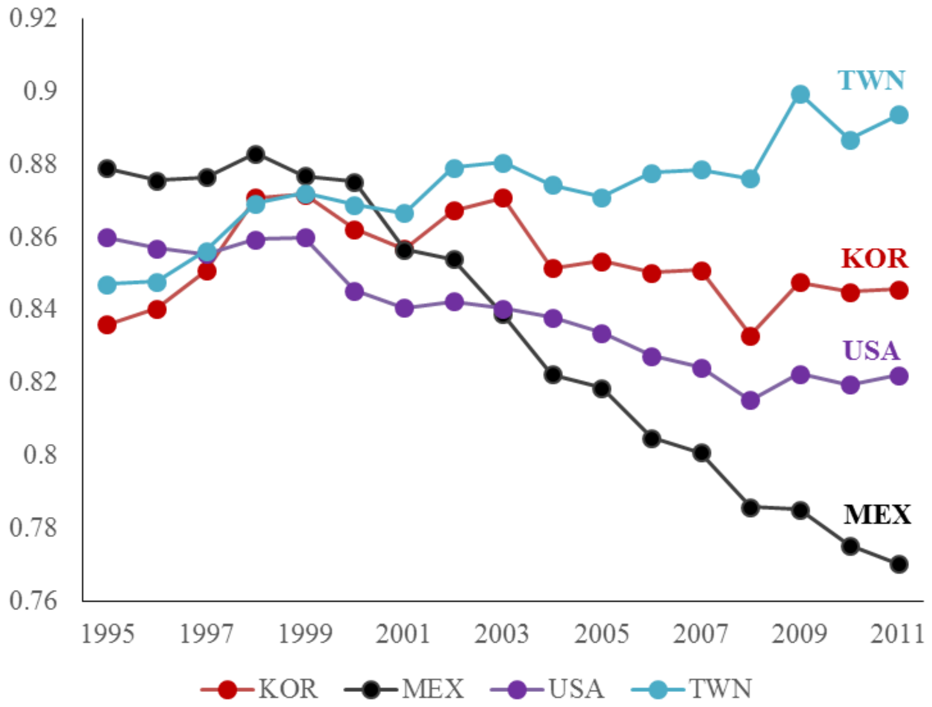
Analyzing changes of GVC position index over time may allow us quantify the evolution path of each country's role and position along a particular production chain. Here we use a typical GVC industries - Electrical and Optical Equipment (WIOD sector 14) as example.

Figure 17 plots out the time trend of GVC position index of Electrical and Optical Equipment for the United States, Mexico, Korea and Taiwan during 1995 to 2011.

As a member of NAFTA since 1994， Mexico gradually become a processing and assembling center of electronic and optical equipment in west hemisphere due to its low cost of production (such as labor cost), proximity and duty free access to the world largest consumer electronic market. As our GVC position index indicates, its forward production linkage based GVC production length become shorter and shorter, its backward industrial linkage based production length become longer and longer, its production line position on Electrical and Optical Equipment value chain have been moved from relative upstream in 1995, the first year of NAFTA in effect, to the most downstream in 2011. Similar to Mexico, U.S. position in this global production chain also moved relative downstream, however, the driven force of such move is different from Mexico. Both forward and backward linkage based U.S production length have become longer since U.S. electric and optical equipment industry has offshoring a large part of its middle production stages and also import a large amount of parts and components from other countries in its final good production. The relative faster lengthening of backward linkage based production length than the lengthening of forward linkage based production length lower US GVC position index.

In 1995, Taiwan and Korea were located relative downstream in the electric and optical equipment production chain compare to the U.S. and Mexico, but as the rapidly developed electronic supply chain cluster in east Asia, particular in China's south coast area, Taiwan and Korea have become major suppliers of electronic parts and component in the world. Their position on the electric and optical equipment production chain increased quickly, exceed both US and Mexico around year 2000. Similar to the US, both forward and backward linkage based production length have become longer for Taiwan and Korea since 1995, but their forward linkage based production length grow much faster, so their production line position move upstream.

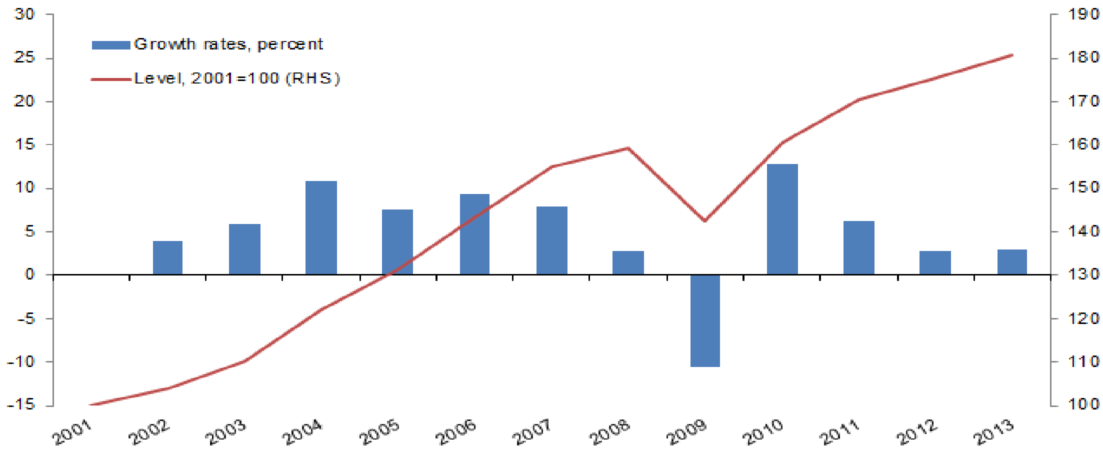
Figure 17 Time Trend of GVC Position Index, Electrical and Optical Equipment



3.5 Index application: participation intensity, production line positions, GVC length, and the economic shocks of the recent global financial crisis

In the aftermath of the Global Financial Crisis, as shown in Figure 18, world trade grew by 6.2% in 2011, 2.8% in 2012, and 3.0% in 2013. This growth in trade volumes is substantially lower than the pre-crisis average of 7.1% (1987–2007), and is slightly below the growth rate of world GDP in real terms.

Figure 18 The Growth of World Trade before and after the Financial Crisis



As we analyzed before, value-added creation activities by a country can be decomposed into four parts: pure domestic production, production of traditional trade, shallow and deep cross country production sharing activities. Then, in financial crisis, are there differences in the degree of effects on the four types of value added?

Figure 19 Different Effects of the Same Economic Shock to Different Value Added Creating Activities – Impact of global Financial Crisis

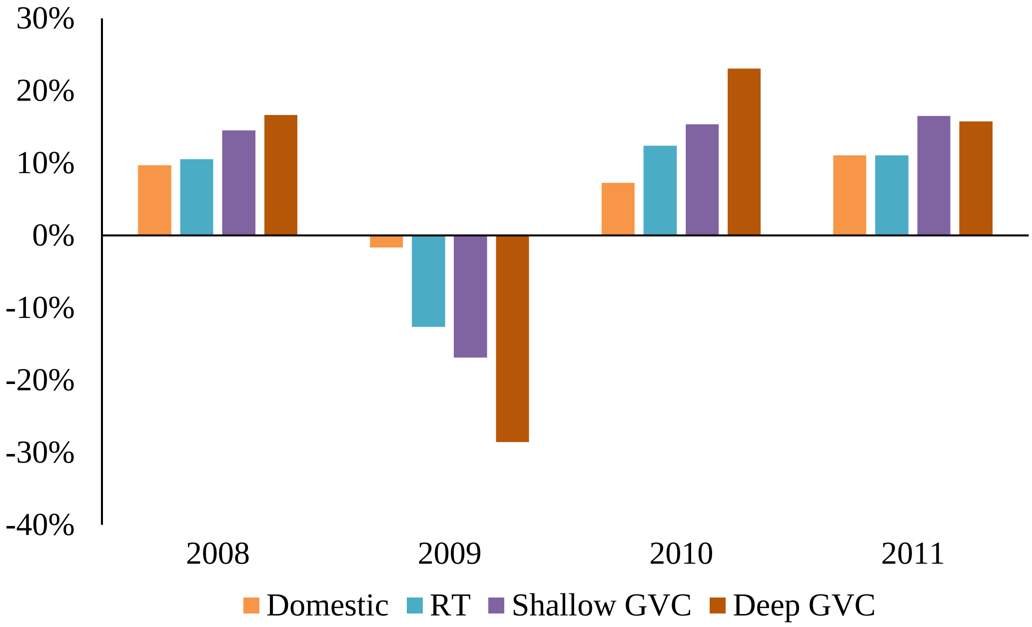


Figure 19 shows the result at the global level. During the financial crisis in 2009, pure domestic production activities were least affected (in comparison with 2008, the fall was only 1.7%), the impact on production of traditional trade rank next, while the cross country GVC production activities were mostly affected, as the fall reached 16.9% in its shallow portion and 28.5% in its deep portion. However, it is also observed that the two portion of GVC related production activities had the fastest after-crisis-recovery.

Divided among different countries and sectors, the above phenomenon also holds. Table 12 shows that: pure domestic production is least affected by the financial crisis (China even continued a positive growth). For most sectors, GVC production and trade activities were most affected. The second issue is this: Are the GVC participation intensity, GVC length and production line positions related to the degree of effects of the financial crisis? To test this, we estimate the following regression model:

$$\Delta \ln(Va_{ic}) = \beta_0 + \beta_1 \times GVCPt_{ic} + \beta_2 \times Position_{ic} + \beta_3 \times PLv\_GVC\_shallow_{ic} + \beta_4 \times PLv\_GVC\_deep_{ic} + \beta_5 \times W_{ic} + \beta_6 \times Z_c + \gamma_i + u_{ic}$$

where

$\Delta \ln(Va_{ic})$ equals to the change of sectoral GDP, $\ln(Va_{ic})$ in year 2009 minus $\ln(Va_{ic})$ in year 2008, which quantifies the degree of effects on this industry/country pair during the financial crisis;

$GVCPt_{ic}$ is the forward (or backward) GVC participation ratio. It can also be divided into shallow and deep portions;

$Position_{ic}$ is the production line position Index, calculated as Total_PLv/Total_PLy. When the value is high, it means that this sector is relatively further from the final consumption end;

$PLv\_GVC\_shallow_{ic}$ and $PLv\_GVC\_deep_{ic}$ is the forward linkage GVC production length for shallow and deep production sharing activities;

W_{ic} represents the country-sector level control variables, including the labor productivity defined as value added per worker, and hours worked by high-skilled workers (share in total hours);

Z_c represents the country level control variable. In the regressions, we use a dummy variable to indicate whether this is a mature economy (=1);

Table 12 The Effects of Financial Crisis to Different Value Added Creating Activities (Sectoral Level)

China					USA				
Sector	Domestic	RT	Shallow GVC	Deep GVC	Sector	Domestic	RT	Shallow GVC	Deep GVC
Agriculture	9.0%	-3.2%	-8.3%	-14.3%	Agriculture	-13.9%	-21.5%	-27.4%	-30.8%
Mining	18.0%	-9.4%	-20.5%	-28.6%	Mining	-23.5%	-33.2%	-19.6%	-38.0%
Food	7.9%	-3.7%	-10.4%	-16.4%	Food	16.0%	8.0%	-2.2%	-12.1%
Textiles Products	23.7%	-6.0%	-5.8%	-11.9%	Textiles Products	-19.9%	-7.9%	-18.5%	-22.1%
Leather and Footwear	18.3%	-6.5%	-6.0%	-10.0%	Leather and Footwear	-19.8%	14.4%	-10.0%	-14.3%
Wood Products	15.4%	-13.4%	-18.5%	-23.9%	Wood Products	-15.8%	-22.0%	-20.7%	-30.3%
Paper and Printing	13.5%	-8.7%	-12.0%	-19.6%	Paper and Printing	-1.7%	-10.5%	-5.1%	-18.2%
Refined Petroleum	16.5%	-11.3%	-21.6%	-23.5%	Refined Petroleum	-21.4%	-25.7%	-24.5%	-37.8%
Chemical Products	17.9%	-6.0%	-14.3%	-22.7%	Chemical Products	10.8%	15.9%	4.7%	-8.3%
Rubber and Plastics	20.3%	-7.2%	-9.2%	-18.3%	Rubber and Plastics	-3.1%	-8.2%	-2.3%	-14.7%
Other Non-Metal	10.4%	-17.5%	-17.9%	-28.5%	Other Non-Metal	-2.5%	-11.5%	0.6%	-18.5%
Basic Metals	22.8%	-10.5%	-22.3%	-33.2%	Basic Metals	-15.6%	-16.9%	-11.6%	-28.1%
Machinery	20.3%	-16.8%	-22.0%	-28.6%	Machinery	-10.6%	-8.6%	2.3%	-15.1%
Electrical Equipment	28.6%	-8.6%	-5.2%	-16.1%	Electrical Equipment	1.1%	4.4%	5.9%	-11.1%
Transport Equipment	14.0%	-12.3%	-16.6%	-25.1%	Transport Equipment	-1.6%	-8.3%	-4.1%	-27.1%
Recycling	41.1%	-5.9%	-10.4%	0.3%	Recycling	-8.3%	-12.6%	3.3%	-0.5%

We also control for the sector fixed effects by including a sector dummy γ_i in the model.

Summary statistics for key variables are provided in Table 13:

Table 13 Summary Statistics for Key Variables

	Variable	Observations	Mean	SD	Min	Max
2008	Va	1400	37009	114850	0	1972298
	Position	1382	1.01	0.31	0.32	2.85
	GVCPt (forward)	1382	0.25	0.20	0	0.94
	GVCPt_shallow (forward)	1382	0.14	0.12	0	0.67
	GVCPt_deep (forward)	1382	0.10	0.09	0	0.59
	GVCPt (backward)	1382	0.24	0.15	0	0.85
	PLv_GVC: International Portion	1361	2.15	0.18	1.44	3.21
	PLv_GVC: Domestic Portion	1361	1.84	0.61	1.00	4.17
2009	Va	1400	35170	114725	0	1902096
	Position	1379	1.03	0.33	0.32	3.41
	GVCPt (forward)	1379	0.24	0.20	0	1.10
	GVCPt_shallow (forward)	1379	0.15	0.13	0	0.86
	GVCPt_deep (forward)	1379	0.09	0.09	0	0.51
	GVCPt (backward)	1379	0.21	0.14	0	0.84
	PLv_GVC: International Portion	1358	2.13	0.18	1.43	3.16
	PLv_GVC: Domestic Portion	1358	1.86	0.62	1.00	4.23

The regression results are shown in Table 14. Regressions (1), (2), (4), (6) and (8) indicate that the forward linkage based GVC participation intensity (GVCPr) has significant impact on the degree of effect of the global financial crisis. The higher the ratio, the greater the degree of negative impact. And as show in regression (2), the impact of backward linkage based GVC participation ratio is not significant.

In regression (3), (5), (7), (9), we further differentiate the GVC participation into two categories to represent the shallow and deep production sharing activities, respectively. Regression results clearly show that the impact of GVC participation on sectoral GDP during the financial crisis mainly come from its deep portion, while the coefficients of its shallow portion are not significant.

Besides that, in all regressions we find that production line position has significant impact on the degree of effect of the global financial crisis. The further is the position from the final consumption end, the less affected the node would be by the financial crisis. In the meanwhile, as show in regressions (6)-(9), the influences of financial crisis tend to be more severe for countries with a longer international portion and shorter domestic portion of forward linkage based GVC production length.

Furthermore, as shown in Regression (4), (5), (8) and (9), sectors in mature economies are less affected, while the negative shocks on sectors with higher ratio of high-skill labor are more severe and the impact of sectoral level labor productivity is not significant.

4. Conclusions

In this paper, we have developed a GVC index system that includes three types of indexes based on both forward and backward inter-industry and cross-country linkages: a participation index for the intensity of a country-sector's engagement in global value chains; a production length index for the average number of production stages and complexity of the global value chain; and a position index for the location of a country sector on a global value chain, or the relative distance of a particular production stage to both ends of a global value chain. While the existing literature has proposed similar measures, our indices contain improvements that we argue are desirable and sensible from the viewpoint of economic intuition.

We thus can provide a comprehensive picture of each country/sector pair's GVC activities from multiple dimensions. All these indexes are built at the decomposition of GDP by industry statistics and can be further divided into different components with clear economic interpretations. By estimating these indexes according to real world data, we produce a large set of indicators. We hope these indexes could be widely used by both theoretical and empirical economists in advancing studies of global supply chains and become a bridge between economic theories of supply chains and GVC measures based on GDP and gross trade accounting.

These new measures can potentially be linked to productivity growth or changing patterns of comparative advantage as well. We leave such investigation for future research.

Table 14 Regression Results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
GVCP (forward)	-26.94***	-26.79***		-25.64***		-20.14***		-18.29***	
	(3.430)	(5.206)		(3.462)		(3.830)		(3.945)	
GVCP (backward)		-0.302							
		(8.278)							
Shallow GVCP (forward)			-4.909		-5.483		-0.228		-0.561
			(8.348)		(8.354)		(8.400)		(8.426)
Deep GVCP (forward)			-61.26***		-56.95***		-51.20***		-45.72***
			(13.41)		(13.33)		(12.77)		(12.51)
Position	8.303**	8.231*	8.633**	8.544**	8.871**	7.744*	7.596*	8.077*	7.931*
	(4.223)	(4.992)	(4.162)	(4.254)	(4.211)	(4.455)	(4.403)	(4.508)	(4.476)
International Portion						-9.240**	-7.108	-10.26**	-8.319*
						(4.608)	(4.427)	(4.660)	(4.481)
Domestic Portion						3.942***	3.890***	4.112***	4.086***
						(1.048)	(1.039)	(1.049)	(1.040)
Mature（=1）				3.402***	3.125***			3.324***	3.063***
				(0.789)	(0.799)			(0.796)	(0.804)
Labor Productivity				-0.00450*	-0.00355			-0.00419	-0.00332
				(0.00272)	(0.00254)			(0.00272)	(0.00257)
High Skill				-17.02***	-17.27***			-17.82***	-18.02***
				(3.707)	(3.692)			(3.770)	(3.751)
Constant	-18.75***	-18.64***	-19.75***	-19.15***	-19.93***	-8.983	-13.40*	-7.893	-11.79
	(4.627)	(5.856)	(4.616)	(4.718)	(4.708)	(8.224)	(8.042)	(8.264)	(8.062)
Sector Fixed Effects	YES	YES	YES	YES	YES	YES	YES	YES	YES
Observations	1,379	1,379	1,379	1,343	1,343	1,358	1,358	1,322	1,322
R-squared	0.229	0.229	0.236	0.245	0.251	0.236	0.241	0.253	0.258

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Definition of major variables

Label	**Description**
X	Gross outputs vector
$\boldsymbol{Y}$	Final products vector
$\boldsymbol{Z}$	Intermediate flow matrix
Va	Value added vector
A	Input coefficient matrix
V	Value added coefficient vector
B	Leontief inverse matrix (Global)
G	Ghosh inverse matrix
δ	dummy variable
$plvy_{ij}$	Production length of value added from sector i embodied in the final products of sector j
Xv	The total value of gross outputs that induced by value added
PLv	Forward linkage based production length
PLy	Backward linkage based production length
L^{ss}	Local Leontief inverse of country s
DV	Domestic value added
FD	Final goods production
GVCPs	The Position of the particular sector in GVC
GVCPt	The Participation of the particular sector in GVC
APL	The Average Propagation Length (APL)

Definition of major label

Label	**Description**
_D	Domestic value added to satisfy domestic final demand that is not related to international trade
_RT	Domestic value added in final product exports
_GVC	Relative to global value chains / intermediate exports
_ GVC_R	Intermediate exports to satisfy importing country's final demand directly
_GVC_D	DVA in intermediate exports that are used to produce either intermediate or final goods and services and shipped back to the source country
_GVC_F	DVA in intermediate exports to indirectly satisfy other countries' final demand
D	Domestic portion
C	Number of border crossing for production
F	International domestic portion

Appendix

Appendix A Detailed mathematical proof of equation (3)

Based on general ICIO model shown in table 1 of main text, classical Leontief inverse equation can be expressed as

$$X = BY\mu \tag{A1}$$

Therefore, the gross exports of country s can be expressed as

$$E\mu = A^F X + Y^F \mu = A^F BY\mu + Y^F \mu \tag{A2}$$

Inserting equation (A2) into the second term of equation (2) in main text.

$$LE\mu = LA^F BY\mu + LY^F \mu \tag{A3}$$

Appendix B Detailed mathematical proof of equation (5)

The gross input production and use balance, or the column balance condition of the ICIO table in Table 1 can be written as:

$$\mu\hat{X} = \mu A\hat{X} + V\hat{X} \tag{B1}$$

Rearranging the equation (B1) yields

$$\mu = V(I - A)^{-1} = VB \tag{B2}$$

Inserting the final products production as a diagonal matrix into equation (B2), the decomposition of final products production based on the Leontief model can be expressed as follows:

$$[Y\mu]' = \mu'\widehat{Y\mu} = VB\widehat{Y\mu} \tag{B3}$$

Expanding equation (B3), final products production at each country/sector pair can be decomposed into five different parts as follows:

$$[Y\mu]' = VB\widehat{Y\mu} = \underbrace{VL\widehat{Y^D\mu}}_{(1)-Y\_D} + \underbrace{VL\widehat{Y^F\mu}}_{(2)-Y\_RT} + \underbrace{VLA^F L\widehat{Y^D\mu}}_{(3a)-Y\_GVC\_R} + \underbrace{[VBA^F L]^D \widehat{Y\mu}}_{(3b)-Y\_GVC\_D}$$
$$+ \underbrace{[VBA^F L]^F \widehat{Y\mu} - VLA^F L\widehat{Y^D\mu}}_{(3c)-Y\_GVC\_F} \tag{B4}$$

Appendix C Forward and backward linkage based GVC participation indexes at global level

As shown in equations (6) and (7), GVC participation indexes based on forward and backward industrial linkage can be defined as

$$GVCPt\_f = \frac{V\_GVC}{\hat{V}X} = \frac{\hat{V}LA^F BY\mu}{\hat{V}X} \quad \text{(C1)}$$

$$GVCPt\_b = \frac{Y\_GVC}{[Y\mu]'} = \frac{VBA^F L\widehat{Y\mu}}{[Y\mu]'} \quad \text{(C2)}$$

Aggregating to the world level

$$\mu GVCPt\_f = \frac{\mu' V\_GVC}{\mu'\hat{V}X} = \frac{\mu'\hat{V}LA^F BY\mu}{\mu' GDP} = \frac{VLA^F BY\mu}{\mu' GDP} = 1 - \frac{VLY\mu}{\mu' GDP} \quad \text{(C3)}$$

$$GVCPt\_b\mu = \frac{Y\_GVC\mu}{[Y\mu]'\mu} = \frac{VBA^F LY\mu}{\mu' Y} = 1 - \frac{VLY\mu}{\mu' GDP} \quad \text{(C4)}$$

Obviously, the numerators in equations (C3) and (C4) are the same. Therefore, GVC participation indexes based on forward and backward industrial linkage equal each other at the global level.

Appendix D Ghosh input-output model and its linkage with Leontief model

We define the output coefficient matrix as $H = \hat{X}^{-1} Z$, and the final products coefficient vector as $F = \hat{X}^{-1}Y$ as in Ghosh (1958). From the input side, gross inputs can be split into intermediate inputs and value added, $X'H + Va = X'$. Rearranging terms, we can reach the classical Ghosh inverse equation, $X' = VaG$, where $G = (I - H)^{-1}$ is the Ghosh inverse matrix. The linkage between value added and final products can also be expressed as: $Y' = X'\hat{F} = VaG\hat{F}$.

It is easy to derive the linkage between the input and output coefficient matrices as: $\hat{X}^{-1}A\hat{X} = \hat{X}^{-1}Z = H$. Similarly, the linkage between the Leontief inverse and the Ghosh inverse matrices are:

$$\hat{X}^{-1}B\hat{X} = \hat{X}^{-1}(I-A)^{-1}\hat{X} = \left[\hat{X}^{-1}(I-A)\hat{X}\right]^{-1}$$

$$= \left(1 - \hat{X}^{-1}A\hat{X}\right)^{-1} = (1-H)^{-1} = G \quad \text{(D1)}$$

Appendix E Detailed mathematical derivation of Upstreamness

As defined in Fally (2012a, 2012b, 2013) and Antras et al (2012, 2013), the Upstreamness of an industry’s output in the value chain can be measured as

$$U = \frac{Y\mu}{X} + 2\frac{AY\mu}{X} + 3\frac{AAY\mu}{X} + \cdots = \frac{Y\mu + 2AY\mu + 3AAY\mu + \cdots}{X} \quad \text{(E1)}$$

The numerator of equation E1 can be expressed in matrix notation as

$$Y\mu + 2AY\mu + 3AAY\mu + \cdots = (Y\mu + AY\mu + AAY\mu + \cdots)$$
$$+A(Y\mu + AY\mu + AAY\mu + \cdots) + AA(Y\mu + AY\mu + AAY\mu + \cdots) + \cdots$$
$$= BY\mu + ABY\mu + AABY\mu + \cdots = BBY\mu = BX$$

Therefore, Upstreamness of an industry's output can be measured as

$$U = \frac{BX}{X} = G\mu \quad \text{(E2)}$$

The right side of equation E2 is the same to equation (11a) of main text.

As defined in Antras and Chor (2013), the Downstreamness of an industry's output in the value chain can be measured as

$$D = \frac{\hat{X}V}{X'} + 2\frac{\hat{X}VA}{X'} + 3\frac{\hat{X}VAA}{X'} + \cdots = V + 2VA + 3VAA + \cdots = VBB = \mu' B \quad \text{(E3)}$$

The equation E3 can be expressed in matrix notation as

$$V + 2VA + 3VAA + \ldots$$
$$= V[(I + A + AA + \ldots) + A(I + A + AA + \ldots) + AA(I + A + AA + \ldots) + \ldots]$$
$$= V[B + \hat{X}VAB + AAB + \ldots] = VBB = uB$$

Therefore, Downstreamness of an industry's output can be measured as

$$D = \mu' B \quad \text{(E4)}$$

The right side of equation E4 is the same to equation (14) of main text.

Appendix F Detailed mathematical proof of Equations (16) and (17)

Multiplying domestic value-added generated from each production stage of section 2.3.2 with production length of that stage and summing all production stages in an infinite stage production process, we can obtain the pure domestic value-added induced gross output as

$$Xv\_D = \hat{V}Y^D\mu + 2\hat{V}A^DY^D\mu + 3\hat{V}A^DA^DY^D\mu + \cdots$$
$$= \hat{V}(I + A^D + A^DA^D + \cdots)Y^D\mu + \hat{V}(A^D + A^DA^D + \cdots)Y^D\mu + \cdots$$
$$= \hat{V}(L + A^DL + A^DA^DL + \cdots)Y^D\mu = \hat{V}LLY^D\mu \quad \text{(F1)}$$

Where $I + A^{ss} + A^{ss}A^{ss} + \ldots = (I - A^{ss})^{-1} = L^{ss}$ $I + A^D + A^DA^D + \cdots = l$

Similarly, production of value-added in "traditional trade" is also entirely take place domestically, the gross output it induced can be expressed as

$$Xv\_RT = \hat{V}Y^F\mu + 2\hat{V}A^DY^F\mu + 3\hat{V}A^DA^DY^F\mu + \cdots$$

$$= \hat{V}(L + A^D L + A^D A^D L + \cdots)Y^F \mu = \hat{V} L L Y^F \mu \qquad \text{(F2)}$$

Appendix G Detailed mathematical derivation of section 2.3.3

Summing all over the international production stages in an infinite stage production process, we have

$$V\_GVC = \hat{V}A^F Y\mu + \hat{V}A^D A^F Y\mu + \hat{V}A^F AY\mu + \hat{V}A^D A^D A^F Y\mu + \hat{V}A^D A^F AY\mu + \cdots$$

$$= \left[\hat{V}A^F Y\mu + \hat{V}A^D A^F Y\mu + \hat{V}A^D A^D A^F Y\mu + \cdots\right]$$

$$+\left[\hat{V}A^F AY\mu + \hat{V}A^D A^F AY\mu + \hat{V}A^D A^D A^F AY\mu + \cdots\right] + \cdots$$

$$= \hat{V}LA^F Y\mu + \hat{V}LA^F AY\mu + \cdots = \hat{V}LA^F BY\mu \qquad \text{(G1)}$$

Where $\sum_u^G B^{ru}$ is the limit of the series $I + \sum_u^G A^{ru} + \sum_k^G A^{rk} \sum_u^G A^{ku}$ .... It measures the amount of domestic value added that can be generated from the production of gross intermediate exports $A^{sr}X^r$ in country *s*, regardless of whether these exports are finally absorbed in importing country *r* or not.

Using the domestic or international production length of each stage of intermediate exports production discussed earlier as weights and summing across all production stages, we can obtain the global gross output generated by GVC related trade as well as its 3 components in any particular bilateral route.

$$Xvd\_GVC = \hat{V}A^F Y + 2\hat{V}A^D A^F Y + \hat{V}A^F AY + 3\hat{V}A^D A^D A^F Y$$

$$+2\hat{V}A^D A^F AY + \hat{V}A^F AAY + \cdots$$

$$= \left[\hat{V}A^F Y + 2\hat{V}A^D A^F Y + 3\hat{V}A^D A^D A^F Y + \cdots\right]$$

$$+\left[\hat{V}A^F AY + 2\hat{V}A^D A^F AY + 3\hat{V}A^D A^D A^F AY + \cdots\right] + \cdots$$

$$= \hat{V}LLA^F Y + \hat{V}LLA^F AY + \hat{V}LLA^F AAY + \cdots = \hat{V}LLA^F BY \qquad \text{(G2)}$$

Similarly, the total international (foreign) gross outputs induced by domestic value-added of country *s* embodied in its GVC related intermediate exports can be expressed as:

$$Xvf\_GVC = \hat{V}A^F Y + \hat{V}A^D A^F Y + 2\hat{V}A^F AY + \hat{V}A^D A^D A^F Y$$

$$+2\hat{V}A^D A^F AY + 3\hat{V}A^F AAY + \cdots$$

$$= \left[\hat{V}A^F Y + \hat{V}A^D A^F Y + \hat{V}A^D A^D A^F Y + \cdots\right]$$

$$+2[\hat{V}A^FAY + \hat{V}A^DA^FAY + \hat{V}A^DA^DA^FAY + \cdots] + \cdots$$

$$= \hat{V}LA^FY + 2\hat{V}LA^FAY + 3\hat{V}LA^FAAY + \cdots = \hat{V}LA^FBBY \quad \text{(G3)}$$

Adding equation (G2) and (3), the total outputs induced by domestic value-added of country *s* embodied in its GVC related intermediate exports can be expressed as:

$$Xv\_GVC = Xvd\_GVC + Xvf\_GVC = \hat{V}LLA^FBY + \hat{V}LA^FBBY$$

$$= \hat{V}L(B-L)Y + \hat{V}(B-L)BY = \hat{V}BBY - \hat{V}LLY \quad \text{(G4)}$$

Summing the numerator of equations (16), (18), (22) and (25) in the main text over all trading partner countries *r*, we obtain

$$Xv\_D + Xv\_RT + Xv\_GVC$$

$$= \hat{V}LLY^D + \hat{V}LLY^F + \hat{V}BBY - \hat{V}LLY = \hat{V}BBY = Xv \quad \text{(G5)}$$

Appendix H Detailed mathematical derivation of section 2.3.4

As we discussed in section 2 of the main text, V_GVC measures domestic value-added embodied in intermediate exports. Based on equation (20) in the main text, it is easy to see that the part of V_GVC which cross border for production only once can be measured as $\hat{V}LA^FLY$, where A^F is the import input use coefficient matrix (with zero block matrix in the diagonal block in the GN by GN input use coefficient matrix A). The part of V_GVC which cross border 2 times for production can be quantified as $\hat{V}LA^FLA^FLY$, and the part of V_GVC which cross border 3 times for production can be quantified as $\hat{V}LA^FLA^FLA^FLY$. The same goes on for the succeeding more border crossing for production.

Summing up all the above production stages, we obtain the total intermediate exports induced by V_GVC as follow

$$Ev\_GVC = \hat{V}LA^FL(I + 2A^FL + 3A^FLA^FL + 4A^FLA^FLA^FL + \cdots)Y$$

$$= \hat{V}LA^FL(I - A^FL)^{-1}(I - A^FL)^{-1}Y \quad \text{(H1)}$$

Based on the definition of Leontief Inverse, $B - L = BA^FL$, rearranging as

$$B = L(I - A^FL)^{-1} = L(I + A^FL + A^FLA^FL + A^FLA^FLA^FL + \cdots)$$

And $(I - A^D)B = (I - A^FL)^{-1}$

Inserting these two equation into the equation (H1),

$$\mathrm{Ev\_GVC} = \hat{V}LA^F B(I - A^D)BY = \hat{V}BA^F L(I - A^D)X = \hat{V}BA^F X \tag{H2}$$

Where $LA^F B = B - L = BA^F L$, and $BY = X$.

The total domestic output of foreign countries induced by V_GVC is the rest of output which equals that the total international (foreign) gross outputs induced by V_GVC minus the total intermediate exports induced by V_GVC. It can be measured as

$$\mathrm{Xvfd\_GVC} = \mathrm{Xvf}\_GVC - Ev\_GVC = \hat{V}LA^F BBY - \hat{V}LA^F B(I - A^D)BY$$
$$= \hat{V}LA^F BA^D BY = \hat{V}LA^F BA^D X \tag{H3}$$

Summing up the total domestic output of foreign countries induced by V_GVC and the total domestic output of source countries induced by V_GVC, we obtain the total domestic output induced by V_GVC.

$$Xvtd\_GVC = Xvfd\_GVC + Xvd\_GVC = \hat{V}LA^F BA^D X + \hat{V}LLA^F BY$$
$$= \hat{V}BA^D X - \hat{V}LX + \hat{V}X + \hat{V}BX - \hat{V}BX + \hat{V}LX - \hat{V}LLY$$
$$= \hat{V}BA^D X + \hat{V}BY - \hat{V}LLY = \hat{V}BBY - \hat{V}LLY - \hat{V}BA^F X \tag{H4}$$

Summing up the equation (H2) and (H4), we obtain the total output induced by V_GVC.

$$Xv\_GVC = \mathrm{Ev\_GVC} + Xvtd\_GVC = \hat{V}BBY - \hat{V}LLY$$
$$= Xvd\_GVC + Xvf\_GVC \tag{H5}$$

Muradov(2016) has proposed a measure of the average number of border crossing. Different from the number of border crossing for production measure defined in this paper, his measure includes not only border crossing for production, but also includes border crossing for consumption (it also accounts border crossing of final goods trade). The final exports induced by value added crossing border can be quantified as $\hat{V}BY^F$. Therefore, the total exports induced by value added based on forward linkage can be measured as

$$Ev\_T = Ev\_GVC + \hat{V}BY^F = \hat{V}BA^F X + \hat{V}BY^F = \hat{V}BE \tag{H6}$$

As we already know from WWZ (2013), domestic value added embodied in gross exports based on forward linkage can be measured as $\hat{V}LE$ at country/sector level, therefore, the average number of border crossing based on forward linkage can be measured as

$$CBv\_T = \frac{\hat{V}BE}{\hat{V}LE} \quad \text{(H7)}$$

Aggregating to world level, the average number of border crossing is the reciprocal of the domestic value added share in world exports

$$CBv\_T^w = \frac{VBE}{VLE} = 1/DVAS^w \quad \text{(H8)}$$

Similarly, as we discussed in section 2 of the main text, Y_GVC measure both domestic and foreign value-added in intermediate imports. It can be seen that the part of Y_GVC which cross border for production only once can be measured by $VLA^F L\hat{Y}$. The part of Y_GVC which cross border 2 times for production can be quantified as $VLA^F LA^F L\hat{Y}$, and the part of Y_GVC which cross border 3 times for production can be quantified as $VLA^F LA^F LA^F L\hat{Y}$. The same goes on for the succeeding more border crossing for production.

Summing up all the above stages, we obtain the total intermediate imports induced by Y_GVC as follow

$$Ey\_GVC = VLA^F L(I + 2A^F L + 3A^F LA^F L + 4A^F LA^F LA^F L + \cdots)\hat{Y}$$

$$= VLA^F L(I - A^F L)^{-1}(I - A^F L)^{-1}\hat{Y} = VBA^F B\hat{Y} = \mu' A^F B\hat{Y} \quad \text{(H9)}$$

Aggregating equation (H9) over all countries and sectors, we obtain total global intermediate imports.

The total domestic output of foreign countries induced by Y_GVC can be measured as

$$Xyfd\_GVC = Xyf\_GVC - Ey\_GVC = \mu BA^F L\hat{Y} - \mu A^F B\hat{Y}$$

$$= \mu LA^F B\hat{Y} - \mu A^F B\hat{Y} = A^D LA^F B\hat{Y} = \mu A^D BA^F L\hat{Y} \quad \text{(H10)}$$

Dividing equations (H9) and (H10) by Y_GVC, we can obtain (1) the average number of border crossing of intermediate imports used in source country final product production activities; (2) the average domestic production length of Y_GVC within countries involved in the GVCs entering the importing country as

$$PLyf\_GVC = CBy\_GVC + PLyfd\_GVC$$

$$= \frac{Ey\_GVC}{Y\_GVC} + \frac{Xyfd\_GVC}{Y\_GVC} \quad \text{(H11)}$$

The total domestic output induced by Y_GVC can be measured as

$$Xytd\_GVC = Xyfd\_GVC + Xyd\_GVC$$

$$= \mu A^D B A^F L\hat{Y} + \mu A^F L L\hat{Y} \quad \text{(H12)}$$

Summing up the equation (H9) and (H12), we obtain the total output induced by Y_GVC.

$$Xy\_GVC = Ev\_GVC + Xvtd\_GVC = \mu A^F B\hat{Y} + \mu A^D B A^F L\hat{Y} + \mu A^F L L\hat{Y}$$

$$= \mu(I + A^D L)A^F B\hat{Y} + \mu A^F L L\hat{Y} = \mu L A^F B\hat{Y} + \mu L\hat{Y} - VLL\hat{Y}$$

$$= \mu B\hat{Y} - \mu L\hat{Y} + \mu L\hat{Y} - VLL\hat{Y} = VBB\hat{Y} - VLL\hat{Y}$$

$$= Xvd\_GVC + Xvf\_GVC \quad \text{(H13)}$$

Adding final product exports, we obtain the total cross country exports induced by gross exports of country s

$$Ey\_T = Ey\_GVC + VB\hat{Y}^F = A^F B\hat{Y} + \hat{Y}^F \quad \text{(H14)}$$

And divide Ey_T by the value of total final goods and services production, we can obtain average border crossing of final production as

$$CBy = \frac{\mu A^F B\hat{Y} + \mu\hat{Y}^F}{\mu\hat{Y}} = \mu A^F B + \frac{\mu\hat{Y}^F}{\mu\hat{Y}} \quad \text{(H15)}$$

Muradov (2016) has proposed a measure of the average number of border crossing:

$$C = \frac{(I - A^F L)^{-2} Y^F + [(I - A^F L)^{-1} - I](I - A^F L)^{-1} Y^D}{(I - A^F L)^{-1} Y^F + [(I - A^F L)^{-1} - I] Y^D} \quad \text{(H16)}$$

He names the denominator as accumulated exports or $Ecum$, and where $L = (I - A^D)^{-1}$. From the definition of Leontief Inverse, we have $BA^F L = B - L$. Rearranging:

$$(I - A^F L)^{-1} = (I - A^D)B = B - I + I - A^D B \quad \text{(H17)}$$

$$= I + B - (I - A)B - A^D B = I + AB - A^D B = I + A^F B$$

Inserting (H17) into the numerator and the denominator of C respectively, we obtain:

$$(I - A^F L)^{-2} Y^F + [(I - A^F L)^{-1} - I](I - A^F L)^{-1} Y^D$$

$$= (I + A^F B)^2 Y^F + A^F B(I + A^F B) Y^D$$

$$= (I + A^F B)^2 Y^F + (I + A^F B) A^F B Y^D$$

$$= (I + A^F B)(Y^F + A^F B Y) \quad \text{(H18)}$$

$$Ecum = (I - A^F L)^{-1} Y^F + [(I - A^F L)^{-1} - I] Y^D$$

$$= (I + A^F B) Y^F + A^F B Y^D = Y^F + A^F B Y \quad \text{(H19)}$$

It is easy to see that the 1st terms in both the nominator (H18) and the denominator (H19) are final exports, the 2nd terms are intermediate exports induced by various final products. Therefore, the definition of C can be can be rearranging as

$$C = [(I + A^F B)(Y^F + A^F BY)] \emptyset (Y^F + A^F BY) \qquad \text{(H20)}$$

Multiply $\widehat{VL}$, the diagonal local value added multiplier (VL) matrix to both the nominator and denominator, we have

$$C = [\widehat{VL}(I + A^F B)Ecum] \emptyset (\widehat{VL}Ecum) \qquad \text{(H21)}$$

Aggregate both nominator and denominator across the column and along the row, of this GN by G matrix, we obtain a GN by 1 vector of gross exports at country/sector level.

$$Ecum\, u' = (I + A^F B)Y^F u' + A^F BY^D u' = Y^F u' + A^F BYu' = E \qquad \text{(H22)}$$

$$C = \frac{(I+A^F B)E}{E} = \frac{\widehat{VL}(I+A^F B)E}{\widehat{VL}E} \qquad \text{(H23)}$$

The average number of border crossing of country s can be measured as

$$C^s = \frac{\sum_{t\neq s}^{G} E^{st} + \sum_{r\neq s}^{G} A^{sr} \sum_{u}^{s} B^{ru} \sum_{t\neq u}^{G} E^{ut}}{\sum_{t\neq s}^{G} E^{st}} = \frac{[V^s L^{ss}]' \# [\sum_{t\neq s}^{G} E^{st} + \sum_{r\neq s}^{G} A^{sr} \sum_{u}^{s} B^{ru} \sum_{t\neq u}^{G} E^{ut}]}{[V^s L^{ss}]' \# [\sum_{t\neq s}^{G} E^{st}]} \text{(H24)}$$

The average number of border crossing based on forward industrial linkage in our method

$$CBv\_T = \frac{\hat{V}BE}{\hat{V}LE} = \frac{\hat{V}LE + \hat{V}LA^F BE}{\hat{V}LE} \qquad \text{(H25)}$$

$$CBv\_T^s = \frac{\hat{V}^s L^{ss} \sum_{t\neq s}^{G} E^{st} + \hat{V}^s L^{ss} \sum_{r\neq s}^{G} A^{sr} \sum_{u}^{s} B^{ru} \sum_{t\neq u}^{G} E^{ut}}{\hat{V}^s L^{ss} \sum_{t\neq s}^{G} E^{st}} \qquad \text{(H26)}$$

Compare equations (H24) and (H26), the two method seems different. The denominator of Muradov (2016) is a country/sector's total gross exports to the world, nominator is this total gross exports plus the sum of this exports have been used to produce exports (repeat counting). While the denominator in our method is domestic value-added embodied in gross exports， nominator is this part of value-added induced gross exports by all countries, they are not equal each other at the country/sector level. However, once we aggregating them to country and global level, these two methods become the same

$$c^s = \frac{V^s L^{ss}\sum_{t\neq s}^{G} E^{st} + \sum_{r\neq s}^{G} A^{sr}\sum_{u}^{s} B^{ru}\sum_{t\neq u}^{G} E^{ut}}{V^s L^{ss}\sum_{t\neq s}^{G} E^{st}} \quad \text{(H27)}$$

$$cbv\_t^s = \frac{V^s L^{ss}\sum_{t\neq s}^{G} E^{st} + V^s L^{ss}\sum_{r\neq s}^{G} A^{sr}\sum_{u}^{s} B^{ru}\sum_{t\neq u}^{G} E^{ut}}{V^s L^{ss}\sum_{t\neq s}^{G} E^{st}} \quad \text{(H28)}$$

$$C^w = \frac{u(I+A^F B)E}{uE} = \frac{u\widehat{VL}(I+A^F B)E}{u\widehat{VL}E} = \frac{VLE+VLA^F BE}{VLE} = \frac{VBE}{VLE} \quad \text{(H29)}$$

$$CBv\_T^w = \frac{u\hat{V}BE}{u\hat{V}LE} = \frac{VBE}{VLE} \quad \text{(H30)}$$

Appendix I Detailed mathematical derivation of section 2.3.5

Based on the decomposition of final goods and services production at each country/sector pair in equation 5, following the same logic of Sections 2.3.2 and 2.3.3, we can compute domestic and international gross outputs derived by different parts of final product production.

In a one stage production process, the domestic value added generated from a particular country/sector (for example, sector *i* of country *s*) is directly embodied in its final products that are consumed at home or exported to country *r* and consumed there. It can be measured as $V^s\hat{Y}^{ss} + V^s\sum_{\mathrm{r}\neq s}^{G}\hat{Y}^{sr}$ and its domestic production length equals 1 and its international production length equals 0.

In a two stage production process, the domestic value added generated from country *s* will be first embodied in its gross output that is used as intermediate input either by home country *s* or other countries (through exports) in the production of final products. It can be measured as $V^s A^{ss}\hat{Y}^{ss} + V^s A^{ss}\sum_{\mathrm{r}\neq s}^{G}\hat{Y}^{sr}$. Their domestic production lengths equal 2 and 1, respectively, and their international production lengths equal 0 and 1, respectively. Besides domestic value-added, production of final products in country s may use other countries value-added, which can be measured as $\sum_{t\neq s}^{G} V^t A^{ts}\sum_{\mathrm{r}}^{G}\hat{Y}^{sr}$. Their domestic and international production lengths equal 1 and 1 respectively.

In a three stage production process, the domestic value added generated from country *s* and foreign value-added generated from country t will be embodied in the final products produced from the third stage and consumed in all possible destination counties. It can be measure as $V^s A^{ss} A^{ss}\hat{Y}^{ss} + V^s A^{ss} A^{ss}\sum_{\mathrm{r}\neq s}^{G}\hat{Y}^{sr}$, $\sum_{t\neq s}^{G} V^t A^{ts} A^{ss}\sum_{\mathrm{r}}^{G}\hat{Y}^{sr}$, and $\sum_{t}^{G} V^t\sum_{u\neq s}^{G} A^{tu} A^{us}\sum_{\mathrm{r}}^{G}\hat{Y}^{sr}$. Their domestic production lengths equal 3, 2, and 1, respectively, and their international production lengths equal 0, 1, and 2, respectively.

The same holds for an *n*-stage production process.

Summing over all production stages in an infinite stage production process, we have the final goods and services produced and consumed domestically with only domestic value added and outputs as follow

$$Y\_D = V\hat{Y}^D + VA^D\hat{Y}^D + VA^DA^D\hat{Y}^D + \cdots = VL\widehat{Y^D}\mu \quad \text{(I1a)}$$

$$y\_D = V\hat{Y}^D + 2VA^D\hat{Y}^D + 3VA^DA^D\hat{Y}^D + \cdots = VLL\widehat{Y^D}\mu \quad \text{(I1b)}$$

The final goods and services produced for traditional exports with only domestic value added and outputs as follow

$$Y\_RT = V\hat{Y}^F + VA^D\hat{Y}^F + VA^DA^D\hat{Y}^F + \cdots = VL\widehat{Y^F}\mu \quad \text{(I2a)}$$

$$Xy\_RT = V\hat{Y}^F + 2VA^D\hat{Y}^F + 3VA^DA^D\hat{Y}^F + \cdots = VLL\widehat{Y^F}\mu \quad \text{(I2b)}$$

The final goods and services production that related to cross border production activities with both domestic (the 1st term) and foreign (the 2nd term)value added and outputs can be expressed as

$$Y\_GVC = [VA^F + VA^FA^D + VAA^F + VA^FA^DA^D + VAA^FA^D + VAAA^F + \cdots]\widehat{Y\mu}$$

$$= VA^FL\widehat{Y\mu} + VAA^FL\widehat{Y\mu} + VAAA^FL\widehat{Y\mu} \ldots = VBA^FL\widehat{Y\mu} \quad \text{(I3a)}$$

$$Xyd\_GVC = VA^F\widehat{Y\mu} + 2VA^FA^D\widehat{Y\mu} + VAA^F\widehat{Y\mu} + 3VA^FA^DA^D\widehat{Y\mu}$$

$$+2VAA^FA^D\widehat{Y\mu} + VAAA^F\widehat{Y\mu} + \cdots$$

$$= VBA^F\widehat{Y\mu} + 2VBA^FA^D\widehat{Y\mu} + 3VBA^FA^DA^DL\widehat{Y\mu} + \cdots = VBA^FLL\widehat{Y\mu} \quad \text{(I3b)}$$

$$Xyf\_GVC = VA^F\widehat{Y\mu} + VA^FA^D\widehat{Y\mu} + 2VAA^F\widehat{Y\mu} + VA^FA^DA^D\widehat{Y\mu}$$

$$+2VAA^FA^D\widehat{Y\mu} + 3VAAA^F\widehat{Y\mu} + \cdots$$

$$= VA^FL\widehat{Y\mu} + 2VAA^FL\widehat{Y\mu} + 3VAAA^FL\widehat{Y\mu} + \cdots = VBBA^FL\widehat{Y\mu} \quad \text{(I3c)}$$

$$Xy\_GVC = Xyd\_GVC + Xyf\_GVC = VBA^FLL\widehat{Y\mu} + VBBA^FL\widehat{Y\mu}$$

$$= V(B-L)L\widehat{Y\mu} + VB(B-L)\widehat{Y\mu} = VBB\widehat{Y\mu} - VLL\widehat{Y\mu} \quad \text{(I3d)}$$

Summing up the equation (I1b), (I2b), and (I3d), we have the total outputs induced by production of country s' final goods and services

$$Xy = Xy\_D + Xy\_RT + Xy\_GVC$$

$$= VLL\widehat{Y^D}\mu + VLL\widehat{Y^F}\mu + VBB\widehat{Y}\mu - VLL\widehat{Y}\mu = VBB\widehat{Y}\mu \quad (I4)$$

Aggregating V_GVC^s and Y_GVC^s to global level

$$V\_GVC^w = \mu'\hat{V}LA^F BY\mu = \mu'\hat{V}BY\mu - \mu'\hat{V}LY\mu = VBY\mu - VLY\mu \quad (I5)$$

$$Y\_GVC^w = VBA^F L\widehat{Y\mu}\mu = VBB\widehat{Y\mu}\mu - VLL\widehat{Y\mu}\mu = VBBY\mu - VLLY\mu \quad (I6)$$

Obviously, V_GVC^w and Y_GVC^w equal each other a global level

Aggregating Xv_GVC^sin equation (G4) and Xy_GVC^s in equation (H3d) to global level

$$V\_GVC^w = \mu'\hat{V}BBY\mu - \mu'\hat{V}LLY\mu = VBBY\mu - VLLY\mu \quad (I7)$$

$$Y\_GVC^w = VBB\widehat{Y\mu}\mu - VLL\widehat{Y\mu}\mu = VBBY\mu - VLLY\mu \quad (I8)$$

Obviously, Xv_GVC^w and Xy_GVC^w are the same at World level.

Therefore, the average production length of V_GVC^w and Y_GVC^w are the same.

Appendix J Difference between production length and APL in mathematical terms

Production length has some similarities to the Average Propagation Length (APL) proposed by Erik et.al (2005), but the two are different in both economic interpretation and mathematical expression. The APL is used to measure the distance between two sectors, which defined as the average number of steps that it takes an exogenous change in one sector to affect the value of production in another sector. Based on equation 11 of Erik et.al paper, The APL can be defined as

$$\text{APL} = \frac{\text{G}(\text{G}-\text{I})}{\text{G}-\text{I}} = \frac{B(B-I)}{B-I} \quad (J1)$$

And the APL from sector i to sector j can be expressed as

$$apl_{ij} = \frac{1}{g_{ij}-\delta_{ij}}\left[\sum_k^n g_{ik}g_{kj} - g_{ij}\right] = \frac{1}{b_{ij}-\delta_{ij}}\left[\sum_k^n b_{ik}b_{kj} - b_{ij}\right] \quad (J2)$$

The average production length we defined in main text,

$$PL = \frac{\hat{V}BB\hat{Y}}{\hat{V}B\hat{Y}} = \frac{BB}{B} \quad (J3)$$

$$pl_{ij} = \frac{v_i \sum_k^n b_{ik}b_{kj}y_j}{v_i b_{ij} y_j} = \frac{\sum_k^n b_{ik}b_{kj}}{b_{ij}} \quad (J4)$$

If sector i ≠ sector j, $\delta_{ij} = 0$, therefore

$$apl_{ij} = \frac{\sum_k^n b_{ik}b_{kj} - b_{ij}}{b_{ij}} = \frac{\sum_k^n b_{ik}b_{kj}}{b_{ij}} - 1 = pl_{ij} - 1 \tag{J5}$$

If sector i = sector j, $\delta_{ij} = 1$, therefore

$$apl_{ii} = \frac{\sum_k^n b_{ik}b_{ki} - b_{ii}}{b_{ii} - 1} = \frac{pl_{ii}b_{ii} - b_{ii}}{b_{ii} - 1} = pl_{ii} + \frac{pl_{ii} - b_{ii}}{b_{ii} - 1} \tag{J6}$$

From definition of Leontief Inverse, $b_{ii} - 1 > 0$. From the definition of production length, $pl_{ii} - b_{ii} > b_{ii}b_{ii} - b_{ii} > 0$.

Therefore, in the off-diagonals, APL are smaller than production length, but in diagonal elements, APL are larger than production length.

We defined the average production length as total output value induced by an unit particular value added or final products, which equals total gross output to GDP ratio. Therefore, if a closed economy's total output and GDP are stable, its' average production length is also robust. However, the APL is the average number of production stages that it takes an exogenous change in one sector to affect the value of production in another sector. APL will change as number of sector classification changes.

Let's use a simple example to illustrate the relationship between APL and PL

Table J1 An ICIO Table with country S and R

	S1	S2	R	S	R	TO
S1	3	1	2	3	1	10
S2	1	2	1	1	0	5
R	1	1	4	2	2	10
VA	5	1	3			
TI	10	5	10			

Table J2 The **input coefficients** matrix

A	S1	S2	R
S1	0.3	0.2	0.2
S2	0.1	0.4	0.1
R	0.1	0.2	0.4

Table J3 The **Leontief Inverse** matrix

B	S1	S2	R
S1	1.63	0.77	0.67
S2	0.34	1.92	0.43
R	0.38	0.77	1.92

Table J4 The Square of Leontief Inverse matrix

BB	S1	S2	R
S1	3.19	3.25	2.73
S2	1.36	4.29	1.89
R	1.63	3.25	4.29

Table J5 The average production length (PL)

PL	S1	S2	R
S1	1.95	4.23	4.05
S2	4.05	2.23	4.37
R	4.23	4.23	2.23

Table J6 Aggregating the average production length (PL)

	S1	S2	R	WLD
PLy	2.36	3.46	3.03	**2.78**
Y	4.00	1.00	4.00	9.00
PLv	2.69	3.5	2.69	**2.78**
VA	5	1	3	9.00

Table J7 Combining the ICIO table to a sector (World level)

	W	Y	TO
W	16	9	25
V	9		
TI	25		

And A=0.64， B=2.78，BB=7.72，PL=2.78

The average production length in ICIO table and a sector model are the same.

Table J8 The indirect input coefficients matrix

B-I	S1	S2	R
S1	0.63	0.77	0.67
S2	0.34	0.92	0.43
R	0.38	0.77	0.92

Table J9 The matrix of indirect input coefficients and Leontief Inverse matrix

B(B-I)	S1	S2	R
S1	1.56	2.49	2.05
S2	1.03	2.37	1.46
R	1.24	2.49	2.37

Table J10 The APL

APL	S1	S2	R
S1	2.45	3.23	3.05
S2	3.05	2.56	3.37
R	3.23	3.23	2.56

Table J11 The Aggregating APL

	S1	S2	R	WLD

APL_b	2.73	2.9	2.82	**2.81**
Z_b	5	4	7	16
APL_f	2.78	2.9	2.78	**2.81**
Z_f	6	4	6	16

Combine the ICIO table to a sector (World level)

And A=0.64， B-I=1.78，B(B-I)=4.94，APL=2.78

When sectors in ICIO are aggregated, the APL changes, while PL stays the same.

Appendix K：The Production Length of the US become longer or shorter over time?

Fally (2012) showed a somewhat puzzling finding that the production chain (or the distance to the final demand) appears to have shortened over time and he concludes such a trend is also a global phenomenon.

Fally's definition of "production length" (or "Upstreamness") is the average number of production stages from a sector's gross output to the final users. His results rely on the US IO tables, which covers 85 industries from 1947 to 2002, or 540 products categories from 1967 to 1992. To estimate the global production length, Fally made a very strong assumption that **"Same industry have the same production length across countries"**. In this part, we will show that **this strong assumption is the main factor that leads to the puzzling finding**.

First of all, consistent with Fally, **our results also shows that the production length of the US is getting shorter**. Table A1 reports the overall production length for US sectors. The production length has decreased for 26 out of 35 sectors from 1995 to 2011.

Table K1 Production Length (Forward Linkage) of US Sectors, 2011

Sector	Year 1995	Year 2011	Become shorter?
Agriculture	2.677	2.583	√
Mining	2.918	2.487	√
Food	1.679	1.688	
Textiles Products	2.227	2.112	√
Leather and Footwear	1.632	1.252	√
Wood Products	2.531	2.597	
Paper and Printing	2.581	2.306	√
Refined Petroleum	2.375	2.305	√

Chemical Products	2.665	2.468	√
Rubber and Plastics	2.659	2.509	√
Other Non-Metal	2.615	2.563	√
Basic Metals	3.025	3.027	
Machinery	1.834	1.784	√
Electrical Equipment	2.187	2.016	√
Transport Equipment	1.802	1.672	√
Recycling	1.570	1.588	
Electricity, Gas and Water	2.061	1.820	√
Construction	1.246	1.295	
Sale of Vehicles and Fuel	1.386	1.324	√
Wholesale Trade	2.154	1.937	√
Retail Trade	1.321	1.204	√
Hotels and Restaurants	1.446	1.435	√
Inland Transport	2.429	2.289	√
Water Transport	2.298	1.740	√
Air Transport	1.806	1.654	√
Other Transport	2.805	2.693	√
Post and Telecommunications	2.266	2.115	√
Financial Intermediation	2.187	2.311	
Real Estate	1.472	1.429	√
Business Activities	2.590	2.453	√
Public Admin	1.103	1.110	
Education	1.254	1.097	√
Health and Social Work	1.036	1.029	
Other Services	1.764	1.785	
Private Households	1.386	1.324	√

Figure K1 Average Production Length for US

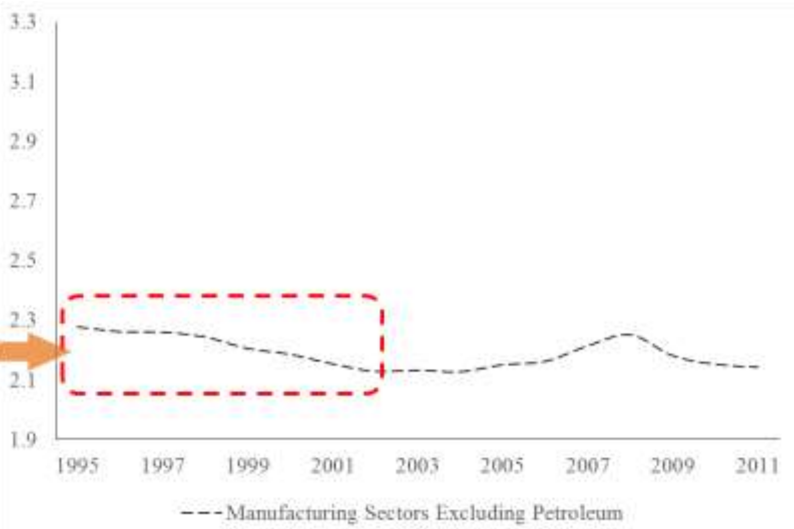
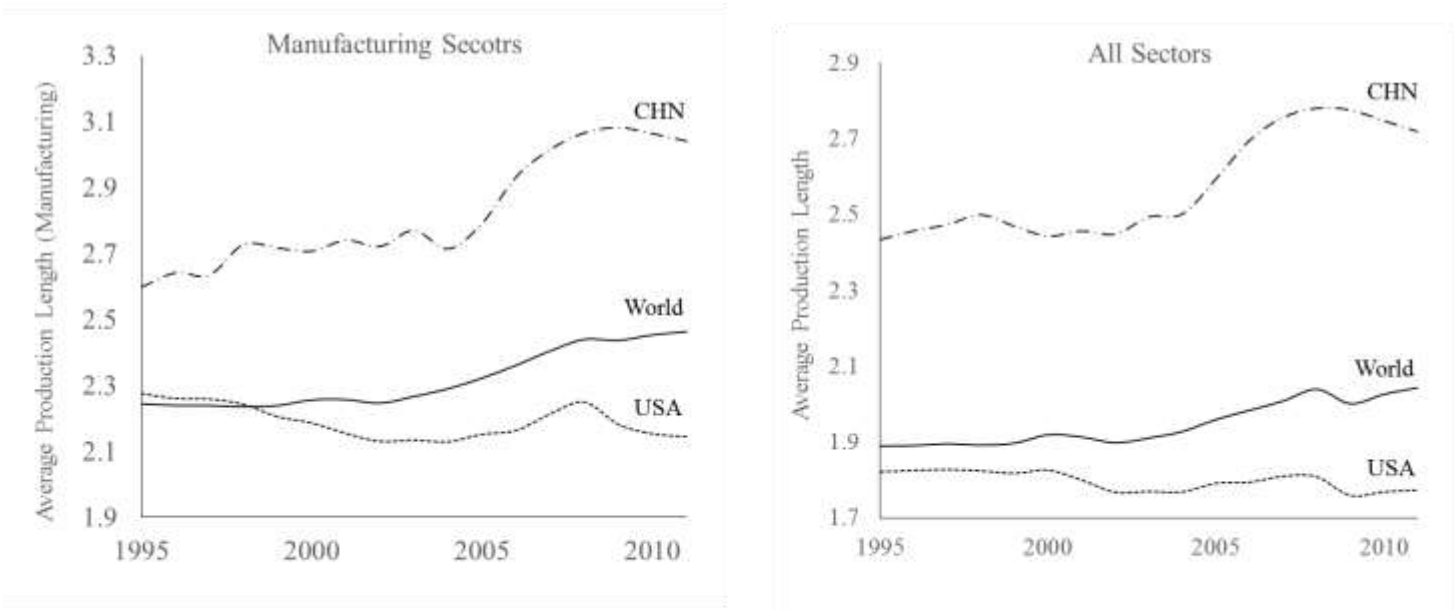


Figure 1 in Fally (2012) "Production Staging: Measurement and Facts"
Aggregate measure of vertical fragmentation (tradable goods excluding petroleum)

Aggregated to the country level, we also find that the average production length for US industries as a whole decreased during the period 1995-2003, but has increased since then until 2008, the global financial crisis, then in a decline trend again

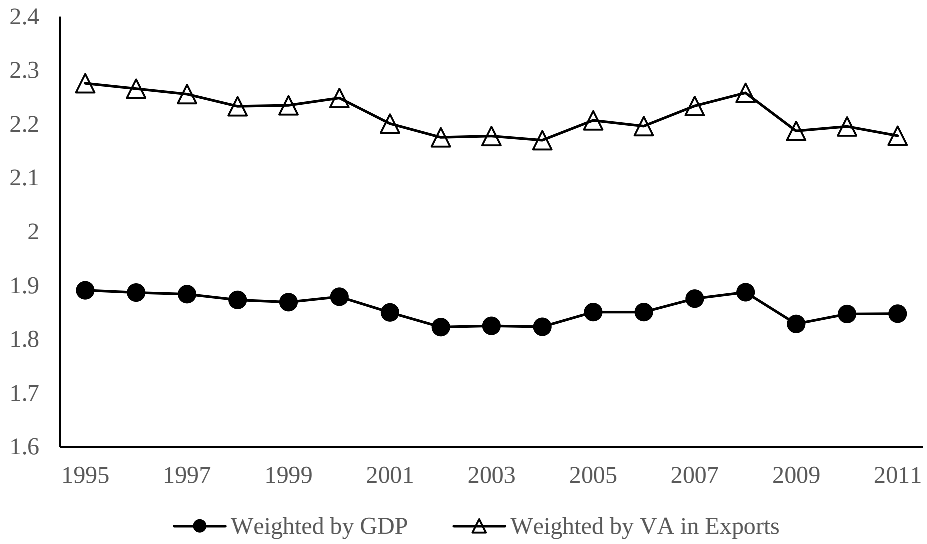
However, this finding is reversed at the global level**.** As shown in Figure A2, the production length for a certain industry may vary considerably across countries. While the length of production in the United States decreased, it has an opposite pattern in China. which means that the assumption “Same industry have the same production length across countries” does not hold in reality. As a results, for the world as a whole, we have observed that the production chain has become longer.

Figure K2   Average Production Length, China, US and the World



To understand why this assumption is crucial to the result, we re-estimate the weighted average global production length with the assumption that the production length of a certain sector is the same across countries and equal to the US. After applying this strong assumption, the upward trend of the global production length in Figure K2 has disappeared, and instead, we see a downward trend in Figure K3.

Figure K3 Global Average Production Length under the "Equal Length Assumption"



Appendix L Changes of production length measure and APL when industry aggregation changes.

Figure L1 The Changes of TPL and APL at Country Level

Forward Linkage

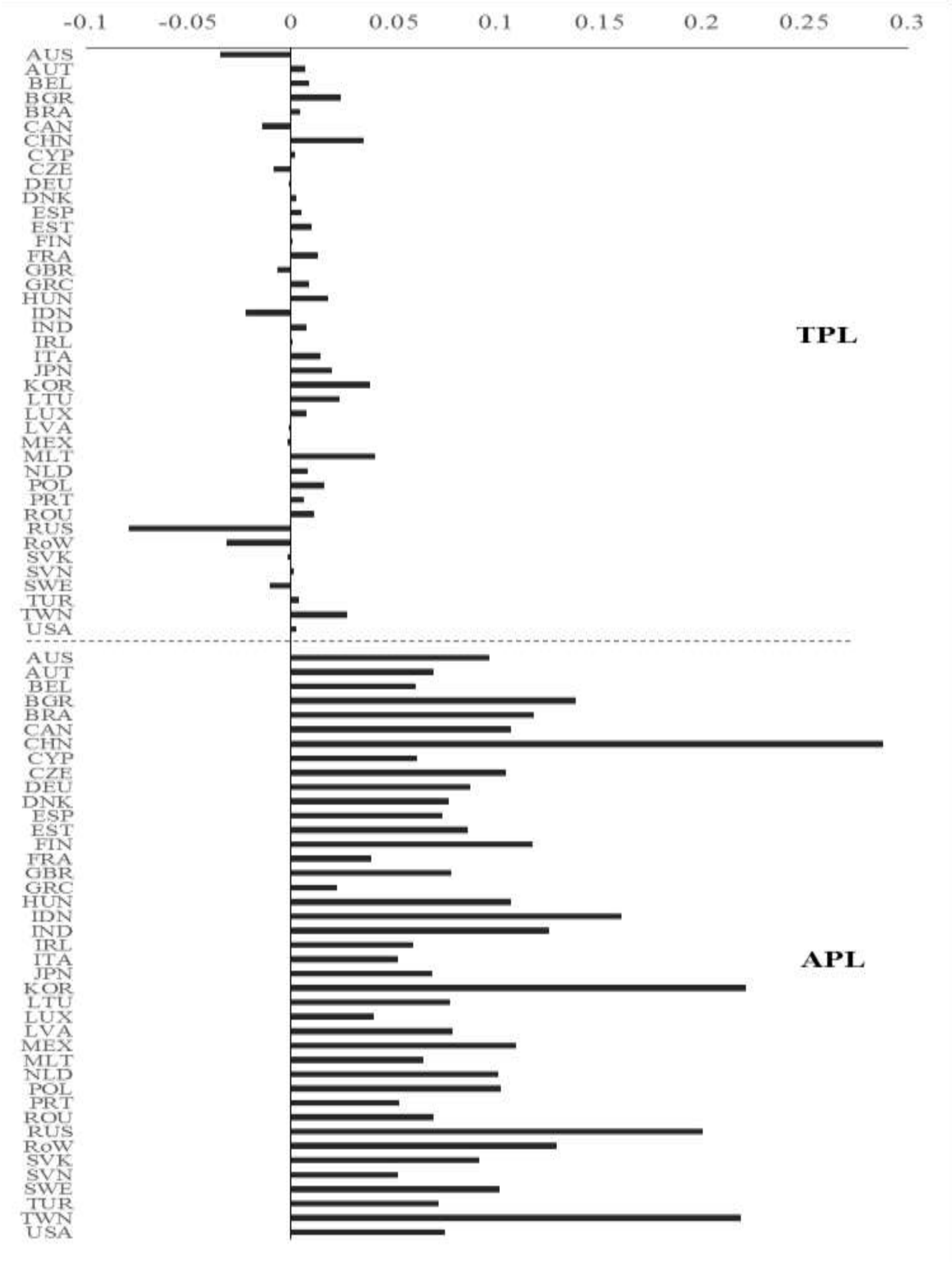


Figure L2 The Changes of TPL and APL at Country Level
Backward Linkage

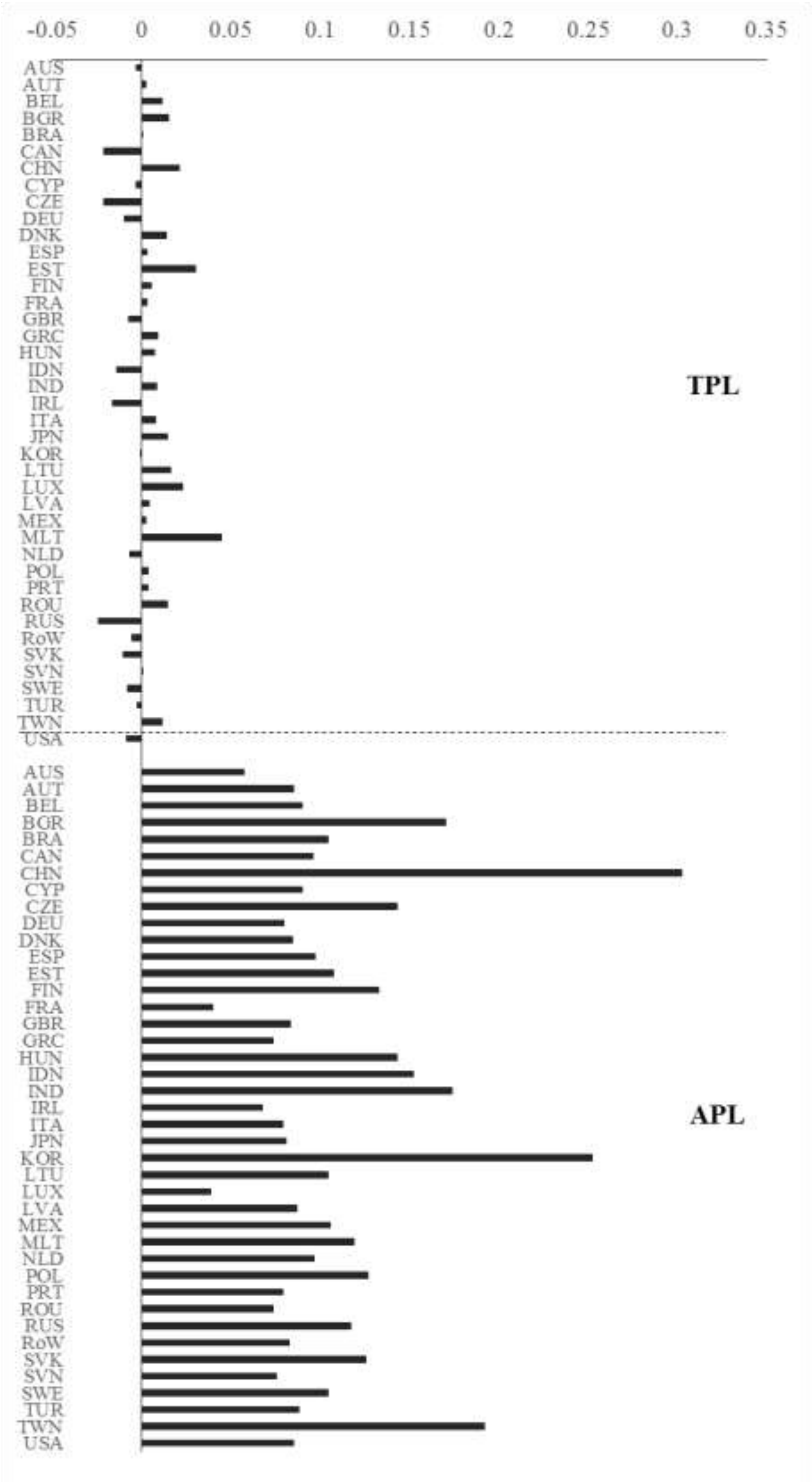


Table A1 WIOD Sectors

Code	NACE	Industry	Description
C01	**AtB**	Agriculture	Agriculture, Hunting, Forestry and Fishing
C02	**C**	Mining	Mining and Quarrying
C03	**15t16**	Food	Food, Beverages and Tobacco
C04	**17t18**	Textiles Products	Textiles and Textile Products
C05	**19**	Leather and Footwear	Leather, Leather and Footwear
C06	**20**	Wood Products	Wood and Products of Wood and Cork
C07	**21t22**	Paper and Printing	Pulp, Paper, Paper, Printing and Publishing
C08	**23**	Refined Petroleum	Coke, Refined Petroleum and Nuclear Fuel
C09	**24**	Chemical Products	Chemicals and Chemical Products
C10	**25**	Rubber and Plastics	Rubber and Plastics
C11	**26**	Other Non-Metal	Other Non-Metallic Mineral
C12	**27t28**	Basic Metals	Basic Metals and Fabricated Metal
C13	**29**	Machinery	Machinery, Nec
C14	**30t33**	Electrical Equipment	Electrical and Optical Equipment
C15	**34t35**	Transport Equipment	Transport Equipment
C16	**36t37**	Recycling	Manufacturing, Nec; Recycling
C17	**E**	Electricity, Gas and Water	Electricity, Gas and Water Supply
C18	**F**	Construction	Construction
C19	**50**	Sale of Vehicles and Fuel	Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel
C20	**51**	Wholesale Trade	Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles
C21	**52**	Retail Trade	Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods
C22	**H**	Hotels and Restaurants	Hotels and Restaurants
C23	**60**	Inland Transport	Inland Transport
C24	**61**	Water Transport	Water Transport
C25	**62**	Air Transport	Air Transport
C26	**63**	Other Transport	Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies
C27	**64**	Post and Telecommunications	Post and Telecommunications
C28	**J**	Financial Intermediation	Financial Intermediation
C29	**70**	Real Estate	Real Estate Activities
C30	**71t74**	Business Activities	Renting of M&Eq and Other Business Activities
C31	**L**	Public Admin	Public Admin and Defense; Compulsory Social Security
C32	**M**	Education	Education
C33	**N**	Health and Social Work	Health and Social Work
C34	**O**	Other Services	Other Community, Social and Personal Services
C35	**P**	Private Households	Private Households with Employed Persons

Table A2 WIOD Country and Region

Label	Country	Region	Label	Country	Region
AUS	Australia	Asia-Pacific	**IRL**	Ireland	Europe
AUT	Austria	Europe	**ITA**	Italy	Europe
BEL	Belgium	Europe	**JPN**	Japan	Asia-Pacific
BGR	Bulgaria	Europe	**KOR**	South Korea	Asia-Pacific
BRA	Brazil	American	**LTU**	Lithuania	Europe
CAN	Canada	American	**LUX**	Luxembourg	Europe
CHN	China	Asia-Pacific	**LVA**	Latvia	Europe
CYP	Cyprus	Europe	**MEX**	Mexico	American
CZE	Czech Republic	Europe	**MLT**	Malta	Europe
DEU	Germany	Europe	**NLD**	Netherlands	Europe
DNK	Denmark	Europe	**POL**	Poland	Europe
ESP	Spain	Europe	**PRT**	Portugal	Europe
EST	Estonia	Europe	**ROM**	Romania	Europe
FIN	Finland	Europe	**RUS**	Russia	Europe
FRA	France	Europe	**SVK**	Slovak Republic	Europe
GBR	United Kingdom	Europe	**SVN**	Slovenia	Europe
GRC	Greece	Europe	**SWE**	Sweden	Europe
HUN	Hungary	Europe0	**TUR**	Turkey	Europe
IDN	Indonesia	Asia-Pacific	**TWN**	Taiwan	Asia-Pacific
IND	India	Asia-Pacific	**USA**	United States	American