Trade and Informality in the Presence of Labor Market Frictions and Regulations*

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Abstract

Motivated by recent work on the labor market effects of trade, we build a model of trade with labor market frictions and regulations that are not perfectly enforced by the government. Heterogeneous firms decide whether to operate formally or informally, allowing for a link between globalization, informality and unemployment. We estimate the model using several data sources from Brazil, including matched employer-employee data from formal and informal firms and workers. We perform counterfactual analyses to understand how increasing trade openness affects informality, unemployment and welfare under different scenarios of labor market regulations and levels of enforcement. Our results suggest that domestic policies leading to a reduction in informality have the potential to strongly increase aggregate productivity and welfare, at the expense of modest increases in unemployment. These policies have a much larger effect on welfare relative to policies aiming to reduce international trade costs. The informal sector works as a buffer in the event of negative economic shocks. However, the welfare gains from eradicating informality are so significant that it is hard to justify lenience toward the informal sector on the basis that it works as a buffer following negative economic shocks.

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1 Introduction

Recent research has shown that shifts into and out of unemployment and informal employment constitute important margins of labor market adjustment in response to trade shocks. In particular, McCaig and Pavcnik (2018) show that Vietnamese manufacturing sectors that benefitted the most from tariff reductions induced by the United States-Vietnam Bilateral Trade Agreement experienced substantial increases in formal-sector employment relative to other sectors. On the other hand, Dix-Carneiro and Kovak (2019) document that, following Brazil's trade liberalization episode of the 1990s, regions more exposed to foreign competition faced increases in unemployment in the medium run relative to the national average. However, in the long run, foreign competition had no effect on unemployment, but a significant positive effect on informal employment at the local level. These results prompted the authors to hypothesize that the presence of a large informal sector may have worked as a buffer to trade-displaced workers. In the absence of a large informal sector, the long-run effect of foreign competition on unemployment could have been much more severe. This view is corroborated by Ponczek and Ulyssea (2018) who document that the medium-run effect of liberalization-induced foreign competition on unemployment was larger in regions where labor market regulations were more strictly enforced.

Although a substantial share of the labor force in developing countries is employed informally (for example, in Latin America, informality ranges from 35 percent in Chile to 80 percent in Peru), trade models have typically abstracted from informality. Therefore, in light of recent results, understanding the labor market and welfare effects of globalization within a model of trade with informality, unemployment and regulations is a first order question.

On the worker side, one can broadly define informality in two ways: The first defines a worker as informal if she does not have permanent and stable employment associated with benefits such as health and social security. The second defines a worker as informal if, in addition to not receiving benefits, she is invisible to the tax authorities and her employer illegally evades labor market regulations (including minimum wages and firing rules). The first definition has become relevant even in developed countries in recent years with the emergence of companies such as Uber, Taskrabbit or Airbnb. The second definition applies primarily to developing countries where informality, and the tax evasion associated with it, is a first-order issue and has been shown to be associated with low productivity and a barrier to growth. On the firm side, informality implies that firms do not comply with taxes or relevant regulations (e.g. labor laws). This can be harmful to the economy for two main reasons. First, it implies substantial tax evasion thus hindering fiscal capacity and the provision of public goods. Second, it might entail substantial misallocation of resources and hamper growth, as non-productive firms can survive by evading taxes and avoiding compliance with labor market regulations. On the other hand, as suggested by the results in Dix-Carneiro and Kovak (2019) and Ponczek and Ulyssea (2018), informality may provide *de facto* flexibility for firms and workers to cope with adverse shocks.

We build on Cosar et al. (2016) and develop a structural equilibrium model with heterogeneous firms that choose whether to operate in the formal or in the informal sector. The model features a rich institutional setting, where formal firms must comply with minimum wages, and are subject to firing costs as well as payroll and revenue taxes. Taxes and labor market regulations are imperfectly enforced by the government, giving rise to incentives for some firms to be informal. The labor market is characterized by labor market frictions and costs of hiring, features leading to unemployment. The economy is composed by tradable and non-tradable sectors, and tradable sector firms are able to export. We estimate the model using several data sources, including matched employer-employee data from formal and informal firms and workers in Brazil, as well as several other sources of firm- and worker-level data such as household surveys, manufacturing and services censuses, and customs data. Next, we conduct a series of counterfactual experiments to better understand the impact of trade shocks on an economy with a large informal sector.

Brazil constitutes a relevant case study for several reasons. First, it has strict and burdensome labor regulations that are imperfectly enforced and a large informal sector: nearly two thirds of businesses, 40 percent of GDP and 35 percent of employees are informal (Ulyssea, 2018). Second, the Brazilian case is typical of developing countries, especially in Latin America, where the urban labor force employed informally averages over 50 percent, with this number varying from 35 percent in Chile to 80 percent in Peru (Perry et al., 2007). Third, it has unique data availability and quality, allowing the direct observation of informality for workers and firms. We define as informal workers those employees who do not hold a formal labor contract, which in Brazil is sharply defined as having a booklet (*carteira de trabalho*) that registers workers' entire employment history in the formal sector. We define as informal firms those not registered with the tax authorities, which means that they do not possess the tax identification number required for Brazilian firms (*Cadastro Nacional de Pessoa Juridica* – CNPJ). We can observe both definitions directly from the data available (more details are provided in the Data section). Finally, even though Brazil experienced a relatively fast and intense trade liberalization episode in early 1990s (e.g. Dix-Carneiro and Kovak, 2017), it remains a relatively closed economy. Therefore, our analyses in this paper are of great policy relevance.

Our results suggest that an increase in globalization via reduced iceberg trade costs substantially reduces informality within the tradable sector. This result is consistent with the empirical results documented in McCaig and Pavcnik (2018) who focus only on the Vietnamese tradable sector. However, as real incomes rise, increased globalization makes it more attractive for firms to enter the non-tradable sector (both formally and informally), leading to a relative increase in an informality-intensive sector. Given that informality in the tradable sector is relatively small to start with, the total informality share in the economy is reduced, but only slightly. The small role of trade on informality predicted by our model is consistent with the casual observation that the informal sector has not substantially shrunk in middle-income economies despite the large-scale liberalization episodes they went through in the 1980s and 1990s.

As suggested by Dix-Carneiro and Kovak (2019) and Ponczek and Ulyssea (2018), our results confirm that the informal sector does work as a buffer during bad times. In the event of an aggregate *negative* shift in the distribution of firm-level productivities (e.g., a supply-driven recession), the benchmark economy experiences a smaller decline in unemployment and welfare relative to an otherwise similar economy, except that the costs of informality are prohibitive. However, our counterfactual experiments show that eliminating informality altogether is associated with very large aggregate productivity and welfare gains (≈42%). These gains are so large that it is hard to justify lenience toward the informal sector on the basis that it works as a buffer during bad times. The gains from eradicating informality are driven by a strong reallocation of workers from less productive informal firms to more productive firms. Although unemployment increases following the shutdown of informal firms, the increase in aggregate productivity vastly offsets this negative effect. One important note of caution behind this effect is that we do not take into account transitional dynamics between steady states. In our model, it takes time for firms to grow as they face convex hiring costs, so that shutting down informality at once is likely to generate large increases in the unemployment rate and declines in aggregate welfare in the short and medium run. We are currently working on counterfactual experiments describing these transitional dynamics.

Given that our results suggest that informality leads to a quantitatively important misallocation of resources, we address the question of how to eradicate or attenuate the important misallocation of resources driven by the existence of a large informal sector. One natural starting point is to investigate how labor market regulations, such as minimum wages and firing costs, and institutional aspects, such as bureaucracy and red tape, affect informality, productivity and welfare. Our findings suggest that deregulation plays a small role, and can, in some cases, even increase informality. We also simulate upward shifts in the distribution of firm-level productivities (uniform productivity growth). Perhaps surprisingly, this technological upgrade can lead to more, *not less*, informality. Indeed, the productivity threshold for informal-sector entry is more easily met, leading to a strong growth in the mass of relatively unproductive informal firms and reducing aggregate *measured* productivity.[1] We conclude that trade policy, more lax labor market regulations and (productivity) growth, by themselves, are unlikely to significantly attenuate the misallocation of resources present in our model. The policy with higher payoff in terms of welfare and productivity gains seems to be more strict enforcement of regulations and taxes (such as larger fines on informal firms and increases in the probability of detection of informal firms). How exactly this policy of more strict enforcement should be implemented, and how costly it would be, is an exciting topic for further research.

Our paper is organized as follows. Section 2 outlines our model. Section 3 discusses the main regulations in place in the Brazilian economy, and Section 4 describes the data we use to estimate the model. Section 5 details the estimation procedure, discusses identification and shows how the model fits key aspects of the data. Section 6 shows our counterfactual experiments and Section 7 presents our main takeaways.

2 Model

2.1 Set Up

The economy is populated by homogeneous, infinitely-lived workers-consumers. Individuals derive utility from the consumption of a composite good of differentiated, tradable sector goods C and from the consumption of a composite good of differentiated, non-tradable sector goods S. Preferences are given by

$$U = \sum_{t=1}^{\infty} \frac{C_t^{\zeta} S_t^{1-\zeta}}{(1+r)^t}, \tag{1}$$

[1]Note that we make a distinction between the distribution of productivities, a primitive of the model, and aggregate measured productivity, and endogenous variable given by the employment-weighted average productivity in the economy.

where

$$C_t = \left(\int_0^{N_{Ct}} c_t(n)^{\frac{\sigma_C - 1}{\sigma_C}} dn \right)^{\frac{\sigma_C}{\sigma_C - 1}} \tag{2}$$

$$S_t = \left(\int_0^{N_{St}} s_t(n)^{\frac{\sigma_S - 1}{\sigma_S}} dn \right)^{\frac{\sigma_S}{\sigma_S - 1}} \tag{3}$$

and $\zeta \in (0,1)$ is the fraction of expenditure on tradable sector goods, $\sigma_k > 1$ is the elasticity of substitution across varieties within sector k, N_{kt} denotes the measure of varieties available in sector $k \in \{C, S\}$ at time t, and $n \in (0, N_{kt})$ indexes varieties. As we will focus on steady state equilibria, we henceforth drop the time subscript for notational convenience.

2.2 Firms

There is a continuum of firms in both tradable and non-tradable sectors. Formal and informal firms coexist in both sectors, and each firm produces a unique variety $n \in (0, N_k)$, $k \in \{C, S\}$. Firms use labor as the single input in a constant returns to scale production function: $q(z, \ell) = z\ell$, where ℓ denotes firm's employment size. Firms' idiosyncratic productivity evolves over time following the AR(1) process below:

$$\ln z' = \rho_k \ln z + \sigma_k^z \varepsilon, \tag{4}$$

where $\rho_k \in (0,1)$, $\varepsilon \sim N(0,1)$ and σ_k^z is the standard deviation of the shocks. It will be convenient to denote $G_k(z'|z)$ the cumulative distribution function of z' conditional on z and $g_k(z'|z)$ its density.[2]

Monopolistic competition implies that revenues in sector $k \in \{C, S\}$ are given by:

$$R_k(z, \ell) = \left(\frac{X_k}{P_k^{1-\sigma}} \right)^{\frac{1}{\sigma}} (z\ell)^{\frac{\sigma - 1}{\sigma}} \tag{5}$$

where X_k is total expenditure in sector k goods, and $P_k = \left(\int_0^{N_k} p_k(n)^{1-\sigma} dn \right)^{\frac{1}{1-\sigma}}$ is the price index for sector $k \in \{C, S\}$. For the tradable sector, $X_C = \zeta I$, where I is aggregate income. For the non-tradable sector, $X_S = (1 - \zeta) I + R$, where R represents

[2] This process is imposed to be the same across formal and informal firms within tradable and non-tradable sectors. Unfortunately, we do not have longitudinal data on firms in the informal sector, so that this process cannot be separately identified for formal and informal firms.

expenditures on non-tradable sector goods made by firms in order to cover hiring, fixed and export costs (which we discuss below). Aggregate income is determined by total wages, government transfers and aggregate firms' profits.

Timing

Every period, formal incumbent firms must choose whether to stay or exit their industry. If the firm decides to stay, it draws its new productivity shock and must decide to adjust or not its labor force. Informal firms face a similar problem but also have one additional option, which is to formalize their businesses. If they decide to formalize, they will then be subject to all regulatory costs faced by formal firms, namely the payroll and revenue taxes, firing costs and minimum wages. After the informal firm decides to stay informal or migrate to the formal sector, it draws its new productivity shock and must also decide whether to adjust its labor force.

The timing of events and firms' behavior is illustrated in Figure 1. Consider an informal firm which starts period t with state (z, ℓ, i). There are three initial possibilities: (i) the firm decides to stay informal and draws a new shock z'; (ii) the firm exits because it decides to, or because it is hit with an exogenous death shock (with probability α_{ki}); or (iii) the firm registers with the authorities, becomes formal, and draws a new shock z'. If the firm decides to stay active (as informal or formal), it must choose how to adjust its workforce in response to the shock z'. To do so, it posts vacancies or fires workers and ends period t with ℓ' workers. At that point, it realizes profits and starts period $t+1$ with state (z', ℓ', i), if it decided to remain informal or with state (z', ℓ', f), if it decided to become formal.

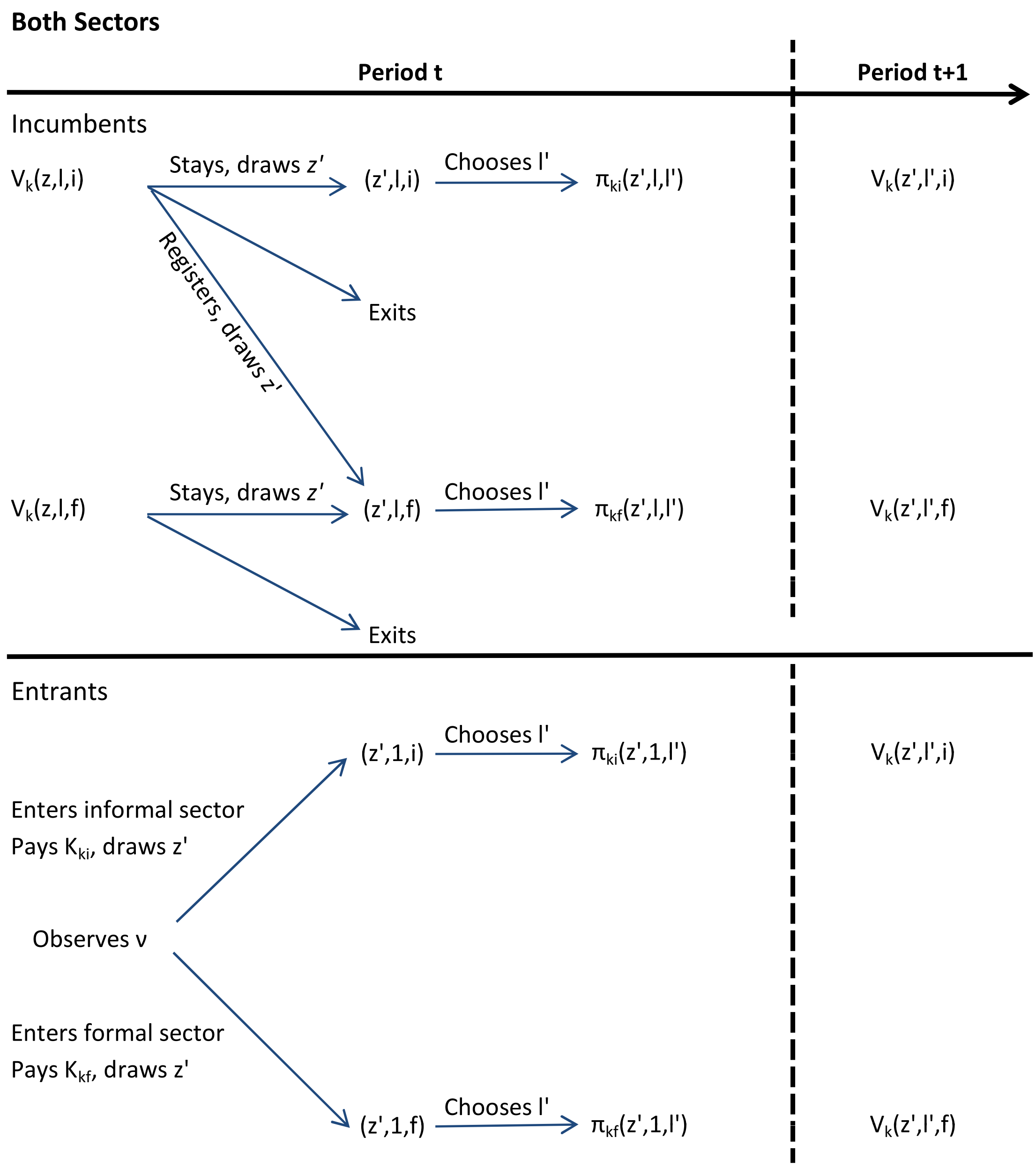
Now, consider a formal firm which starts period t with state (z, ℓ, f). The timing and sequence of events is the same as for informal firms. The only difference is that we do not allow for formal firms to become informal, and the exogenous death shock arrives with probability α_{kf}.

Hiring and Firing Costs

When deciding employment levels, both formal and informal firms in tradable and non-tradable sectors face hiring costs. These are defined by the costs of posting vacancies, which are given by the following function:

$$C^h_{kj}(\ell, v) = \left(\frac{h_{kj}}{\gamma_{k1}}\right)\left(\frac{v}{\ell^{\gamma_{k2}}}\right)^{\gamma_{k1}} \tag{6}$$

Figure 1: Diagram of Firms' Behavior



where h_{kj}, γ_{k1} and $\gamma_{k2} \in [0,1]$ for $k \in \{C,S\}$ and $j = i,f$, are parameters. The γ_{kj1} determines the convexity in hiring costs and γ_{k2} captures economies of scale in hiring.

Expanding from ℓ to ℓ' therefore requires posting $v = \frac{\ell'-\ell}{\mu_{kj}^{v}}$ vacancies, where μ_{kj}^{v} is the probability of filling a vacancy faced by a firm of type j in sector k. The cost of expanding from ℓ to ℓ' workers for a formal firm is therefore given by:

$$H_{kj}(\ell,\ell') = \left(\mu_{kj}^{v}\right)^{-\gamma_{k1}} \left(\frac{h_{kj}}{\gamma_{k1}}\right) \left(\frac{\ell'-\ell}{\ell^{\gamma_{k2}}}\right)^{\gamma_{k1}} \tag{7}$$

The functional form of the hiring cost function is important for a couple of reasons. First, depending on the estimate of the scale parameter γ_{k2} , it is possible to generate the stylized fact that firm-level growth rates in employment decline with size. To obtain some intuition, suppose that $\gamma_{k2} = 0$. In that case, all firms posting v vacancies face the same hiring costs, irrespective of their size. On the other hand, if $\gamma_{k2} = 1$, then all firms face the same cost of a given employment growth rate. For values of γ_{k2} between 0 and 1, larger firms face a higher cost if they want to grow their employment by a particular rate. So, in the event of a positive shock, larger firms will grow less, and in the event of a negative shock, they will also downsize less (as they anticipate large hiring costs if they are hit with a positive shock in the future). Second, the parameter γ_{k1} governs the convexity of the hiring function. If $\gamma_{k1} > 1$, then hiring costs are convex. Allowing for convexity is important for the model to be able to generate wage dispersion. In this type of model, linear hiring costs lead to no wage dispersion. This is because, as we show later when we discuss the wage determination process, in our framework wages are proportional to average revenue per worker, which is – by virtue of our assumptions - proportional to marginal worker revenue. Optimizing firms set marginal revenue equal to marginal cost of an additional worker. But with linear hiring costs, the marginal cost is constant and equal across firms, so that wages will also be equalized across firms. In contrast, with convex hiring costs, the marginal cost of an additional worker is increasing in the growth of employment, so that expanding firms will pay higher wages.

Regarding firing costs, since they are entirely driven by labor market regulation, we assume that only formal firms are subject to them and they are determined as follows:

$$F(\ell,\ell') = \kappa(\ell-\ell') \tag{8}$$

where $\kappa > 0$ is the parameter governing the firing cost function. We assume that firing costs are equal across the C and S sectors, which is consistent with the Brazilian labor regulation. We also assume that firing costs are collected by the government and are rebated back to consumers, while the hiring costs are incurred in terms of the non-tradable sector composite good.

Profit and Value Functions

Formal firms are subject to payroll and revenue taxes, firing costs and the minimum wage regulation. The profit function of a formal firm in sector $k \in \{C, S\}$ is given by:

$$\pi_{kf}\left(z', \ell, \ell'\right) = \left(1 - \tau_y\right) R_k\left(z', \ell'\right) - C_{kf}\left(z', \ell, \ell'\right) - \bar{c}_{kf}, \tag{9}$$

where $R_k\left(z', \ell'\right)$ denotes the revenue function; $\bar{c}_{kf}$ denotes a per-period, fixed cost of operation, which we assume that it is incurred in terms of the non-tradable sector composite good; τ_y is a sales/revenue tax, collected by the government and rebated to consumers.

Due to hiring and firing costs, the total cost function for a formal firm adjusting from ℓ to ℓ' workers is given by the following expression:

$$C_{kf}\left(z', \ell, \ell'\right) = \begin{cases} \left(1 + \tau_w\right) \max\left\{w_{kf}\left(z', \ell'\right), \underline{w}\right\} \ell' + H_{kf}\left(\ell, \ell'\right) & \text{if } \ell' > \ell \\ \left(1 + \tau_w\right) \max\left\{w_{kf}\left(z', \ell'\right), \underline{w}\right\} \ell' + \kappa\left(\ell - \ell'\right) & \text{if } \ell' \leq \ell \end{cases} \tag{10}$$

where $w_{kf}\left(z', \ell'\right)$ denotes the wage of workers in a formal firm with productivity z' and size ℓ', $\underline{w}$ denotes the minimum wage and τ_w is the payroll tax, which is assumed to be collected by the government and rebated to consumers. The wage schedule $w_{kf}\left(z', \ell'\right)$ is the result of a bargaining problem between the firm and its workers that will be detailed in section 2.4.

Since formal firms have to choose to stay or leave their industry, their value function is given by:

$$V_k\left(z, \ell, f\right) = \left(1 - \alpha_{kf}\right) \max\left\{0, E_{z'|z} \max_{\ell'}\left\{\pi_{kf}\left(z', \ell, \ell'\right) + \frac{1}{1+r} V_k\left(z', \ell', f\right)\right\}\right\} \tag{11}$$

where α_{kf} denotes the exogenous death probability that firms face every period for $k = C, S$. The solution of (11) leads to the employment policy function $\ell' = L_k(z', \ell, f)$ and to the vacancy posting policy function $v_{kf}(z', \ell) = \frac{L_k(z', \ell, f) - \ell}{\mu^v_{kj}}$ (as well as to other policies such as exit and stay-active decisions).

Even though informal firms do not have to incur any of the regulatory costs (taxes, minimum wages, firing costs), they face a probability of detection by the government, which is (presumably) increasing in their number of employees. Therefore, we allow the expected cost of being informal to depend on firm size, which is a common formulation in the literature (see Ulyssea, 2018, and the references therein). The intuition for this assumption is that as firms grow larger, they become more visible to the government and therefore are inspected with higher probability, which entails costs in the form of fines and bribes, or can lead to the firm shutting down its operations. Similarly, this assumption captures the idea that the opportunity costs of informality increase as the firm becomes larger because it might want to access the formal financial market (e.g. credit lines), issue invoices and expand its costumers base. Informal firm's profit function is thus given by:

$$\pi_{ki}\left(z',\ell,\ell'\right)=\left(1-p_{ki}\left(\ell'\right)\right)R_k\left(z',\ell'\right)-C_{ki}\left(z',\ell,\ell'\right)-\bar{c}_{ki}, \tag{12}$$

where $p_{ki}\left(\ell'\right)$ summarizes the costs associated to informality, which are assumed to be proportional to firm's revenues. We impose that

$$p_{ki}\left(\ell'\right)=\max\left\{\min\left\{a_k+b_k\left(\ell'\right),1\right\},0\right\}. \tag{13}$$

Since informal firms are not subject to firing costs, their cost function is given by:

$$C_{ki}\left(z',\ell,\ell'\right)=\begin{cases} w_{ki}\left(z',\ell'\right)\ell'+H_{ki}\left(\ell,\ell'\right) & \text{if } \ell'>\ell \\ w_{ki}\left(z',\ell'\right)\ell' & \text{if } \ell'\leq\ell \end{cases} \tag{14}$$

where $w_{ki}\left(z',\ell'\right)$ denotes the wage of workers in an informal firm with productivity z' and size ℓ'. This wage schedule will be determined by a bargaining problem between the firm and its workers as we describe in section 2.4.

Informal firms' value functions are similar to formal firms', except that they have the additional option to formalize their businesses. The informal value functions are therefore given by:

$$V_k\left(z,\ell,i\right)=\left(1-\alpha_{ki}\right)\max\left\{\begin{array}{c} 0, E_{z'|z}\max\limits_{\ell'}\left\{\pi_{ki}\left(z',\ell,\ell'\right)+\frac{1}{1+r}V_k\left(z',\ell',i\right)\right\}, \\ E_{z'|z}\max\limits_{\ell'}\left\{\pi_{kf}\left(z',\ell,\ell'\right)+\frac{1}{1+r}V_k\left(z',\ell',f\right)\right\} \end{array}\right\}. \tag{15}$$

The solution of (15) leads to the employment policy function $\ell'=L_k(z',\ell,i)$ and to the vacancy posting policy function $v_{ki}(z',\ell)=\frac{L_k(z',\ell,i)-\ell}{\mu_{kj}^v}$ (as well as to other policies such as exit, change to formal and stay informal decisions).

Entry

Firm entry is illustrated in the lower panel of Figure 1. Every period there is a pool of potential entrants into the tradable and non-tradable sectors. After incurring a cost $c_{e,k}$ of entry into sector k, these potential entrants observe a pre-entry signal of how productive they will be if they decide to enter, denoted by ν, which is drawn from the ergodic distribution of z'. They can choose to enter as a formal or an informal firm, and the decision to enter is made solely based on ν. Once they enter, they draw their actual productivity, z', from:

$$\ln z' = \rho_k \ln \nu + \sigma_k^z \varepsilon.$$

which is analog to incumbents' productivity process, described in expression (4). We adopt this structure to allow for the fact that there may be an overlap of productivity of entrants in both the formal and informal sectors (Meghir et al., 2015).

Once entry occurs and entrants draw their actual productivity, z', they start behaving as incumbents. Formal and informal entrants start their first period with workforce 1 and we assume that the recruitment costs of these initial workforces are included in the fixed entry costs. The value functions for entrants in either sector are given by:

$$V_k^e(\nu, j) = E_{z'|\nu} \max_{\ell' \geq 1} \left\{ \pi_{kj}(z', 1, \ell') + \frac{1}{1+r} V_k(z', \ell', j) \right\} \tag{16}$$

where $j = i, f$. The entry conditions into the informal and formal sectors are given by the following inequalities, respectively:

$$V_k^e(\nu, i) - K_{ki} \geq \max\{0, V_k^e(\nu, f) - K_{kf}\} \tag{17}$$

$$V_k^e(\nu, f) - K_{kf} \geq \max\{0, V_k^e(\nu, i) - K_{ki}\} \tag{18}$$

The solution to equations (17) and (18) lead to policy entry functions $I^{informal}(\nu)$ and $I^{formal}(\nu)$. The equilibrium mass M_k of entrants in each sector $k = C, S$ is pinned down by the free entry condition below.

$$V_k^e = \int \left[V_k^e(\nu, i) I_k^{informal}(\nu) + V_k^e(\nu, f) I_k^{formal}(\nu) \right] g_k^e(\nu) d\nu \leq c_{e,k}, \tag{19}$$

where $c_{e,k}$ is the cost of entry in sector k – the cost of drawing a ν signal. If entry in sector k is positive, then (19) holds with equality.

2.3 Labor Market Frictions

Formal and informal labor markets are characterized by search and matching frictions, which prevent unemployed workers to immediately find open vacancies. We assume undirected search, and therefore unemployed workers form a unique pool of individuals who are randomly matched with formal or informal firms in one of the sectors $k = C, S$. Thus, formal and informal firms operating in tradable and non-tradable sectors compete for workers in the labor market. Given the total number of vacancies posted in each sector and type of firm $(V_{Cf}, V_{Ci}, V_{Sf}, V_{Si})$, and the mass of unemployed workers searching for jobs, L_u, the total number of matches that are formed is given by:[3]

$$m\left(V_{Cf}, V_{Ci}, V_{Sf}, V_{Si}, L_u\right) = \phi \widetilde{V}^{\eta} L_u^{1-\eta}, \tag{20}$$

Where $\widetilde{V} = \xi_{Cf}V_{Cf} + \xi_{Ci}V_{Ci} + \xi_{Sf}V_{Sf} + \xi_{Si}V_{Si}$ aggregates vacancies across sectors and types of firms, $\xi_{kj} > 0$ is a mesure of efficiency/visibility of vacancies posted by firms of type j in sector k, $\phi > 0$, and $0 < \eta < 1$. ξ_{Cf} is normalized to 1. Matches are split across sectors according to the following proportionality rule:

$$m_{kj} = \frac{\xi_{kj}V_{kj}}{\widetilde{V}} m\left(V_{Cf}, V_{Ci}, V_{Sf}, V_{Si}, L_u\right). \tag{21}$$

This implies that firms of type j in sector k face probability of filling a vacancy given by:

$$\mu_{kj}^{v} = \frac{m_{kj}}{V_{kj}} = \xi_{kj}\phi\left(\frac{L_u}{\widetilde{V}}\right)^{1-\eta} = \xi_{kj}\mu^{v}, \tag{22}$$

where $\mu^{v} \equiv \phi\left(\frac{L_u}{\widetilde{V}}\right)^{1-\eta}$. Equation (22) highlights that formal firms directly compete with informal firms in the labor market. Finally, unemployed workers face job finding probabilities given by:

$$\mu_{kj}^{e} = \frac{m_{kj}}{L_u} = \xi_{kj}\phi\frac{V_{kj}}{\widetilde{V}}\left(\frac{\widetilde{V}}{L_u}\right)^{\eta}. \tag{23}$$

2.4 Wages

We assume that workers collectively bargain with their employer, after hiring costs are sunk and matching has taken place. More concretely, we assume that workers collectively bargain with their firms in a "all in or all out" fashion. To simplify exposition, we refer

[3] In principle, and depending on ξ_{kj}, the probability of filling a vacancy by firms in sector k of type j can be greater than 1. Our estimation procedure ensures that we only search for parameters leading to $\mu_{kj}^{v} \leq 1$. On the other hand, μ_{kj}^{e} in equation (23) is always bounded by 1.

to workers as "unions". The surpluses of a formal firm in sector k, and the union it faces are given by, respectively:

$$S^e_{kf}(z,\ell) = (1-\tau_y) R_k(z,\ell) - (1+\tau_w) w_{kf}(z,\ell)\ell - \bar{c}_{kf} + \frac{1}{1+r} V_k(z,\ell,f) \tag{24}$$

$$S^u_{kf}(z,\ell) = \left[w_{kf}(z,\ell) + \frac{1}{1+r} J^e_k(z,\ell,f) - \left(b + b^u + \frac{1}{1+r} J^u \right) \right] \ell \tag{25}$$

where b denotes the utility flow from being unemployed; b^u denotes the value of unemployment benefits, which are only received by formal workers; $w_{kf}(z,\ell)$ is the unrestricted wage for formal workers (who nevertheless cannot receive a lower wage than the minimum wage); $J^e_k(z,\ell,f)$ is the future expected value of a job in a formal-sector firm in sector k with current productivity z and workforce ℓ; and J^u is the present value of searching for a job.

We assume that if all workers leave, the firm exits, and that fixed operating costs are incurred after the bargaining process. Let β_f be the bargaining power of workers in the formal sector, the outcome of bargaining is given by:

$$(1-\beta_f) S^e_{kf}(z,\ell) = \beta_f S^u_{kf}(z,\ell) \tag{26}$$

Substituting expressions (24) and (25) into (26), and assuming that the current surplus is shared the same way as future surpluses (Bertola and Garibaldi, 2001; Cosar et al., 2016), one obtains the following (unrestricted) wage functions for formal workers:

$$w_{kf}(z,\ell) = \frac{(1-\beta_f)(b+b^u)}{1+\beta_f\tau_w} + \frac{\beta_f(1-\tau_y)}{1+\beta_f\tau_w}\frac{R_k(z,\ell)}{\ell} - \frac{\beta_f}{1+\beta_f\tau_w}\frac{\bar{c}_{kf}}{\ell} \tag{27}$$

and we again note that formal workers always receive the maximum between the unrestricted wage, $w_{kf}(z,\ell)$, and the minimum wage, $\underline{w}$.

Wages in the informal sector are determined in a similar way. Let the bargaining power parameter be denoted by β_i, where we allow the bargaining power of formal and informal workers to be different. These could differ due to institutional reasons, such as the existence of a centralized union or labour courts, or because informal workers and firms have greater flexibility to negotiate wages. Since these will be directly estimated, the question of whether these bargaining power parameters are indeed different is an empirical one. Following the same steps as above, it is straightforward to obtain:

$$w_{ki}(z,\ell) = (1-\beta_i)\, b + \beta_i (1 - p_{ki}(\ell)) \frac{R_k(z,\ell)}{\ell} - \beta_i \frac{\overline{c}_{ki}}{\ell} \tag{28}$$

where the major differences relatively to expression (27) are the absence of unemployment benefits (b^u), payroll and revenue taxes (τ_w and τ_y, respectively); and the presence of the cost of informality function, $p_{ki}(\ell)$.

Expressions (27) and (28) are intuitive: wages are directly increasing with sales per worker, and the slope is larger if bargaining power is larger. An alternative to this wage setting would be to assume a somewhat more common structure *a la* Stole and Zwiebel (1996), where firms bargain with all of their workers simultaneously and continuously in a one-to-one basis, treating each worker as the marginal one. However, the present formulation generates a richer wage distribution that fits much better the degree of wage dispersion found in the data. Frameworks *a la* Stole and Zwiebel (1996) tend to generate less realistic distributions, as they imply that, for example, all firms that are willing to downsize pay the same wage to all workers (which is equal to workers' reservation wage). Additionally, the present wage setting framework implies wage schedules that are very close to those in the rent sharing literature (e.g. Card et al., 2018) and commonly found in trade models, such as Helpman and Itskhoki (2010).

2.5 Open Economy

We now extend the model to the open economy case. We assume that the home country is small relative to the rest of the world and therefore foreign conditions do not react to its policies. In the following analysis, we drop the formal/informal qualifier in order to simplify notation, as we assume throughout that informal firms cannot export.[4] In what follows, it will be convenient to re-write domestic revenues (Equation (5)) as $R_k(z,\ell) = D_{H,k}^{\frac{1}{\sigma_k}} q(z,\ell)^{\frac{\sigma_k - 1}{\sigma_k}}$, where $k \in \{C,S\}$ $q(z,\ell) = z\ell$, and $D_{H,k} = \frac{X_k}{P_k^{1-\sigma_k}}$. Since the focus in this section lies on the tradable sector only, and for the sake of notation simplicity, we drop the subscript $k \in \{C,S\}$ for the reminder of this subsection.

Price Indices and Aggregates

The price index in the non-tradable sector remains the same, but in the tradable sector it is modified to account for trade. First, we characterize the price index of imports

[4] This assumption comes from the fact that firms that are not registered cannot undertake the necessary legal and bureaucratic procedures to export.

denominated in home-currency:

$$P_F = \epsilon \tau_a \tau_c \left(\int_0^{N_F} p^* (n)^{1-\sigma} \, dn \right)^{\frac{1}{1-\sigma}} = \epsilon \tau_a \tau_c \tag{29}$$

where $p^*(n)$ is the free on board (FOB) price of imported variety n, denominated in foreign currency; N_F denotes the mass of imported varieties; ϵ is the exchange rate, $\tau_a - 1 > 0$ is the ad-valorem tariff and $\tau_c > 1$ the iceberg trade cost. The second equality in the above expression comes from the normalization $\left(\int_0^{N_F} p^* (n)^{1-\sigma} \, dn \right)^{\frac{1}{1-\sigma}} \equiv 1$. This is without loss of generality, as this term is exogenous to our model given the small open economy assumption. The price index of domestically produced varieties $n \in (N_F, N]$ is given by:

$$P_H = \left(\int_{N_F}^{N} p (n)^{1-\sigma} \, dn \right)^{\frac{1}{1-\sigma}} \tag{30}$$

and the price index for the composite tradable sector good is given by

$$P = \left[P_H^{1-\sigma} + P_F^{1-\sigma} \right]^{\frac{1}{1-\sigma}} = \left[\int_{N_F}^{N} p (n)^{1-\sigma} \, dn + (\epsilon \tau_a \tau_c)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{31}$$

The domestic demand for domestically produced goods is given by $Q_H(n) = D_H p(n)^{-\sigma}$, for $n \in (N_F, N]$; and the domestic demand for foreign produced goods is given by $Q_H(n) = D_H (\epsilon \tau_a \tau_c p^*(n))^{-\sigma}$, for $n \in [0, N_F]$. Finally, foreign demand for domestically produced goods is given by $Q_F(n) = D_F^* (p_x^*(n))^{-\sigma}$, for $n \in (N_F, N]$, where $p_x^*(n)$ is the price of domestic variety n in the foreign country, denominated in foreign currency, and D_F^* is an exogenous foreign demand shifter. If $\mathcal{I}^x(n)$ denotes an indicator function that equals one if variety n is exported, we have that the value of aggregate imports (before import tariffs) and exports are given by the following expressions:

$$Imports = \frac{D_H}{\tau_a} \int_0^{N_F} (\epsilon \tau_a \tau_c p^* (n))^{1-\sigma} \, dn = \frac{D_H P_F^{1-\sigma}}{\tau_a} = \frac{D_H (\epsilon \tau_a \tau_c)^{1-\sigma}}{\tau_a} \tag{32}$$

$$Exports = D_F^* \epsilon \int_{N_F}^{N} \mathcal{I}^x (n) \, p_x^* (n)^{1-\sigma} \, dn \tag{33}$$

Exporters

Given the expression of foreign demand for home variety n just described, $Q_F(n)$, revenues from exports are given by $\epsilon D_F^{*\frac{1}{\sigma}} (q_x/\tau_c)^{\frac{\sigma-1}{\sigma}}$, where q_x is the total quantity exported. If a firm exports, it must decide which fraction η of its product to sell abroad.

Conditional on being an exporter, total gross revenue is given by

$$R^x(z,\ell,\eta) = D_H^{\frac{1}{\sigma}}\left[(1-\eta)q(z,\ell)\right]^{\frac{\sigma-1}{\sigma}} + \epsilon D_F^{*\frac{1}{\sigma}}\left(\frac{\eta q\left(z,\ell\right)}{\tau_c}\right)^{\frac{\sigma-1}{\sigma}}$$
$$= q(z,\ell)^{\frac{\sigma-1}{\sigma}}\exp\left(d_H + d_F(\eta)\right) \tag{34}$$

where $d_H = \ln\left(D_H^{\frac{1}{\sigma}}\right)$ and $d_F(\eta) = \ln\left((1-\eta)^{\frac{\sigma-1}{\sigma}} + \epsilon\left(\frac{D_F^*}{D_H}\right)^{\frac{1}{\sigma}}\left(\frac{\eta}{\tau_c}\right)^{\frac{\sigma-1}{\sigma}}\right)$.

The optimal share of exports is given by:

$$\eta^o = \arg\max_{\eta} d_F(\eta) = \left(1 + \frac{\tau_c^{\sigma-1}}{\epsilon^\sigma}\frac{D_H}{D_F^*}\right)^{-1} \tag{35}$$

which shows that, conditional on exporting, all firms choose to export the same share of their output. The revenue functions for non-exporters (R^d) and exporters (R^x) are then given by:

$$R^d(z,\ell) = (z\ell)^{\frac{\sigma-1}{\sigma}}\exp\left(d_H\right) \tag{36}$$

$$R^x(z,\ell) = R^d(z,\ell)\Delta(z,\ell) \tag{37}$$

where $\Delta(z,\ell) = \exp\left(d_F(\eta^o)\right)$, and $d_F(\eta^o)$ is obtained by substituting the expression of the optimal η^o into $d_F(\eta)$.[5] The export policy is then given by:

$$I_C^x\left(z,\ell\right) = \left\{\begin{array}{ll} 1 & \text{if } R_C^x\left(z,\ell\right) - f_x > R_C^d\left(z,\ell\right) \\ 0 & \text{otherwise} \end{array}\right\} \tag{38}$$

where $f_x > 0$ denotes the fixed cost of exporting, which is denominated in terms of the non-tradable composite good.

Since $\Delta(z,\ell) > 1$, being an exporter magnifies firms' revenues and also makes them more sensitive to productivity shocks, for any given state (z,ℓ). Thus, as in Cosar et al. (2016), reducing trade costs will produce two opposing forces: (i) there will be a reallocation of workers toward larger and higher productivity firms, which tend to be more stable and have lower worker turnover (e.g. they face larger costs of growing the workforce); (ii) due to the term $\Delta(z,\ell)$, both new and old exporters become more sensitive to idiosyncratic shocks, which tends to increase turnover. We follow Cosar et al. (2016) and refer to these two forces as the "distribution effect" and "sensitivity

[5] When one substitutes η^o into $d_F(\eta)$, one obtains $d_F \equiv d_F(\eta^o) = \log\left(\left(1 + \frac{D_F^*}{D_H}\epsilon^\sigma\tau_c^{1-\sigma}\right)^{\frac{1}{\sigma}}\right)$.

effect", respectively. Turnover is tightly linked to unemployment, as workers who are fired must spend at least one period in unemployment. In turn, workers transition from unemployment to formal and informal sector jobs. In addition to these forces, we also have “Melitz effects”, where trade liberalization affects the “productivity/size threshold” for firms to export, but it will also affect the thresholds for operating formally, informally and exit. An attractive feature of this model is that it can accommodate both a increase or a decrease in informality. The net effect of the forces in the model is ultimately an empirical question.

2.6 Equilibrium

- Firms act optimally and make entry and exit decisions and post vacancies according to equations (11), (15), (16), (17) and (18). If entry is positive in sector k, the free entry condition (19) holds with equality.

- Wages solve the bargaining problem between workers and the firm, as in equations (27) and (28).

- Labor markets clear, that is, the sum of employment levels across sectors and the number of unemployed workers must be equal to the total labor force $\overline{L}$:

$$L_{Ci} + L_{Cf} + L_{Si} + L_{Sf} + L_u = \overline{L}. \tag{39}$$

- The government runs a balanced budget. Government’s revenues come from tax collection (including tariff revenues and fines on informal firms) and firing costs, while it pays unemployment benefits to all unemployed who come from formal employment. We assume that any surplus is directly rebated to consumers.

- Aggregate income I is given by the sum of all revenues across firms and sectors $Rev_C + Rev_S$ plus the revenue from tariffs $(\tau_a - 1) \times Imports$ minus total costs incurred by hiring firms, fixed costs of operation and fixed costs of exporting. The sum of these costs is R and is incurred in terms of the non-tradable sector good.

- Product markets clear. Expenditure on nontradable goods is divided between final goods expenditure – given by $(1-\zeta)\,I$ – and intermediate goods expenditure R and must equal to S-sector total revenues Rev_S.

$$(1-\zeta)\,I + R = Rev_S \tag{40}$$

Expenditure on tradable goods is given by ζI and is given by total C-sector revenue Rev_C, minus Exports, plus Imports accrued by the import tariff.

$$\zeta I = Rev_C - Exports + \tau_a Imports \tag{41}$$

- Trade is balanced: $Imports = Exports$.
- We focus on steady state equilibria, where all aggregates remain constant. In particular, no sector can be expanding or contracting, which implies that: (i) the flow of workers out of unemployment and into the formal/informal and tradable/non-tradable sectors must be the same as the flow out of these sectors and into unemployment; (ii) the mass of firms entering the informal sector must be equal to the mass of informal firms that decide to exit or to formalize their businesses in either sector $k \in \{C, S\}$; and (iii) the sum of the number of firms entering the formal sector and those formalizing their businesses must be equal to the mass of formal firms that decide to exit either sector $k \in \{C, S\}$.

Appendix A.1 details all of the equilibrium conditions.

3 Background: The cost of labor regulations in Brazil

The relevant laws and regulations that apply to formal labor relations in Brazil are contained in the the Brazilian Labor Code (*Consolidacao das Leis Trabalhistas* – CLT), which dates back to 1943. In 1988, the new Federal Constitution was enacted and extended the range of labor regulations and workers' benefits, which substantially increased both the variable labor costs associated to formal employment and firing costs (De Barros and Corseuil, 2004).[6] As a result of the changes in 1988, the regulatory framework of the Brazilian labor market became quite burdensome and costly, and that has remained unaltered since then. According to the employment index in Botero et al. (2004), the cost of labor regulations in Brazil is around 20 percent above the mean and median of 85 countries and more than 2.5 times as large as in the United States.

The main aspects of the labor regulations in Brazil, in terms of their magnitude and potential impacts on labor market functioning, are the following: the presence of

[6] Among the changes introduced by the new Constitution, one can highlight the following: regular working hours went from 48 to 44 hours per week; overtime premium increased from 20 to 50 percent; maternity leave increased from three to four months; and the value of paid vacations increased from one to, at least, 4/3 of the regular monthly wage (see De Barros and Corseuil, 2004, for a more detailed description of the changes).

a national minimum wage, sizeable payroll taxes, unemployment insurance that is only available to formal workers, and substantial firing costs. Since these play an important role in our model and counterfactuals, we provide a brief background discussion on each of them individually and refer the reader to existing studies that provide a more in depth analysis of these different institutional aspects.

Starting by the national minimum wage, since 1995 (with the end of hyper-inflation) its nominal value is determined by the federal government once a year and is typically quite binding. In 2003, for example, the minimum wage corresponded to 49 percent of the national average wage and 81.3 percent of the national median wage.[7] As for the unemployment insurance, its rules remained unaltered from 1994 to 2015 but substantial changes have been implemented since then. Since our empirical analysis focuses on the period prior to the UI reforms, we discuss the rules in place until 2015.[8] In terms of eligibility, generally a formal worker who is laid off and who has at least 6 months of job tenure is eligible to receive UI benefits for up to 5 months.[9] The actual duration of the benefit depends on the worker's accumulated tenure across her formal jobs in the 36 months prior to layoff. In practice, most workers receive between 4 and 5 months of UI benefits, with the mean and median number of monthly payments per UI spell equal to 4.3 and 4.7 months, respectively. Finally, the value of the benefit depends on the worker's average wage in the three months prior to layoff and the replacement rate is 100 percent for individuals who earn one minimum wage, with an average replacement rate of 64 percent (all data comes from Gerard and Gonzaga, 2018).

As for the firing costs, the Brazilian labor regulation states that all formal workers "dismissed with no just cause" should receive a monetary compensation paid by the employer. Since labour courts are extremely favourable to workers, *de facto* all workers are entitled to receive this compensation upon an involuntary separation. The magnitude of this compensation is determined proportionally to the funds accumulated in the worker's *Fundo de Garantia por Tempo de Servico* (FGTS), which is a job security fund accumulated while the worker remains employed at a given firm. This is a private and individual fund that is specific to the worker, and to which employers must contribute, every month,

[7]The mean and the median wages are computed using micro data from the National Household Survey (PNAD) and pooling together all formal and informal employees who are between 18 and 64 years old and worked at least 20 hours per week.

[8]See Carvalho et al. (2018) for a discussion of the reform, which substantially changed the eligibility criteria of unemployment benefits, and its impacts on layoffs in Brazil.

[9]There are some nuances to eligibility that depend upon the elapsed time since worker's last successful application to UI benefits. See Gerard and Gonzaga (2018) for a more detailed discussion of the UI program in Brazil.

the equivalent of 8 percent of worker's monthly wage. Hence, the worker's FGTS funds are proportional to her tenure and accumulate at a rate of roughly one monthly wage per year. Although these resources are owned by the worker, the fund is run by the government and the real return rates are typically below market rates, when not negative. Moreover, workers only have access to their own fund when they are laid off or upon retirement. In addition to the totality of their fund, workers who are laid off also receive a penalty, paid by their employer, which amounts to 40 percent of total resources accumulated in their fund during the duration of the job they are being laid off from. Firms must also pay an additional 10 percent of the FGTS in fines, which go directly to the federal government. In addition to this severance payment of 50 percent (40 plus 10 percent) of the FGTS, firms must provide a one-month advance notice, which *de facto* means that workers receive an additional monthly wage and are dismissed immediately.[10]

Finally, Brazil has a burdensome tax system, which is not only characterized by high tax rates but also by a complex structure that implies large compliance costs. For example, the estimated cost in terms of time required to comply with the tax system in Brazil is 2,600 hours, which is the highest in the world, and more than 8 times larger than the cost that a firm faces in the U.S. Even though a substantial part of this cost is not due to the payment of labor taxes, the time required to comply with labor taxes in Brazil is almost 5 times higher than in the U.S. (491 and 100 hours, respectively).[11] In terms of the tax rate, even though we use the statutory values for both payroll and revenue taxes in our estimation, it is useful to provide a comparison to other countries, which is done in Doing Business (2007): The labor tax computed as a share of commercial profits amounts to 42.1 percent in Brazil, while it is 12.9 percent in Canada and 10 percent in the U.S. Hence, not only labor taxes seem to be quite high in Brazil, but also they imply substantial compliance costs.

4 Data and Facts

4.1 Firms

In this paper we make use of 6 datasets that contain information on formal and informal firms and their workers. The first is the *Relacao Anual de Informacoes Sociais* (RAIS), which is a matched employer-employee dataset assembled by the Brazilian Min-

[10] Gonzaga et al. (2003) provide an in depth discussion of the legislation on dismissal costs in Brazil.

[11] These data come from Doing Business (2007), which is the earliest report available on paying taxes in the Doing Business Initiative that provides comparability across a comprehensive set of countries.

istry of Labor every year since 1976. RAIS is a high quality panel that contains the universe of formal firms and workers.[12] It provides information on firms' 5-digit industry, location and ownership (i.e public vs. private enterprises), among others. At the worker level, the main variables are gender, age, level of education, monthly wage, number of hours in the contract, tenure at the firm, occupation, month of accession into the job (if accession occurred during the current year), and month of separation (if any). We use the matched employer-employee structure to compute firm size and firm-level average wages over time.

We also use three economic surveys that cover the *formal* manufacturing, retail and service sectors: *Pesquisa Industrial Anual* (PIA), *Pesquisa Anual de Comercio* (PAC), and *Pesquisa Anual de Servicos* (PAS), respectively. These surveys collect detailed information about firms' inputs, output and revenues, and are a combination of a census for larger firms and a representative sample for smaller firms. In the manufacturing sector (PIA), all firms with at least 30 employees are part of the census and are surveyed every year, while firms with 5 to 29 employees are randomly sampled.[13] The PAC (retail sector) and PAS (services) follow similar designs, although they have lower size thresholds for firms to be included in the census: firms with 20 employees or more are part of the census, while firms with up to 19 employees are randomly sampled.

The fifth data source used is Customs data from *Secretaria de Comercio Exterior* (SECEX), which give us the list of every export and import transaction (and values) made from and by Brazilian firms every year since 1990 and until 2007. Importantly for this study, there is a unique firm identifier across these 5 data sets, which allows us to merge the production information from PIA, PAS and PAC with the information about firms' labor and wages coming from RAIS, and the customs data from SECEX.

These six data sets provide a comprehensive coverage of the formal sector, but are completely silent about the informal sector (by design). We therefore use a sixth data source, which is the *Pesquisa de Economia Informal Urbana* (ECINF). This survey was collected by the Brazilian Bureau of Statistics (IBGE) in 1997 and 2003, and was designed to be representative of the universe of *urban* firms with up to five employees (both formal and informal). It is a matched employer-employee data set that contains information on entrepreneurs, their businesses and employees. Firms are directly asked whether they are registered with the tax authorities and whether each of their workers has a formal labor

[12]The RAIS data set has been increasingly used in different applications. For recent examples see Dix-Carneiro (2014), Helpman et al. (2017), Alvarez et al. (2018), Ulyssea (2018), among others.

[13]The main source of information used by IBGE to design its sample is the RAIS data set described above.

contract. Thus, it is possible to directly observe both firms' and workers' formal status. Given that the formality/informality statuses are self-reported, one could have concerns about measurement error and under-reporting. However, the IBGE has a long tradition of accurately measuring labor informality, and it has very strict confidentiality clauses, so the information cannot be used for auditing purposes by other government branches, in particular those responsible for enforcing the relevant laws and regulations. These characteristics, associated to the high levels of informality observed in the data, make us confident that respondents are not systematically underreporting their informality status.[14]

In all seven data sets we exclude public sector firms and those in agriculture, mining, coal, oil and gas industries. We do so because our focus lies on private sector, urban firms. Moreover, our model is not well suited to describe sectors with very large economies of scale and dominated by few very large firms, such as oil and gas. In the data, as well as in the model, the tradable sector is comprised by manufacturing firms and the non-tradable sector is comprised by services and retail firms. In sum, information on the formal tradable sector comes from RAIS, PIA and SECEX; on the non-tradable formal sector comes from RAIS, PAS and PAC; and data on both tradable and non-tradable informal sectors come from the ECINF survey. These datasets are summarized in Table 1.

Since 2003 is the last year available for the ECINF survey, we use it as the reference year for all other data sets. Table 2 shows the size distribution (measured as number of employees) in the tradable and non-tradable sectors for formal and informal firms. As expected, the number of observations is much larger for formal firms, as these come from a census (the RAIS data). Nevertheless, the share of tradable sector firms is quite similar in the formal and informal sectors (13.1 and 14.2 percent, respectively). The size difference between formal exporters and non-exporters in the tradable sector is quite remarkable, with exporting firms being more than 8 times larger than non-exporting firms, on average. Figure 2 shows this fact from a different angle, as the share of exporters increases steeply moving up in the size distribution.

Formal firms in the tradable sector are also larger on average than those in the non-tradable sector and the distribution is more skewed to the left. The size difference between informal firms in tradable and non-tradable sectors is almost null, which is expected: the ECINF survey has a size cap, which mechanically limits the size differential. More

[14] Additionally, Ulyssea (2018) shows that the ECINF reproduces very well the RAIS in all the dimensions that are common to both data sets (e.g. size and sectoral distributions), which is reassuring of ECINF's quality.

Table 1: Summary of Datasets

Dataset	Source	Description
Relacao Anual de Informacoes Sociais *RAIS*	Ministry of Labor	Administrative matched employer-employee dataset. Covers all formal firms and workers. Detailed information on firms and workers, but no information on firm-level sales, capital and expenditures with intermediate inputs.
Pesquisa Industrial Anual *PIA*	IBGE	Survey data on Manufacturing firms. Firm-level information such as revenues, capital, investment, expenditures with intermediate inputs. Covers all firms with 30 employees or more; random sample of smaller firms.
Pesquisa Anual dos Servicos *PAS*	IBGE	Survey data on Service-sector firms. Firm-level information such as revenues, capital, investment, expenditures with intermediate inputs. Covers all firms with 20 employees or more; random sample of smaller firms.
Pesquisa Anual do Comercio *PAC*	IBGE	Survey data on Retail and Commerce firms. Firm-level information such as revenues, capital, investment, expenditures with intermediate inputs. Covers all firms with 20 employees or more; random sample of smaller firms.
Secretaria de Comercio Exterior *SECEX*	Ministry of Industry, Foreign Trade and Services	Administrative customs data. Export and import values at the firm level.
Economia Informal Urbana *ECINF*	IBGE	Survey data. Matched employer-employee data set of formal and informal firms and their workers. Representative sample of small businesses (firms with 5 employees or less). Information on formal status of the firm and its workers.
Pesquisa Mensal de Emprego *PME*	IBGE	Survey data. Rotating panel of households that covers the 6 main metropolitan areas in Brazil.

Table 2: Firm Size Distribution in Number of Employees

	Formal		Informal	
	Sector C	Sector S	Sector C	Sector S
All Firms				
Mean (Log-Employment)	1.78	1.18	0.10	0.10
Variance (Log-Employment)	1.82	1.26	0.09	0.08
Exporters				
Mean (Log-Employment)	3.9	–	–	–
Variance (Log-Employment)	2.7	–	–	–
Employment Distribution				
Pct. 20	2	1	1	1
Pct. 40	4	2	1	1
Pct. 60	7	4	1	1
Pct. 80	17	8	1	1
Pct. 90	35	14	2	2
Pct. 95	67	25	2	2
Pct. 99	298	109	4	3
# Observations	216,467	1,430,633	1,069	6,192

Notes: To compute the moments for the formal tradable sector, we use the PIA; for non-tradable formal sector, the PAS and PAC data sets; and for both tradable and non-tradable informal sectors, we use the ECINF survey.

substantially, informal firms cannot grow much without becoming too visible to the authorities and cannot export either, which limits their ability to grow.

Table 3 shows the same information as in Table 2 but focusing on firms' revenues. The same patterns found in Table 2 arise, but it is worth noting that the size differences across percentiles are much larger when one uses revenues instead of employment as the size measure. For example, the 99th percentile of the size distribution measured as number of employees is nearly three times larger in the formal tradable than in the formal non-tradable. The same ratio is more than 30 when one uses revenues. Interestingly, this relationship is inverted in the informal sector, where firms in the non-tradable sector earn higher revenues than firms in the tradable sector. This is intuitive, as one would expect that the penalty for remaining small (and informal) is lower in the non-tradable sector.

Figure 3 shows that there is a substantial size-wage premium in both tradable and non-tradable formal sectors, but the same is not true for informal firms. This is somewhat mechanical, as most informal firms have only one employee. As for employment, wage and revenue growth, Tables 4 and 5 show different patterns moving up the firm size distribution. Table 4 shows that, on average, expanding firms tend to present higher wage growth, but this relationship is not constant across different percentiles of the size

Figure 2: Share of Exporters by Firm Size Percentiles

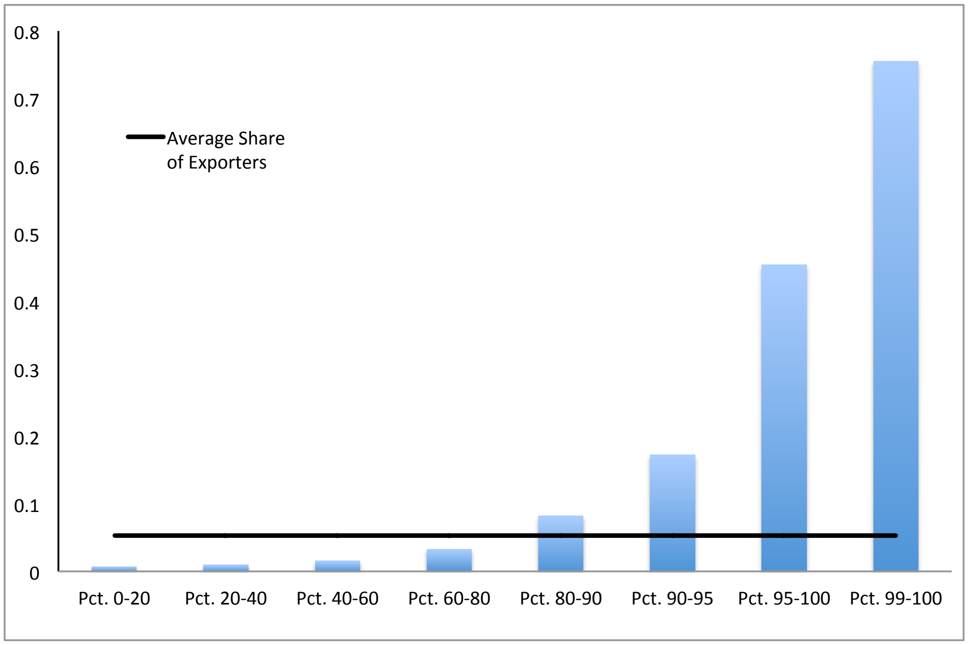


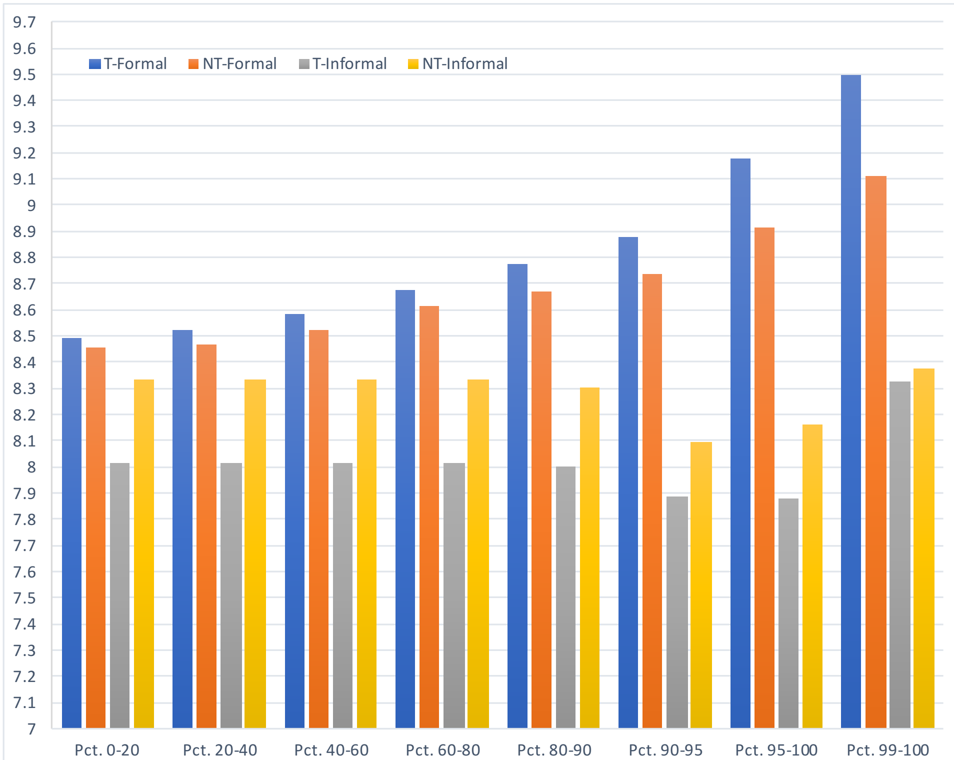
Table 3: Revenue Distribution

	Formal		Informal	
	Tradable	Non-Tradable	Tradable	Non-Tradable
All Firms				
Mean (Log-Revenue)	12.73	10.81	8.53	8.95
Variance (Log-Revenue)	3.51	2.07	1.44	1.30
Exporters				
Mean	15.46	–	–	–
Variance	4.45	–	–	–
Revenue Distribution (in 2003 R$)				
Pct. 20	77,962	15,897	1,920	3,600
Pct. 40	166,110	31,102	4,200	6,000
Pct. 60	407,595	59,492	6,600	9,600
Pct. 80	1,143,359	137,162	13,428	19,200
Pct. 90	4,038,112	288,717	24,000	32,160
Pct. 95	12,494,325	558,989	36,000	49,200
Pct. 99	103,287,792	3,229,837	72,000	108,000

Notes: To compute the moments for the formal tradable sector, we use the PIA; for non-tradable formal sector, the PAS and PAC data sets; and for both tradable and non-tradable informal sectors, we use the ECINF survey.

distribution (for none of the groups considered in the table). On the contrary, Table 5 shows a clear pattern that is in line with other available evidence in the literature: yearly employment and revenues growth rates decrease with size, except at the very top of the distribution (top 5 and one percent of the size distribution).

Figure 3: Average Log-Wages by Firm Size Percentiles



4.2 Workers

In order to complement the information on firms, we use the Monthly Employment Survey (PME – *Pesquisa Mensal de Emprego*) to obtain information on worker allocations and labor market flows. This is a rotating panel with a similar design to that of the Current Population Survey in the U.S.: individuals in a given household are interviewed for 4 consecutive months, they "rest" for 8 months and are then re-interviewed for additional 4 consecutive months, which implies a maximum panel length of 16 months. This employment survey covers the six main metropolitan areas in Brazil and contains detailed information on individuals' socio-demographic characteristics and labor market outcomes, including informal employment and self employment.[15]

We exploit the panel structure of PME to estimate one-year labor market transitions between formal employment, informal employment (in both tradable and non-tradable

[15]See Meghir et al. (2015) for a more detailed description of the PME data.

Table 4: Formal Firms' Average Wage Growth

	Surviving Firms		Expanding Firms		Contracting Firms	
	T	N-T	T	N-T	T	N-T
All Firms	0.051	0.044	0.063	0.059	0.042	0.037
Pct. 0-20	0.043	0.032	0.065	–	0.040	0.032
Pct. 20-40	0.055	0.042	0.072	0.075	0.048	0.038
Pct. 40-60	0.050	0.049	0.066	0.064	0.040	0.042
Pct. 60-80	0.051	0.042	0.062	0.057	0.039	0.032
Pct. 80-90	0.048	0.042	0.060	0.053	0.033	0.031
Pct. 90-95	0.055	0.047	0.072	0.056	0.032	0.036
Pct. 95-100	0.074	0.060	0.050	0.061	0.109	0.059
Pct. 99-100	0.059	0.123	0.029	0.094	0.110	0.168

Notes: We compute firm-level yearly average growth using the RAIS data for the years of 2002 and 2003. Surviving firms are those that are alive in 2002 and 2003. Expanding firms are those for which employment in 2003 is strictly larger than 2002. Contracting firms are those whose labor force remains constant or decreases between 2002 and 2003. *T* and *N-T* denote the tradable and non-tradable sectors, respectively.

Table 5: Formal Firms' Employment and Revenue Growth

	Employment Growth		Revenue Growth	
	Tradable	Non-Tradable	Tradable	Non-Tradable
All Firms	0.156	0.121	0.201	0.229
Pct. 0-20	0.362	0.318	0.216	0.242
Pct. 20-40	0.155	0.231	0.212	0.227
Pct. 40-60	0.096	0.073	0.210	0.235
Pct. 60-80	0.072	0.036	0.201	0.236
Pct. 80-90	0.072	0.031	0.178	0.215
Pct. 90-95	0.071	0.036	0.169	0.212
Pct. 95-100	0.088	0.046	0.146	0.149
Pct. 99-100	0.101	0.060	0.148	0.112

Notes: We compute firm-level yearly employment and revenue growth using the years of 2003 and 2004. We compute employment growth using the RAIS data set. For revenue growth in the formal tradable sector, we use the PIA data set; for the non-tradable formal sector, we use the PAS and PAC data sets.

sectors) and unemployment statuses.[16] As in the firm-level data, we exclude individuals employed in the public sector, agriculture, mining, coal, oil and gas industries. In addition to these filters, we also exclude individuals younger than 17 and older than 65 years old.

Panel A of Table 6 shows worker allocations in 2003. It is noteworthy that 15 percent

[16] A worker is defined to be unemployed if she is not working – regardless of whether she is searching for a job or not.

of the working age population is unemployed (or more precisely, not employed), and that approximately 20 percent of employed workers are in the C-sector. These numbers also indicate that 48 percent of the labor force is employed in the informal sector. In addition, 35 percent of C-sector workers are informal, whereas 51 percent of S-sector workers are. Panel B of Table 6 shows the relevant transition matrix for the purposes of our model. Even though we estimate the full transition matrix in the data, we only report the transitions that are accounted for in our model, while the remaining ones are omitted (such as from the formal tradable sector to the informal non-tradable sector). We start by noting that the table confirms two well-known facts: (i) most of the labor force is in the non-tradable sector (69.3 percent); and (ii) informality is very high in Brazil, accounting for 41 percent of the labor force. As for the probabilities of transition, the rate of retention (main diagonal in the transition matrix) is highest in the non-tradable formal sector (68.5 percent) and is lowest in the informal tradable sector (27 percent). Unemployed workers are most likely to exit to a non-tradable informal sector (38 percent), while the formal tradable sector is the least likely destination of those who are unemployed.

Table 6: Sectoral Shares and 12-month Transition Rates

Panel A: Workers Allocation

	Unemp.	Tradable Inf.	Tradable Form.	Non-Trad. Inf.	Non-Trad. Form.
Shares†	0.137	0.058	0.112	0.352	0.341

Panel B: 12-month Transition Rates

	Unemp.	Tradable Inf.	Tradable Form.	Non-Trad. Inf.	Non-Trad. Form.
Unemp.	0.335	0.062	0.049	0.381	0.159
Tradable - Inf.	0.087	0.270	0.135	0.369	0.119
Tradable - Form.	0.039	0.044	0.530	0.074	0.300
Non-Tradable - Inf	0.086	0.062	0.035	0.610	0.177
Non-Tradable - Form.	0.044	0.016	0.100	0.132	0.685

Notes: Authors' own calculations from the Monthly Employment Survey (PME), years 2003 and 2004. We use the first and 4th interviews to compute 4-months transition rates for the full transition matrix, M. We then annualize M by computing M^3. † We use sampling weights to compute these shares using the entire sample.

5 Estimation

Our estimation procedure follows two steps. First, we fix a subset of parameters using a combination of aggregate data, estimates from previous papers and the statutory value of institutional parameters, such as revenue and payroll taxes. Then, we estimate the remaining parameters of the model using an Indirect Inference estimator, which allow us to combine information from the different data sources discussed in the previous section.

As discussed in Section 3, labor regulations are quite costly and cumbersome in Brazil, so we need to make a few simplifying assumptions. We follow Ulyssea (2018) and set τ_w so that it reflects the main taxes that are proportional to firms' wage bill, namely, employer's social security contribution (20 percent), payroll tax (9 percent), and severance contributions to FGTS (8.5 percent). τ_y includes only the federal VAT taxes, IPI (20 percent) and PIS/COFINS. We exclude state-level value-added taxes because these vary greatly across states and there is a cumbersome system of tax substitution across the production chain, which would be impossible to properly capture.[17]

Firing costs are set following Heckman et al. (2000), who compute the expected discounted cost of dismissing a worker for several Latin American countries, including Brazil. This is done taking into account the main characteristics of dismissal costs in Brazil, as discussed in Section 3, and the expected cost is expressed as a multiple of the monthly wage. To make this parameter compatible with our model, we convert it to a fixed monetary value using the average formal wage found in the data in 2003. The minimum wage corresponds to the annualized value of the national monthly minimum wage in 2003. The unemployment benefit is set assuming that all workers receive the maximum number of benefits (5 monthly payments), which is very close to both the mean and median number of benefits (Section 3), while we use the mean monthly value paid in 2003 reported by the Ministry of Labor, which is denominated in multiples of the minimum wage. Finally, the matching function elasticity η is brought from Meghir et al. (2015) who calibrate it to Brazilian data and is within the range surveyed by Petrongolo and Pissarides (2001). Table 7 shows these parameter values and their sources.

In a second step, we take the parameters described in Table 7 as given and estimate the remaining parameters using an Indirect Inference estimator with equilibrium con-

[17] As discussed in Ulyssea (2018), these taxes can be large in some states, which would imply that we underestimate the overall tax burden that firms face. However, we do not include intermediate inputs, which implies that we might be overestimating the actual tax burden faced by some firms. The net effect of these forces is *a priori* unclear.

Table 7: Calibrated Parameters

Parameter	Description	Source	Value
τ_c	Iceberg Trade Costs	Cosar, Guner and Tybout (2016)	2.50
ζ	Share of expend. C	World Input-Output Database	0.283
r	Interest rate	Ulyssea (2010)	0.08
τ_y	Sales Tax	Ulyssea (2017)	0.293
τ_w	Payroll Tax	Ulyssea (2017)	0.375
τ_a	Import Tariff	TRAINS	1.12
κ	Firing Costs	Heckman and Pages (2000) (in R$)	1,956.7
$\underline{w}$	Min. Wage	Annualized 2003 value (in R$)	2,880
b_u	Unemp. Benefit	$1.37 \times 5 = 6.85$ monthly MW	1,644
η	Matching Function	Meghir, Narita and Robin (2015)	0.34

straints.[18] The estimation algorithm is described in details in Appendix B.1. In this step we estimate 35 parameters using 139 data moments and auxiliary parameters. This version of the model imposes $\xi_{kj} = 1$ and $K_{kj} = 0$.

5.1 Identification

The choice of parameters from auxiliary regressions (and moments) to be matched by the model is crucial to achieve identification. Given the high non-linearity and dimension of the model, it is not possible to provide a direct proof of identification. Nevertheless, we provide a heuristic discussion of which variation in the data provides information on different sets of parameters to be estimated.

We start by noting that even though one can directly use micro data to estimate the parameters of the AR(1) processes for productivity (ρ_k and σ_k^z), we estimate them within our Indirect Inference procedure and use the persistence and volatility of firm revenues and labor force sizes to obtain information about these parameters. This information is obtained with PIA (Manufacturing Survey), PAS (Services Survey), PAC (Retail-trade Survey) and RAIS (all sectors). We choose to proceed in this way because the production functions typically assumed by the existing estimators (e.g. Olley and Pakes, 1996; Ackerberg et al., 2007, 2015) are not compatible with our setting where firms use labor as their only input and there is no investment decision.[19] The CES parameters σ_k are

[18]This is the usual Indirect Inference estimator (e.g. Gourieroux and Monfort, 1996; Smith, 2008), but we also penalize deviations from the model's equilibrium constraints in the objective function.

[19]As a cross-check, we use Olley and Pakes (1996)'s estimator to obtain a measure of firm-level

identified by matching the coefficient on log-employment in a regression of log-revenues on log-employment.

The parameters of the hiring costs function (h, γ_1 and γ_2 in equation (6)) are identified using information on growth rates of formal firms, and how these depend on firm size. The convexity in hiring is also important for the model to generate dispersion in wages across firms. Therefore, the relationship between wages and size provide useful information on γ_1. The matching function's parameters ϕ and η are identified from worker transitions out of unemployment and into formal and informal employment in the tradable and non-tradable sectors. We estimate those from the monthly employment survey (PME) and annualize the transitions to make them compatible with the model's period.

The exogenous death shocks for formal firms can be identified off the exit rates of very large firms. Because we cannot observe exit rates of informal firms, we set the exogenous death shocks to be equal for formal and informal firms within sectors. The fixed costs of operation of formal firms ($\bar{c}_{kf}$) are disciplined by how exit rates decline with firm size. In addition, average firm-level revenues help the identification of fixed operating costs of formal firms but also those of informal firms ($\bar{c}_{ki}$). Average firm-level revenues are available from PIA, PAS and PAC for formal firms and ECINF (for informal firms). Larger fixed costs force low-revenue firms to exit, and thereby increase average revenues among survivors.

The fixed cost of exporting, f_x, is identified by the fraction of exporters among formal firms, which is available merging information from RAIS and SECEX. The foreign demand shifter d_F is identified using information on the average size and revenue of exporters (RAIS, PIA and SECEX), fraction of revenues in the tradable sector coming from exports (PIA and SECEX), and the log-wage premium (regression of firm-level log-wages on log-size and exporter indicator, using RAIS and SECEX).

The identification of bargaining power parameters β_f and β_i is straightforward in light of equations (27) and (28). We target the coefficients of a linear regression of firm-level log-wages on revenue per worker (for both the formal and informal sectors, using data from RAIS and ECINF respectively). Lastly, the cost of informality function $p_{ki}(\ell)$ is identified off the size distribution of firms in the informal sector (ECINF), and the share of informal firms by employment size (ECINF).

The exogenous exit rates in the informal sector α_{ki} are necessary to achieve a better fit for the transitions from unemployment to the informal sector. Small values of α_{ki}

productivity for manufacturing firms and use it to estimate a simple AR(1) process. The estimate for the persistence parameter,ρ_C, is remarkably close to the one we obtain in our Indirect Inference estimator. These results are available upon request.

cannot lead to a good match of the high transitions to the informal sector. With small fixed costs of operation (which are necessary to match the small size of informal firms), there would be too little exit from informal sector firms, leading to few new vacancies being created every period, and low transitions from unemployment to informality.

We still need to think more carefully if we can separately identify α_{ki}, h_{ki}, $\bar{c}_{ki}$, and $p_{ki}(\ell)$. The current version estimates these parameters separately as they were essential for a better fit of the data, but it is still not entirely clear to us why we need all of those parameters jointly and how they interact with each other in generating model-based moments.

5.2 Estimates

Table 8 shows our preliminary estimation results. We now discuss the magitude and plausibility of some of these estimates. First, notice that the value of leisure is estimated at $b = 4,515$, quite a bit over the annualized value of the minimum wage of R\$ 2,880. This relatively high value is necessary for a good fit of wages. Also important for the fit of wages are the bargaining parameters $\beta_f = 0.355$ and $\beta_i = 0.999$. Note that our estimates point toward a very large bargaining power of informal sector workers. Perhaps surprising, this result is not necessarily unreasonable. Firm-level revenues in the informal sector are typically very low – the geometric mean in the informal C-sector is of \$5,080 – so, the only way the model can assign wages in the informal sector that are closer to those in the data is by assigning a high value of the bargaining power parameter of informal workers. In the formal sector, the value of $\beta_f = 0.355$ is quite close to the value CGT estimate in Colombia, which amounts to 0.4.

The expected cost of informality – as a share of revenue – is different across sectors. For informal firms in the C-sector, we estimate that firms of size 1 face an expected cost of about half their revenue, but this cost does not increase much with size. On the other hand, the expected cost of informality is negligible for firms of size 1 in the S-sector, but it increases quite steeply with size.

The hiring cost function presents quite a bit of convexity in both sectors ($\gamma_{1C} = 3.28$ and $\gamma_{1S} = 1.81$) and a fair degree of scale economies ($\gamma_{2C} = 0.38$ and $\gamma_{2S} = 0.26$). For comparison, CGT obtain estimates of $\gamma_1 = 3.1$ and $\gamma_2 = 0.39$. To illustrate the magnitude of hiring costs, consider a firm with 10 employees in the C-sector. It will cost this firm R\$394 to expand to 11 employees, or 0.03 times the annual average wage in the formal sector (which is of $R\$12,230$), $R\$3,830$ to expand to 12, and $R\$77,360$ (or 6 times the annual average wage) to expand to 15. On the other hand, it will cost $R\$36,000$ – or 3 times the annual average wage – for a firm with size 50 to expand to 60 employees.

We note that the fixed costs of operation in the formal sector ($\bar{c}_{Cf} = 3,730$ and $\bar{c}_{Sf} = 2,806$) are much larger than the fixed costs of operation in the informal sector ($\bar{c}_{Ci} = 53$ and $\bar{c}_{Si} = 880$). This is expected, given the large perceived costs of operating a firm in the formal sector (compliance with regulations, bureaucracy, bribes, etc.).

Finally, remember that μ_v, d_{HC}, d_{HS} are actually endogenous objects and not parameters. As we discuss in Appendix B.1, we treat these objects as parameters in the estimation procedure, but penalize the deviations from their equilibrium values in the objective function. What is noteworthy is that, in the estimated equilibrium, the vacancy filling rate is $\mu_v = 0.88$, so that firms need to post, on average, 1.13 vacancies to be able to hire 1 worker.

5.3 Model Fit

Tables 9 through 14 compare the moments and statistical relationships generated by the model (under the parameterization described in Tables 7 and 8) with those found in the data. Several key features of the data are well matched by the model.

Table 9 shows the allocation of workers across sectors and unemployment, worker transitions from unemployment, and the distribution of firm size for firms in the C and S sectors. The model is able to match all of these moments reasonably well.

Table 10 shows trade-related moments, as well as firm- exit and turnover moments. The model matches well the share of exports in exporters' revenue and the ratio between total exports and total C-sector revenue. On the other hand, the model overestimates a bit the share of sector-C firms that export and the correlation between log-employment and export status. The model matches well average firm exit rates and the fact that exit rates are much larger among smaller firms. However, the model exclusively attributes exit to death shocks for firms that are above the 20^{th} percentile of the size distribution (that is, firms of size 3 or larger). The firm-exit rates in the S-sector are generally better matched. Firm-level turnover is measured as the absolute value of the employment growth rates popularized by Davis and Haltiwanger (1990) – so it takes firm exit inton account.[20] The model matches well the fact that larger firms tend to experience lower employment turnover, but that conditional on size, exporters experience larger turnover. The model matches well the pattern of turnover even when we condition on employment expansions or contractions.

[20] The growth rate in Davis and Haltiwanger (1990) is defined as $g = \frac{x_{t+1} - x_t}{0.5x_{t+1} + 0.5x_t}$, so it is well defined when $x_{t+1} = 0$.

Table 8: Parameter Estimates

μ^v	Vacancy Filling Rate	0.883
b	Value of Leisure	4,515
β_f	Bargaining Power in the Formal Sector	0.355
β_i	Bargaining Power in the Informal Sector	0.999
ϕ	Matching Function Parameter	0.736
a_C	Cost of Informality: Intercept, C sector	0.539
b_C	Cost of Informality: Slope, C sector	0.033
a_S	Cost of Informality: Intercept, S sector	0.017
b_S	Cost of Informality: Slope, S sector	0.390
h_{Cf}	Hiring Cost Function, level	14,828
h_{Ci}	Hiring Cost Function, level	20,733
γ_{1C}	Hiring Cost Function, convexity	3.280
γ_{2C}	Hiring Cost Function, scale	0.377
h_{Sf}	Hiring Cost Function, level	5,081
h_{Si}	Hiring Cost Function, level	261.0
γ_{1S}	Hiring Cost Function, convexity	1.809
γ_{2S}	Hiring Cost Function, scale	0.260
d_F	Foreign demand shifter	0.063
f_x	Fixed costs of exporting	69,348
$d_{H,C}$	Domestic demand shifter	7.794
σ_C	CES parameter	4.908
ρ_C^Z	AR(1) process, persistence	0.972
σ_C^Z	AR(1) process, volatility	0.367
α_{Cf}	Death Shocks	0.058
$\bar{c}_{Cf}$	Fixed operating costs	3,730
α_{Ci}	Death Shocks	0.039
$\bar{c}_{Ci}$	fixed operating costs	53.32
$d_{H,S}$	Domestic demand shifter	8.047
σ_S	CES parameter	5.234
ρ_S^Z	AR(1) process, persistence	0.949
σ_S^Z	AR(1) process, volatility	0.350
α_{Sf}	Death Shocks	0.022
$\bar{c}_{Sf}$	Fixed operating costs	2,806
α_{Si}	Death Shocks	0.071
$\bar{c}_{Si}$	fixed operating costs	880.4

Notes: ξ_{kj}'s are imposed to be equal to 1, K_{kj}'s are imposed to be equal to 0. μ_v, $d_{H,C}$, $d_{H,S}$, d_F are endogenous variables, but treated as parameters to be estimated. Deviations from these estimates and their revealed equilibrium values are penalized in the objective function. See Appendix B.1 for details.

Table 11 shows moments related to wages in the formal sector and to the firm-size distribution in the informal sector. The wage moments are generally well matched. Perhaps the exception is the exporter-wage premium, which is underestimated by the model. However, that may be explained by the fact that our model does not allow for skill heterogeneity and capital, but in the data exporters tend to be more skill and capital intensive than non-exporters. The distribution of firm-size in the informal sector is very well matched by the model.

Tables 12 and 13 show remaining moments related to firm-level revenues, informal-sector wages and firm-level serial correlation of employment and revenues. The model replicates the fact that transitions from unempoyment to the informal sector are much more likely than transitions from unemployment to the formal sector.

Finally, Table 14 shows the fraction of informal firms conditional on firm size, for size varying from 1 to 5. The model is able to replicate the fact that this fraction is very large for small firm sizes, but decreases steeply with firm-level employment.

Table 9: Model Fit – Panel A

Employment Allocations	Dataset	Model	Data
Share of Workers in informal-C	PME	0.045	0.059
Share of Workers in formal-C	PME	0.105	0.106
Share of Workers in informal-S	PME	0.355	0.351
Share of Workers in formal-S	PME	0.319	0.334
Share of Workers in Unemployment	PME	0.177	0.150
Worker Yearly Transition Rates	Dataset	Model	Data
From Unemployment to informal-C	PME	0.027	0.063
From Unemployment to formal-C	PME	0.070	0.050
From Unemployment to informal-S	PME	0.380	0.387
From Unemployment to formal-S	PME	0.194	0.161
From Unemployment to Unemployment	PME	0.329	0.338
Distribution of Employment across formal-C Firms	Dataset	Model	Data
20th Percentile Employment Distribution	RAIS	2	2
40th Percentile Employment Distribution	RAIS	5	4
60th Percentile Employment Distribution	RAIS	9	7
80th Percentile Employment Distribution	RAIS	21	17
90th Percentile Employment Distribution	RAIS	36	35
95th Percentile Employment Distribution	RAIS	61	67
Mean log-employment	RAIS	1.990	1.779
Variance log-employment	RAIS	1.420	1.821
Mean log-employment formal-C | Exporter	RAIS + SECEX	4.285	3.936
Distribution of Employment across formal-S Firms	Dataset	Model	Data
20th Percentile Employment Distribution	RAIS	1	1
40th Percentile Employment Distribution	RAIS	3	2
60th Percentile Employment Distribution	RAIS	4	4
80th Percentile Employment Distribution	RAIS	9	8
90th Percentile Employment Distribution	RAIS	14	14
95th Percentile Employment Distribution	RAIS	24	25
Mean log-employment	RAIS	1.271	1.178
Variance log-employment	RAIS	0.988	1.262

Table 10: Model Fit – Panel B

Trade-Related Moments formal-C	Dataset	Model	Data
Correlation log-employment and Exporter Status	RAIS + SECEX	0.529	0.378
Fraction of formal-C firms that Export	RAIS + SECEX	0.070	0.053
Mean Exports / Revenue | Exporter (firm-level)	SECEX + IBGE	0.264	0.264
Total Exports / Total Revenue formal-C	SECEX + IBGE	0.131	0.136
Firm Exit - formal-C	Dataset	Model	Data
Mean Firm Exit Rate	RAIS	0.101	0.096
Firm Exit Rate Employment $\leq$ 20th Percentile	RAIS	0.235	0.208
Firm Exit Rate Employment 20th-40th Percentile	RAIS	0.058	0.108
Firm Exit Rate Employment 40th-60th Percentile	RAIS	0.058	0.063
Firm Exit Rate Employment 60th-80th Percentile	RAIS	0.058	0.041
Firm Exit Rate Employment 80th-90th Percentile	RAIS	0.058	0.026
Firm Exit Rate Employment 90th-95th Percentile	RAIS	0.058	0.021
Firm Exit Rate Employment $\geq$ 95th Percentile	RAIS	0.058	0.020
Firm Exit - formal-S	Dataset	Model	Data
Mean Firm Exit Rate	RAIS	0.138	0.113
Firm Exit Rate Employment $\leq$ 20th Percentile	RAIS	0.280	0.218
Firm Exit Rate Employment 20th-40th Percentile	RAIS	0.197	0.181
Firm Exit Rate Employment 40th-60th Percentile	RAIS	0.037	0.092
Firm Exit Rate Employment 60th-80th Percentile	RAIS	0.085	0.053
Firm Exit Rate Employment 80th-90th Percentile	RAIS	0.022	0.043
Firm Exit Rate Employment 90th-95th Percentile	RAIS	0.022	0.037
Firm Exit Rate Employment $\geq$ 95th Percentile	RAIS	0.034	0.030
Turnover formal-C Firms	Dataset	Model	Data
Mean Turnover	RAIS	0.343	0.499
Regression Turnover: Constant	RAIS	0.622	0.731
Regression Turnover: log-Employment	RAIS	-0.145	-0.134
Regression Turnover: Exporter Status	RAIS	0.147	0.108
Regression Turnover | Expansion: Constant	RAIS	0.502	0.712
Regression Turnover | Expansion: log-Employment	RAIS	-0.118	-0.146
Regression Turnover | Expansion: Exporter Status	RAIS	0.125	0.141
Regression Turnover | Contraction: Constant	RAIS	0.673	0.730
Regression Turnover | Contraction: log-Employment	RAIS	-0.129	-0.117
Regression Turnover | Contraction: Exporter Status	RAIS	0.285	0.099
Turnover Formal-S Firms	Dataset	Model	Data
Mean Turnover	RAIS	0.437	0.504
Regression Turnover: Constant	RAIS	0.599	0.642
Regression Turnover: log-Employment	RAIS	-0.127	-0.117
Regression Turnover | Expansion: Constant	RAIS	0.596	0.695
Regression Turnover | Expansion: log-Employment	RAIS	-0.153	-0.156
Regression Turnover | Contraction: Constant	RAIS	0.591	0.622
Regression Turnover | Contraction: log-Employment	RAIS	-0.106	-0.095

Table 11: Model Fit – Panel C

Firm-level Wages Formal-C	Dataset	Model	Data
Mean log-Wages	RAIS	8.880	8.637
Mean log-Wages | Exporter	RAIS + SECEX	9.269	9.276
Regression 1 log-Wages: Constant	RAIS	8.671	8.443
Regression 1 log-Wages: log-Employment	RAIS	0.099	0.094
Regression 1 log-Wages: Exporter Status	RAIS	0.174	0.462
Regression 2 log-Wage: Constant	IBGE	2.826	6.334
Regression 2 log-Wage: log-Revenue/Worker	IBGE	0.620	0.235
Firm-level Wages Formal-S	Dataset	Model	Data
Mean log-Wages	RAIS	8.761	8.562
Regression 1 log-Wages: Constant	RAIS	8.601	8.434
Regression 1 log-Wages: log-Employment	RAIS	0.126	0.108
Regression 2 log-Wage: Constant	IBGE	3.143	7.417
Regression 2 log-Wage: log-Revenue/Worker	IBGE	0.586	0.109
Distribution of Employment across Informal-C Firms (truncated at size 5)	Dataset	Model	Data
20th Percentile Employment Distribution	ECINF	1	1
40th Percentile Employment Distribution	ECINF	1	1
60th Percentile Employment Distribution	ECINF	1	1
80th Percentile Employment Distribution	ECINF	1	1
90th Percentile Employment Distribution	ECINF	2	2
95th Percentile Employment Distribution	ECINF	2	2
99th Percentile Employment Distribution	ECINF	4	4
Mean log-Employment	ECINF	0.096	0.105
Variance log-Employment	ECINF	0.093	0.092
Distribution of Employment across Informal-S Firms (truncated at size 5)	Dataset	Model	Data
20th Percentile Employment Distribution	ECINF	1	1
40th Percentile Employment Distribution	ECINF	1	1
60th Percentile Employment Distribution	ECINF	1	1
80th Percentile Employment Distribution	ECINF	1	1
90th Percentile Employment Distribution	ECINF	2	2
95th Percentile Employment Distribution	ECINF	2	2
99th Percentile Employment Distribution	ECINF	3	3
Mean log-Employment	ECINF	0.086	0.097
Variance log-Employment	ECINF	0.061	0.075

Table 12: Model Fit – Panel D

Firm-level Revenues informal-C	Dataset	Model	Data
Mean log-Revenue	ECINF	8.720	8.533
Variance log-Revenue	ECINF	0.585	1.444
Correlation log-Revenue and log-Employment	ECINF	0.641	0.339
Firm-level Revenues informal-S	Dataset	Model	Data
Mean log-Revenue	ECINF	8.733	8.952
Variance log-Revenue	ECINF	0.409	1.298
Correlation log-Revenue and log-Employment	ECINF	0.747	0.318
Firm-level Wages Informal-C	Dataset	Model	Data
Mean log-Wages	ECINF	7.850	8.014
Regression 1 log-Wage: Constant	ECINF	7.809	8.006
Regression 1 log-Wage: log-Employment	ECINF	0.427	0.079
Regression 2 log-Wage: Constant	ECINF	0.147	3.777
Regression 2 log-Wage: log-Revenue/Worker	ECINF	0.893	0.397
Firm-level Wages Informal-S	Dataset	Model	Data
Mean log-Wages	ECINF	8.357	8.415
Regression 1 log-Wage: Constant	ECINF	8.348	8.413
Regression 1 log-Wage: log-Employment	ECINF	0.103	0.020
Regression 2 log-Wage: Constant	ECINF	0.122	3.912
Regression 2 log-Wage: log-Revenue/Worker	ECINF	0.952	0.379

Table 13: Model Fit – Panel E

Firm-level Revenues formal-C	Dataset	Model	Data
Mean log-Revenue	IBGE	11.747	12.726
Variance log-Revenue	IBGE	2.084	3.511
Mean log-Revenue | Exporter	IBGE	14.640	15.465
Variance log-Revenue | Exporter	IBGE	0.223	4.448
Regression log-Revenue: Constant	IBGE	9.501	10.118
Regression log-Revenue: log-Employment	IBGE	1.116	1.000
Regression log-Revenue: Exporter	IBGE	0.356	1.462
Firm-level Revenues formal-S	Dataset	Model	Data
Mean log-Revenue	IBGE	10.851	10.814
Variance log-Revenue	IBGE	1.387	2.074
Regression log-Revenue: Constant	IBGE	9.382	10.004
Regression log-Revenue: log-Employment	IBGE	1.156	0.872
Longitudinal Relationships	Dataset	Model	Data
Correlation log-Emp(t) and log-Emp($t+1$) – formal-C	RAIS	0.922	0.918
Correlation log-Emp(t) and log-Emp($t+1$) – formal-S	RAIS	0.906	0.908
Correlation log-Revenue(t) and log-Revenue($t+1$) – formal-C	IBGE	0.884	0.929
Correlation log-Revenue(t) and log-Revenue($t+1$) – formal-S	IBGE	0.673	0.845
Miscellaneous	Dataset	Model	Data
Transitions from U to Informal / Transitions from U to formal	PME	1.541	2.130
Share of Informal Employed Workers	PME	0.485	0.482

Table 14: Model Fit – Panel F

Fraction of Informal Firms – C and S Sectors Pooled	Dataset	Model	Data
Among Firms with 1 Worker	ECINF	0.956	0.933
Among Firms with 2 Workers	ECINF	0.877	0.711
Among Firms with 3 Workers	ECINF	0.340	0.491
Among Firms with 4 Workers	ECINF	0.117	0.261
Among Firms with 5 Workers	ECINF	0.055	0.372
Fraction of Informal Firms – C Sector Firms Only	Dataset	Model	Data
Among Firms with 1 Worker	ECINF	0.985	0.962
Among Firms with 2 Workers	ECINF	0.641	0.755
Among Firms with 3 Workers	ECINF	0.544	0.411
Among Firms with 4 Workers	ECINF	0.568	0.508
Among Firms with 5 Workers	ECINF	0.539	0.497
Fraction of Informal Firms – S Sector Firms Only	Dataset	Model	Data
Among Firms with 1 Worker	ECINF	0.954	0.929
Among Firms with 2 Workers	ECINF	0.901	0.705
Among Firms with 3 Workers	ECINF	0.330	0.509
Among Firms with 4 Workers	ECINF	0	0.218
Among Firms with 5 Workers	ECINF	0	0.341

6 Counterfactual Experiments

After having estimated our model and having checked that it matches several key features of the data, we move to the counterfactual experiments designed to understand the effects of trade openness in the presence of labor market frictions, regulations, and a large informal sector. We start by simulating the economy for different values of the import tariff. The benchmark case has $\tau_a = 1.12$, reflecting the average import tariff of 12 percent in place in 2003. We will compute new equilibria for $\tau_a = 1$ (free trade), $\tau_a = 1.33$ (average import tariffs before the 1990s trade liberalization), $\tau_a = 1.5$ (very high tariffs) and $\tau_a = \infty$ (closed economy). Throughout this set of simulations, the exchange rate ϵ adjusts to balance trade. In a second set of simulations, we fix the exchange rate at its benchmark value (equilibrium with $\tau_a = 1.12$), so that trade imbalances arise in response to import tariff changes. Trade deficits are modeled as income transfers from the rest of the world to Brazil and trade surpluses are modeled as income transfers from Brazil to the rest of the world – see Appendix C.1 for details.

In a second set of counterfactual experiments, we study how iceberg trade costs affect labor market outcomes. The benchmark case has $\tau_c = 2.5$, which is the value for iceberg trade costs imposed throughout estimation. In the simulations, we gradually reduce iceberg trade costs to $\tau_c = 1.2$, leading to a reduction of over 50 percent in iceberg trade costs. In these simulations, we keep tariffs at the benchmark value of $\tau_a = 1.12$.

Next, we investigate how the economy responds to tougher labor market regulations. Specifically, we simulate two changes in labor market regulations: (1) an increase of 100 percent in the minimum wage $\underline{w}$; and (2) an increase in 50 percent in firing costs κ.[21] We also study how declines in iceberg trade costs affect the economy in a scenario where minimum wages are twice those in the benchmark case (those implemented in practice in 2003). Our goal with this exercise is to understand how the impact of trade liberalization depends on the stringency of labor market regulations in place.

In an important set of counterfactuals, we study how the economy behaves in response to aggregate productivity shocks. In this case, the goal is to understand the role of the informal sector as a labor market smoothing device when the economy is hit with adverse shocks, and to what extent growth can reduce informality.

All of the simulations above are done under two scenarios. The first scenario is the benchmark scenario, where only tariffs, iceberg trade costs, labor market regulations, or aggregate productivity vary, one at a time. The second scenario performs the same

[21] These increases are relative to the "benchmark" parameters in Table 7.

counterfactuals, but we consider an economy without informality. In other words, we consider an economy for which the cost of informality, from the point of view of firms, is infinity – so that no firm chooses to be informal. The goal is to understand the role of the informal sector in mediating the labor market effects of trade, labor market regulations, and aggregate productivity shocks.

A key result of these analyses is that the eradication of informality can have large positive welfare effects. In a last set of counterfactual experiments, we turn to the question of how to reduce informality. We investigate if more lax labor market regulations and a reduction in red tape in the economy is effective in reducing informality and increasing welfare.

6.1 Import Tariffs

Table 15 shows the labor market effects of import tariffs τ_a. What is remarkable in these simulations is the very minor effect of import tariffs on labor market outcomes and welfare. As a matter of fact, the economy is barely affected if its import tariffs increase to infinity – so that it is under autarky – or if they are reduced to zero – so that it is under free trade. In these simulations, import tariffs have large effects on trade volumes, on the share of exporting firms and on the exchange rate, but not on real outcomes such as the distribution of employment across sectors and types of firms, aggregate productivity and welfare. Interestingly, and although the effect of import tariffs on welfare is negligible, the simulations suggest an optimal import tariff between 12 percent and 50 percent.

Table 16 reproduces the simulations of Table 15, but under a scenario where informality is infinitely costly. This is operationalized by, for example, taking the fixed costs of operation in the informal sector c_{ki}, for $k \in \{C, S\}$, to infinity, so that no firm chooses to operate informally. As in Table 15, import tariffs have negligible effects on the real economy, apart from its effects on trade volumes and on the share of exporting firms.[22]

However, note that, at the benchmark with import tariffs of 12 percent ($\tau_a = 1.12$), eradicating informality leads to an increase in welfare of 42 percent. This is an important result that merits a few observations. Note that the mass of formal firms operating in either sector (N_{fC} and N_{fS}) increases by a factor of over 2.2 once informality is eliminated. Indeed, many informal firms could in fact profitably operate formally, but chose not to, remaining artificially small and less visible to the government. Once informality

[22]Note the decline in the share of exporters among formal C-sector firms when informality is shut down. This decline is driven by the strong increase in the mass of formal sector firms, so the pool of formal sector firms is larger.

Table 15: Import Tariffs and Labor Market Outcomes

Variable	$\tau_a = 1$	$\tau_a = 1.12$	$\tau_a = 1.33$	$\tau_a = 1.5$	$\tau_a = \infty$
μ_v	0.883	0.883	0.885	0.888	0.901
$d_{H,C}$	7.794	7.794	7.791	7.789	7.778
$d_{H,S}$	8.047	8.047	8.046	8.046	8.044
Exchange Rate ϵ	21.446	20.318	18.766	17.802	–
Share Emp. iC	0.044	0.045	0.045	0.045	0.046
Share Emp. fC	0.106	0.105	0.105	0.106	0.110
Share Emp. iS	0.355	0.355	0.354	0.353	0.350
Share Emp. fS	0.319	0.319	0.318	0.318	0.316
Share Unemp.	0.177	0.177	0.177	0.177	0.178
Share Informality	0.484	0.485	0.485	0.485	0.482
C-sector Sh. Exporters	0.091	0.070	0.045	0.032	0
Ratio Exports Rev. fC	0.182	0.131	0.077	0.052	0
Avg. Turnover fC	0.346	0.343	0.346	0.346	0.344
Avg. Turnover fS	0.437	0.437	0.437	0.437	0.437
N_{fC}	104,826	105,734	107,319	108,003	110,748
N_{iC}	530,189	534,779	542,796	546,257	560,138
N_{fS}	835,165	835,176	833,260	831,445	823,551
N_{iS}	4,922,038	4,922,101	4,911,304	4,900,868	4,855,100
Std Dev log Wages fC	0.266	0.263	0.258	0.256	0.250
Std Dev log Wages iC	0.573	0.573	0.573	0.573	0.570
Agg. Productivity C	33.551	33.237	33.138	33.088	33.365
Std Dev log Wages fS	0.160	0.160	0.159	0.159	0.159
Std Dev log Wages iS	0.523	0.523	0.523	0.523	0.524
Agg. Productivity S	9.085	9.085	9.095	9.099	9.119
Real Income p/c	123.557	123.760	123.887	123.838	123.184

Notes: Aggregate productivity is the employment-weighted average of the productivities z. Real Income per capita $=\frac{I/\bar{L}}{P}$, where $P = \frac{P_C^{\zeta} P_S^{1-\zeta}}{\zeta^{\zeta}(1-\zeta)^{(1-\zeta)}}$. "Ratio Exports Rev. fC" is the ratio between total exports and total revenues among formal C-sector firms.

is abolished, aggregate productivity increases sharply, but unemployment increases only modestly – from 17.7 percent to 18.4 percent in the benchmark case ($\tau_a = 1.12$). These results suggest that the existence of the informal sector can lead to a substantial misallocation of resources. Indeed, the informal sector leads to a situation where smaller and less productive informal firms compete for workers with larger and more productive formal firms. Once informal firms are not allowed to operate, they free up resources that are reallocated toward more productive uses, raising productivity, incomes and welfare. It is important to note that we are finding a lower bound on the welfare effect of abolishing informality as we are not modelling the provision of public goods.

Table 16: Import Tariffs and Labor Market Outcomes – No Informality

Variable	τ_a=1	$\tau_a = 1.12$	$\tau_a = 1.33$	$\tau_a = 1.5$	$\tau_a = \infty$
μ_v	0.889	0.888	0.890	0.891	0.899
$d_{H,C}$	7.839	7.839	7.837	7.835	7.828
$d_{H,S}$	8.115	8.115	8.115	8.115	8.113
Exchange Rate ϵ	21.385	20.325	18.821	17.906	–
Share Emp. fC	0.170	0.169	0.170	0.172	0.176
Share Emp. fS	0.646	0.647	0.646	0.645	0.638
Share Unemp.	0.184	0.184	0.184	0.183	0.186
C-sector Sh. Exporters	0.043	0.032	0.020	0.014	0
Ratio Exports Rev. fC	0.143	0.101	0.059	0.039	0
Avg. Turnover fC	0.319	0.319	0.319	0.319	0.320
Avg. Turnover fS	0.482	0.482	0.482	0.482	0.483
N_{fC}	288,551	293,113	296,408	297,568	303,840
N_{fS}	1,829,956	1,832,442	1,829,490	1,827,291	1,828,667
Std Dev log Wages C	0.291	0.289	0.285	0.284	0.279
Agg. Productivity C	43.029	42.918	42.724	42.658	42.977
Std Dev log Wages S	0.172	0.172	0.172	0.172	0.172
Agg. Productivity S	15.227	15.226	15.227	15.227	15.308
Real Income p/c	175.102	175.577	175.667	175.652	174.605

Notes: No Informality: firms face prohibitive costs of choosing informality, e.g., $\bar{c}_{ki} = \infty$. Otherwise, all other parameters as in Tables 7 and 8. Aggregate productivity is the employment-weighted average of the productivities z. Real Income per capita $= \frac{I/\bar{L}}{P}$, where $P = \frac{P_C^{\varsigma} P_S^{1-\varsigma}}{\varsigma^{\varsigma}(1-\varsigma)^{(1-\varsigma)}}$. "Ratio Exports Rev. fC" is the ratio between total exports and total revenues among formal C-sector firms.

The small effect of import tariffs on allocations and welfare may be partly explained by the fact that the exchange rate perfectly adjusts to balance trade. Once import tariffs are reduced, increased imports lead to a competition effect. As in Melitz (2003), less productive formal firms exit or become informal, less productive informal firms exit.

This effect leads to a reduction in income and revenues in the economy. However, the increase in imports must be matched by an equal increase in exports, in turn leading to an increase in revenues and incomes (propelled by a devalued exchange rate – larger ϵ), offsetting the competition effect. Indeed, Tables 15 and 16 show a very small effect of τ_a on μ_v and $d_{H,C}$ – the demand shifter in the C-sector.

Table 17 investigates the effect of import tariffs under a fixed exchange rate ϵ. In this scenario, a decline in import tariffs leads to an increase in imports. As the exchange rate is fixed, changes in exports do not match the increase in imports, leading to a trade deficit (which is modeled as an income transfer from the rest of the world to Brazil). Note that with a fixed exchange rate, there should be a strong competition effect. However, the competition effect is more than offset by the income transfer from the rest of the world, leading to a measurable increase in welfare if tariffs go to zero ($\tau_a = 1$). A competition effect is probably more accurately captured by an increase in import tariffs, as this leads to a decline in the demand shifter $d_{H,C}$, an increase in unemployment and an aggregate decline in welfare – an increase in import tariffs leads to an income transfer from Brazil to the rest of the world, hurting domestic workers. Table 18 replicates Table 17 under the scenario abolishing informality. The conclusions are similar.

Table 17: Import Tariffs and Labor Market Outcomes – Fixed Exchange Rate

Variable	$\tau_a = 1$	$\tau_a = 1.12$	$\tau_a = 1.33$	$\tau_a = 1.5$	$\tau_a = 1.75$
μ_v	0.831	0.883	0.914	0.929	0.943
$d_{H,C}$	7.817	7.794	7.779	7.772	7.767
$d_{H,S}$	8.054	8.047	8.042	8.041	8.039
Exchange Rate ϵ	20.319	20.319	20.319	20.319	20.319
Trade Deficit Over Rev_C	0.144	0	-0.085	-0.111	-0.128
Share Emp. iC	0.030	0.045	0.047	0.050	0.053
Share Emp. fC	0.102	0.105	0.113	0.113	0.114
Share Emp. iS	0.370	0.355	0.347	0.347	0.344
Share Emp. fS	0.325	0.319	0.314	0.310	0.308
Share Unemp.	0.173	0.177	0.180	0.179	0.181
Share Informality	0.484	0.485	0.480	0.484	0.485
C-sector Sh. Exporters	0.040	0.070	0.076	0.085	0.088
Ratio Exports Rev. fC	0.106	0.131	0.146	0.154	0.160
Avg. Turnover fC	0.284	0.343	0.345	0.346	0.351
Avg. Turnover fS	0.440	0.437	0.438	0.433	0.433
N_{fC}	153,497	105,738	112,064	105,034	105,182
N_{iC}	479,046	534,800	566,794	589,988	590,822
N_{fS}	808,855	835,167	816,137	835,219	827,636
N_{iS}	4,999,986	4,922,048	4,812,658	4,869,033	4,826,236
Std Dev log Wages fC	0.278	0.263	0.260	0.257	0.254
Std Dev log Wages iC	0.564	0.573	0.570	0.575	0.559
Agg. Productivity C	33.771	33.237	33.933	34.166	34.031
Std Dev log Wages fS	0.159	0.160	0.159	0.160	0.160
Std Dev log Wages iS	0.540	0.523	0.524	0.524	0.524
Agg. Productivity S	9.091	9.085	9.136	9.051	9.067
Real Income p/c	128.438	123.758	121.196	119.745	118.740

Notes: Aggregate productivity is the employment-weighted average of the productivities z. Real Income per capita $=\frac{I/\bar{L}}{P}$, where $P = \frac{P_C^{\zeta} P_S^{1-\zeta}}{\zeta^{\zeta}(1-\zeta)^{(1-\zeta)}}$. "Ratio Exports Rev. fC" is the ratio between total exports and total revenues among formal C-sector firms. Exchange rate ϵ is fixed throughout. Trade deficits are modeled as transfers from the rest of the world to Brazil.

Table 18: Import Tariffs and Labor Market Outcomes – Fixed Exchange Rate, No Informality

Variable	$\tau_a = 1$	$\tau_a = 1.12$	$\tau_a = 1.33$	$\tau_a = 1.5$
μ_v	0.864	0.888	0.907	0.917
$d_{H,C}$	7.850	7.839	7.831	7.827
$d_{H,S}$	8.120	8.115	8.112	8.110
Exchange Rate ϵ	20.325	20.325	20.325	20.325
Trade Deficit Over Rev_C	0.105	0	-0.068	-0.090
Share Emp. fC	0.158	0.169	0.178	0.182
Share Emp. fS	0.661	0.647	0.635	0.630
Share Unemp.	0.181	0.184	0.187	0.188
C-sector Share Exporters	0.030	0.032	0.033	0.034
Ratio Exports Rev. fC	0.093	0.101	0.108	0.111
Avg. Turnover fC	0.319	0.319	0.320	0.320
Avg. Turnover fS	0.481	0.482	0.483	0.483
N_{fC}	280,130	293,113	307,690	313,001
N_{fS}	1,868,825	1,832,442	1,818,589	1,799,464
Std Dev log Wages C	0.291	0.289	0.287	0.286
Aggregate Productivity C	42.426	42.918	43.320	43.473
Std Dev log Wages S	0.173	0.172	0.172	0.171
Aggregate Productivity S	15.182	15.226	15.310	15.336
RealIncome p/c	179.959	175.577	172.207	170.737

Notes: No Informality: firms face prohibitive costs of choosing informality, e.g., $\bar{c}_{ki} = \infty$. Otherwise, all other parameters as in Table 8. Aggregate productivity is the employment-weighted average of the productivities z. Real Income per capita $= \frac{I/\bar{L}}{P}$, where $P = \frac{P_C^{\zeta} P_S^{1-\zeta}}{\zeta^{\zeta}(1-\zeta)^{(1-\zeta)}}$. "Ratio Exports Rev. fC" is the ratio between total exports and total revenues among formal C-sector firms. Exchange rate ϵ is fixed throughout. Trade deficits are modeled as transfers from the rest of the world to Brazil.

6.2 Iceberg Trade Costs

Tables 15 and 16 show that import tariffs have a negligible effect on labor market outcomes and welfare if trade balance is imposed. Maybe this is because import tariffs are small relative to iceberg trade costs. In this section, we investigate the effect of large "globalization shocks", varying iceberg trade costs from $\tau_c = 2.5$ to $\tau_c = 1.2$, leading to a reduction of more than 50 percent in trade costs. In all of the subsequent simulations, we keep import tariffs at the benchmark value of $\tau_a = 1.12$.

Table 19 shows that reductions in iceberg trade costs τ_c lead to more substantial labor market and welfare effects than reductions in import tariffs τ_a. We first focus on a reasonable (albeit large) reduction in trade costs, from $\tau_c = 2.5$ to $\tau_c = 2$. In that case, the number of workers employed in the informal tradable sector is reduced by about 30%. This result is consistent with the empirical results documented in McCaig and Pavcnik (2018) who focus only on the Vietnamese tradable sector. However, as real incomes rise, increased globalization makes it more attractive for firms to enter the non-tradable sector (both formally and informally), leading to a relative increase in an informality-intensive sector. Given that informality in the tradable sector is relatively small to start with, the total informality share in the economy is reduced, but only slightly (from 48.5% to 47.9%). The small role of trade on informality predicted by our model is consistent with the casual observation that the informal sector has not substantially shrunk in middle-income economies despite the large-scale liberalization episodes they went through in the 1980s and 1990s.

Interestingly, the effect of τ_c on informality is quite non-linear. Reductions in τ_c from very large values tend to reduce informality in the C-sector, but further reductions tend to increase it. The behavior of informality in the S-sector in response to τ_c seems to be monotone: informality increases as τ_c declines. Overall, informality tends to increase and unemployment to decline with globalization.

Table 20 investigates if the labor market effects of τ_c differ much if informality is abolished. We also find that unemployment declines with reductions in trade costs, and that these numbers are not much larger than unemployment rates in the presence of informality. Gains from reducing iceberg trade costs from $\tau_c = 2.5$ to $\tau_c = 1.2$ are larger in the "benchmark" economy allowing for informality ($100 \times \left(1 - \frac{143.490}{123.184}\right)$=16.4 percent vs. $100 \times \left(1 - \frac{201.254}{174.605}\right)$=15.2 percent), but the difference in gains does not seem to be large. Removing informality, though, has a much larger welfare effect than reducing trade costs. Finally, note that a reduction of iceberg trade costs from $\tau_c = 2.5$ to $\tau_c = 1.2$ is quite unrealistic, but helpful to understand magnitudes involved.

Table 19: Iceberg Trade Costs and Labor Market Outcomes

Variable	$\tau_c = 1.2$	$\tau_c = 1.5$	$\tau_c = 2$	$\tau_c = 2.19$	$\tau_c = 2.5$	$\tau_c = \infty$
μ_v	0.808	0.818	0.849	0.863	0.883	0.901
$d_{H,C}$	7.534	7.735	7.803	7.802	7.794	7.778
$d_{H,S}$	8.057	8.055	8.051	8.049	8.047	8.044
Exchange Rate ϵ	18.659	20.385	20.355	20.272	20.318	–
Share Emp. iC	0.041	0.035	0.031	0.032	0.045	0.046
Share Emp. fC	0.084	0.094	0.107	0.109	0.105	0.110
Share Emp. iS	0.377	0.374	0.364	0.364	0.355	0.350
Share Emp. fS	0.329	0.327	0.324	0.320	0.319	0.316
Share Unemp.	0.170	0.171	0.174	0.176	0.177	0.178
Share Informality	0.503	0.493	0.479	0.480	0.485	0.482
C-sector Sh. Exporters	0.615	0.319	0.111	0.082	0.070	0
Ratio Exports Rev. fC	0.923	0.700	0.307	0.218	0.131	0
Avg. Turnover fC	0.310	0.338	0.347	0.348	0.343	0.344
Avg. Turnover fS	0.439	0.439	0.435	0.435	0.437	0.437
N_{fC}	42,246	75,717	131,220	136,444	105,734	110,748
N_{iC}	309,701	425,309	487,433	506,837	534,779	560,138
N_{fS}	823,098	816,670	856,672	857,890	835,176	823,551
N_{iS}	5,087,376	5,047,933	5,044,783	5,049,570	4,922,101	4,855,100
Std Dev log Wages fC	0.303	0.302	0.282	0.275	0.263	0.250
Std Dev log Wages iC	0.518	0.573	0.561	0.561	0.573	0.570
Agg. Productivity C	35.088	34.544	35.034	34.933	33.237	33.365
Std Dev log Wages fS	0.159	0.159	0.161	0.161	0.160	0.159
Std Dev log Wages iS	0.540	0.540	0.529	0.523	0.523	0.524
Agg. Productivity S	9.062	9.077	9.012	8.975	9.085	9.119
Real Income p/c	143.490	132.433	125.837	124.666	123.760	123.184

Notes: Aggregate productivity is the employment-weighted average of the productivities z. Real Income per capita $=\frac{I/\overline{L}}{P}$, where $P = \frac{P_C^{\zeta} P_S^{1-\zeta}}{\zeta^{\zeta}(1-\zeta)^{(1-\zeta)}}$. "Ratio Exports Rev. fC" is the ratio between total exports and total revenues among formal C-sector firms.

Table 20: Iceberg Trade Costs and Labor Market Outcomes – No Informality

Variable	$\tau_c = 1.2$	$\tau_c = 1.5$	$\tau_c = 2.0$	$\tau_c = 2.19$	$\tau_c = 2.5$	$\tau_c = \infty$
μ_v	0.820	0.845	0.873	0.880	0.888	0.899
$d_{H,C}$	7.640	7.791	7.843	7.842	7.839	7.828
$d_{H,S}$	8.128	8.123	8.118	8.117	8.115	8.113
Exchange Rate ϵ	19.244	20.213	20.165	20.139	20.325	–
Share Emp. fC	0.137	0.150	0.163	0.166	0.169	0.176
Share Emp. fS	0.686	0.671	0.655	0.651	0.647	0.638
Share Unemp.	0.177	0.179	0.182	0.183	0.184	0.186
C-sector Sh. Exporters	0.270	0.143	0.067	0.051	0.032	0
Ratio Exports Rev. fC	0.854	0.616	0.256	0.180	0.101	0
Avg. Turnover fC	0.407	0.318	0.318	0.318	0.319	0.320
Avg. Turnover fS	0.481	0.481	0.482	0.482	0.482	0.483
N_{fC}	144,382	226,301	271,847	281,779	293,113	303,840
N_{fS}	1,986,312	1,909,748	1,856,561	1,843,899	1,832,442	1,828,667
Std Dev log Wages C	0.329	0.323	0.299	0.294	0.289	0.279
Agg. Productivity C	44.747	43.038	42.959	42.981	42.918	42.977
Std Dev log Wages S	0.175	0.174	0.173	0.173	0.172	0.172
Agg. Productivity S	15.050	15.146	15.212	15.222	15.226	15.308
Real Income p/c	201.254	186.022	177.657	176.585	175.577	174.605

Notes: No Informality: firms face prohibitive costs of choosing informality, e.g., $\overline{c}_{ki} = \infty$. Otherwise, all other parameters as in Tables 7 and 8. Aggregate productivity is the employment-weighted average of the productivities z. Real Income per capita $= \frac{I/\overline{L}}{P}$, where $P = \frac{P_C^{\zeta} P_S^{1-\zeta}}{\zeta^{\zeta}(1-\zeta)^{(1-\zeta)}}$. "Ratio Exports Rev. fC" is the ratio between total exports and total revenues among formal C-sector firms.

6.3 Tougher Labor Market Regulations

This section investigates how increases in the minimum wage $\underline{w}$ and in firing costs κ affect labor market outcomes. We also investigate how these effects change if informality is eradicated.

Table 21 simulates a situation where the minimum wage is doubled. As expected, once the minimm wage increases, unemployment increases (from 17.7 percent to 18.2 percent) and informal employment increases (from 48.5 percent to 50.9 percent). Welfare declines by less than 1 percent. If informality is banned, the effect of the minimum wage on unemployment is larger – it increases from 18.4 percent to 19.8 percent – but welfare slightly increases! The latter result might be explained by the fact that the wage solution of our bargaining process does not lead to efficient outcomes. Food for thought! Also, there seems to be a trade off between the effect of minimum wages on aggregate productivity (pushing less productive firms out of the market) and on unemployment. If the effect of minimum wages on unemployment is not significant, increasing the minimum wage can improve welfare.

Table 22 studies the effect of an increase of 50 percent in firing costs. The policy change does not affect informality rates, but does lead to an increase in the unemployment rate. In both scenarios (with and without informality), increases in firing costs lead to substantial welfare declines.

As a last exercise, we study how a reduction in iceberg trade costs affects the economy depending on the level of minimum wages. We compare the effect of a reduction of iceberg trade costs in the benchmark economy (parameters given in Tables 7 and 8) with those in an economy with a twice as large minimum wage. Table 23 shows that, overall, the effects of reducing iceberg trade costs from $\tau_c = 2.5$ to $\tau_c = 1.5$ do not look very different according to the level of the minimum wage. However, the gains from this reduction are a bit larger in the benchmark economy with the smaller minimum wage: 7 percent compared to 6 percent.

6.4 Aggregate Productivity Shocks

The empirical results documented in Dix-Carneiro and Kovak (2019) are based on a difference-in-difference strategy isolating the competition effect of trade liberalization. Regions that were more exposed to import competition experienced relative increases in unemployment in the medium run, but there are no effects on unemployment in the long run. However, more exposed regions did experience substantial relative increases in

Table 21: Labor Market Effects of Doubling the Minimum Wage $\underline{w}$

Variable	Benchmark: Min Wage $\underline{w}$	Benchmark: Min Wage $2 \times \underline{w}$	No Informality: Min Wage $\underline{w}$	No Informality: Min Wage $2 \times \underline{w}$
μ_v	0.883	0.947	0.888	0.950
$d_{H,C}$	7.794	7.766	7.839	7.822
$d_{H,S}$	8.047	8.043	8.115	8.114
Exchange Rate ϵ	20.318	19.678	20.325	19.802
Share Emp. iC	0.045	0.048	0	0
Share Emp. fC	0.105	0.103	0.169	0.167
Share Emp. iS	0.355	0.369	0	0
Share Emp. fS	0.319	0.298	0.647	0.635
Share Unemp.	0.177	0.182	0.184	0.198
Share Informality	0.485	0.509	0	0
C-sector Share Exporters	0.070	0.076	0.032	0.046
Ratio Exports Rev. fC	0.131	0.134	0.101	0.104
Avg. Turnover fC	0.343	0.351	0.319	0.402
Avg. Turnover fS	0.437	0.524	0.482	0.528
N_{fC}	105,734	94,328	293,113	206,512
N_{iC}	534,779	529,852	0	0
N_{fS}	835,176	608,405	1,832,442	1,801,059
N_{iS}	4,922,101	5,424,880	0	0
Std Dev log Wages fC	0.263	0.249	0.289	0.260
Std Dev log Wages iC	0.573	0.559	–	–
Aggregate Productivity C	33.237	34.047	42.918	45.934
Std Dev log Wages fS	0.160	0.140	0.172	0.150
Std Dev log Wages iS	0.523	0.581	–	–
Aggregate Productivity S	9.085	9.125	15.226	15.831
RealIncome p/c	123.760	122.882	175.577	176.046

Notes: Benchmark economy features the parameters in Tables 7 and 8. No Informality: firms face prohibitive costs of choosing informality, e.g., $\bar{c}_{ki} = \infty$. Otherwise, all other parameters as in Tables 7 and 8. Aggregate productivity is the employment-weighted average of the productivities z. Real Income per capita $= \frac{I/\overline{L}}{P}$, where $P = \frac{P_C^{\zeta} P_S^{1-\zeta}}{\zeta^{\zeta}(1-\zeta)^{(1-\zeta)}}$. "Ratio Exports Rev. fC" is the ratio between total exports and total revenues among formal C-sector firms. Minimum wage $\underline{w}$ as in Table 7.

Table 22: Labor Market Effects of a 50 percent Increase in Firing Costs κ

	Benchmark		No Informality	
Variable	Firing Costs κ	Firing Costs $1.5 \times \kappa$	Firing Costs κ	Firing Costs $1.5 \times \kappa$
μ_v	0.883	0.988	0.888	0.972
$d_{H,C}$	7.794	7.753	7.839	7.810
$d_{H,S}$	8.047	8.037	8.115	8.105
Exchange Rate ϵ	20.318	19.453	20.325	19.690
Share Emp. iC	0.045	0.046	0	0
Share Emp. fC	0.105	0.103	0.169	0.167
Share Emp. iS	0.355	0.350	0	0
Share Emp. fS	0.319	0.319	0.647	0.644
Share Unemp.	0.177	0.182	0.184	0.189
Share Informality	0.485	0.484	0	0
C-sector Share Exporters	0.070	0.079	0.032	0.034
Ratio Exports Rev. fC	0.131	0.136	0.101	0.105
Avg. Turnover fC	0.343	0.308	0.319	0.323
Avg. Turnover fS	0.437	0.444	0.482	0.471
N_{fC}	105,734	88,534	293,113	268,179
N_{iC}	534,779	509,961	0	0
N_{fS}	835,176	652,131	1,832,442	1,605,891
N_{iS}	4,922,101	5,395,034	0	0
Std Dev log Wages fC	0.263	0.259	0.289	0.284
Std Dev log Wages iC	0.573	0.555	–	–
Aggregate Productivity C	33.237	34.033	42.918	43.862
Std Dev log Wages fS	0.160	0.169	0.172	0.184
Std Dev log Wages iS	0.523	0.587	–	–
Aggregate Productivity S	9.085	8.886	15.226	14.999
RealIncome p/c	123.760	118.567	175.577	168.218

Notes: Benchmark economy features the parameters in Tables 7 and 8. No Informality: firms face prohibitive costs of choosing informality, e.g., $\overline{c}_{ki} = \infty$. Otherwise, all other parameters as in Tables 7 and 8. Aggregate productivity is the employment-weighted average of the productivities z. Real Income per capita $=\frac{I/\overline{L}}{P}$, where $P = \frac{P_C^{\zeta} P_S^{1-\zeta}}{\zeta^{\zeta}(1-\zeta)^{(1-\zeta)}}$. "Ratio Exports Rev. fC" is the ratio between total exports and total revenues among formal C-sector firms. Firing cost κ as in Table 7.

Table 23: Iceberg Trade Costs and Labor Market Outcomes – Doubling the Minimum Wage

Variable	Benchmark w/ Min Wage = $\underline{w}$			Min Wage = $2\times\underline{w}$		
	$\tau_c = 1.5$	$\tau_c = 2$	$\tau_c = 2.5$	$\tau_c = 1.5$	$\tau_c = 2$	$\tau_c = 2.5$
μ_v	0.818	0.849	0.883	0.898	0.921	0.947
$d_{H,C}$	7.735	7.803	7.794	7.691	7.770	7.766
$d_{H,S}$	8.055	8.051	8.047	8.050	8.047	8.043
Exchange Rate ϵ	20.385	20.355	20.318	19.420	19.673	19.678
Share Emp. iC	0.035	0.031	0.045	0.035	0.042	0.048
Share Emp. fC	0.094	0.107	0.105	0.094	0.100	0.103
Share Emp. iS	0.374	0.364	0.355	0.388	0.382	0.369
Share Emp. fS	0.327	0.324	0.319	0.303	0.300	0.298
Share Unemp.	0.171	0.174	0.177	0.180	0.175	0.182
Share Informality	0.493	0.479	0.485	0.516	0.515	0.509
C-sector Sh. Exporters	0.319	0.111	0.070	0.312	0.151	0.076
Ratio Exports Rev. fC	0.338	0.347	0.343	0.360	0.350	0.351
Avg. Turnover fC	0.439	0.435	0.437	0.536	0.537	0.524
Avg. Turnover fS	0.700	0.307	0.131	0.702	0.320	0.134
N_{fC}	75,717	131,220	105,734	73,349	89,271	94,328
N_{iC}	425,309	487,433	534,779	412,011	501,449	529,852
N_{fS}	816,670	856,672	835,176	645,844	634,877	608,405
N_{iS}	5,047,933	5,044,783	4,922,101	5,315,226	5,225,236	5,424,880
Std Dev log Wages fC	0.302	0.282	0.263	0.283	0.263	0.249
Std Dev log Wages iC	0.573	0.561	0.573	0.556	0.574	0.559
Agg. Productivity C	34.544	35.034	33.237	35.974	34.866	34.047
Std Dev log Wages fS	0.159	0.161	0.160	0.141	0.140	0.140
Std Dev log Wages iS	0.540	0.529	0.523	0.529	0.529	0.581
Agg. Productivity S	9.077	9.012	9.085	8.983	9.015	9.125
Real Income p/c	132.433	125.837	123.760	130.211	124.997	122.882

Notes: Benchmark economy features the parameters in Tables 7 and 8. No Informality: firms face prohibitive costs of choosing informality, e.g., $\overline{c}_{ki} = \infty$. Otherwise, all other parameters as in Tables 7 and 8. Aggregate productivity is the employment-weighted average of the productivities z. Real Income per capita $= \frac{I/\overline{L}}{P}$, where $P = \frac{P_C^{\zeta} P_S^{1-\zeta}}{\zeta^{\zeta}(1-\zeta)^{(1-\zeta)}}$. "Ratio Exports Rev. fC" is the ratio between total exports and total revenues among formal C-sector firms. Minimum wage $\underline{w}$ as in Table 7.

informality. These results suggest that the informal sector worked as a buffer to trade-displaced workers. The hypothesis is that in the absence of a large informal sector, the long-run effect of liberalization on unemployment wold have been larger.

Our single-region equilibrium model is not well suited to directly address the question above. Tariffs and iceberg trade cost reductions do lead to more competition, but they also benefit exporters and lead to income effects, which mask and potentially offset competition effects. The parallel we can draw with Dix-Carneiro and Kovak (2019) is by simulating a negative economic shock in our model – which mimicks the pure competition effect captured in that paper. This section studies how the economy behaves in response to aggregate productivity shocks. We model an aggregate productivity shock as a shift of the entire distribution of productivities z in both sectors C and S. A productivity decline of x percent uniformly multiplies firm-level productivities z by $1 - \frac{x}{100}$, in all sectors.

Table 24 simulates **negative** shocks of 1 percent and 1.5 percent under two scenarios. One considers the benchmark economy, with informality, and the other considers the economy without informality. A negative aggregate productivity shock of 1.5 percent leads to an increase in unemployment of $100 \times \left(1 - \frac{0.187}{0.177}\right)$=5.6 percent in the benchmark economy with informality. However, that same shock leads to a larger increase in unemployment ($100 \times \left(1 - \frac{0.198}{0.184}\right)$=9.8 percent) in the economy without informality. The effect of the productivity shock on welfare is also larger in the economy *without* informality. In that scenario, welfare declines by $100 \times \left(1 - \frac{172.041}{175.577}\right)$=2 percent in response to an aggregate productivity shock of 1.5 percent, whereas welfare declines by only $100 \times \left(1 - \frac{122.701}{123.76}\right)$=0.86 percent in the scenario that allows for informality. These results suggest that abolishing informality leads to a significant amplification of the response of unemployment and welfare to productivity shocks. They also illustrate that the informal sector can indeed work as a buffer, as it can smooth the adverse consequences of competition and aggregate productivity shocks. On the other hand, our results suggest that although the effect of recessions on welfare and unemployment is amplified in an economy without an informal sector, abolishing the informal sector has a more substantial and dominant effect on welfare.

We turn to the investigation of how the economy responds to *positive* aggregate productivity shocks. Table 25 simulates 1 and 1.5 percent increase in aggregate productivity under the benchmark scenario and under the scenario without informality. Unemployment declines under both scenarios. When productivity is shifted up by 1.5 percent, unemployment in the benchmark economy decreases by 8 percent, whereas in the case without informality it decreases by 6.5 percent. It is interesting that (measured) ag-

Table 24: Negative Aggregate Productivity Shocks and Labor Market Outcomes

	Benchmark			No Informality		
Variable	Initial	1% $\downarrow$ in Prod.	1.5% $\downarrow$ in Prod.	Initial	1% $\downarrow$ in Prod.	1.5% $\downarrow$ in Prod.
μ_v	0.883	0.958	0.998	0.888	0.958	1.000
$d_{H,C}$	7.794	7.768	7.756	7.839	7.818	7.807
$d_{H,S}$	8.047	8.045	8.044	8.115	8.112	8.109
Exchange Rate ϵ	20.318	19.700	19.426	20.325	19.818	19.543
Share Emp. iC	0.045	0.048	0.047	0	0	0
Share Emp. fC	0.105	0.104	0.105	0.169	0.171	0.171
Share Emp. iS	0.355	0.338	0.332	0	0	0
Share Emp. fS	0.319	0.328	0.329	0.647	0.631	0.627
Share Unemp.	0.177	0.182	0.187	0.184	0.198	0.202
Share Informality	0.485	0.472	0.466	0	0	0
C-sector Sh. Exporters	0.070	0.076	0.076	0.032	0.033	0.033
Ratio Exports Rev. fC	0.131	0.134	0.134	0.101	0.103	0.104
Avg. Turnover fC	0.343	0.351	0.352	0.319	0.324	0.326
Avg. Turnover fS	0.437	0.447	0.446	0.482	0.515	0.515
N_{fC}	105,734	94,350	93,984	293,113	280,961	276,546
N_{iC}	534,779	529,976	527,920	0	0	0
N_{fS}	835,176	897,144	938,885	1,832,442	1,823,430	1,794,794
N_{iS}	4,922,101	5,209,685	5,140,064	0	0	0
Std Dev log Wages fC	0.263	0.251	0.248	0.289	0.281	0.276
Std Dev log Wages iC	0.573	0.558	0.551	–	–	–
Agg. Productivity C	33.237	33.869	34.267	42.918	43.578	44.091
Std Dev log Wages fS	0.160	0.158	0.157	0.172	0.167	0.165
Std Dev log Wages iS	0.523	0.587	0.581	–	–	–
Agg. Productivity S	9.085	9.185	9.218	15.226	15.589	15.585
Real Income p/c	123.760	122.986	122.701	175.577	173.391	172.041

Notes: Benchmark economy features the parameters in Tables 7 and 8. No Informality: firms face prohibitive costs of choosing informality, e.g., $\overline{c}_{ki} = \infty$. Otherwise, all other parameters as in Tables 7 and 8. Aggregate productivity is the employment-weighted average of the productivities z. Real Income per capita $= \frac{I/\overline{L}}{P}$, where $P = \frac{P_C^{\zeta} P_S^{1-\zeta}}{\zeta^{\zeta}(1-\zeta)^{(1-\zeta)}}$. "Ratio Exports Rev. fC" is the ratio between total exports and total revenues among formal C-sector firms. A productivity decline of $x\%$ uniformly multiplies firm-level productivities z by $1 - \frac{x}{100}$, in all sectors.

Table 25: Positive Aggregate Productivity Shocks and Labor Market Outcomes

	Benchmark			No Informality		
Variable	Initial	1% ↑ in Prod.	1.5% ↑ in Prod.	Initial	1% ↑ in Prod.	1.5% ↑ in Prod.
μ_v	0.883	0.782	0.756	0.888	0.826	0.796
$d_{H,C}$	7.794	7.833	7.843	7.839	7.860	7.871
$d_{H,S}$	8.047	8.053	8.053	8.115	8.119	8.122
Exchange Rate ϵ	20.318	21.203	21.479	20.325	20.816	21.087
Share Emp. iC	0.045	0.034	0.034	0	0	0
Share Emp. fC	0.105	0.111	0.110	0.169	0.169	0.170
Share Emp. iS	0.355	0.370	0.378	0	0	0
Share Emp. fS	0.319	0.318	0.316	0.647	0.654	0.658
Share Unemp.	0.177	0.167	0.163	0.184	0.177	0.172
Share Informality	0.485	0.484	0.491	0	0	0
C-sector Sh. Exporters	0.070	0.046	0.046	0.032	0.032	0.032
Ratio Exports Rev. fC	0.131	0.124	0.124	0.101	0.101	0.100
N_{fC}	105734	170435	171583	293113	301507	309219
N_{iC}	534779	531911	535493	0	0	0
N_{fS}	835176	808894	758560	1832442	1877845	1930132
N_{iS}	4922101	4996834	5104015	0	0	0
Std Dev log Wages fC	0.263	0.286	0.289	0.289	0.296	0.302
Std Dev log Wages iC	0.573	0.568	0.571	–	–	–
Agg. Productivity C	33.237	33.312	32.963	42.918	42.171	41.546
Std Dev log Wages fS	0.160	0.160	0.163	0.172	0.174	0.177
Std Dev log Wages iS	0.523	0.540	0.540	–	–	–
Agg. Productivity S	9.085	9.097	8.933	15.226	15.238	15.164
Real Income p/c	123.760	126.045	125.530	175.577	177.713	179.028

Notes: Benchmark economy features the parameters in Tables 7 and 8. No Informality: firms face prohibitive costs of choosing informality, e.g., $\bar{c}_{ki} = \infty$. Otherwise, all other parameters as in Tables 7 and 8. Aggregate productivity is the employment-weighted average of the productivities z. Real Income per capita $= \frac{I/\overline{L}}{P}$, where $P = \frac{P_C^{\zeta} P_S^{1-\zeta}}{\zeta^{\zeta}(1-\zeta)^{(1-\zeta)}}$. "Ratio Exports Rev. fC" is the ratio between total exports and total revenues among formal C-sector firms. A productivity increase of $x\%$ uniformly multiplies firm-level productivities z by $1 + \frac{x}{100}$, in all sectors.

gregate productivity declines if the *distribution* of firm-level productivities shifts right by 1.5 percent. This decline in *measured* aggregate productivity occurs in both scenarios, that is, with the benchmark economy and with the economy without informality. This is driven by a strong entry of less productive firms, pushing aggregate productivity down. If aggregate conditions improve with the shift in the distribution of productivities, the demand shifters $d_{H,C}$ and $d_{H,S}$ also increase, allowing less productive firms to enter the market. Although unemployment declines (in both scenarios), informality slightly increases under the benchmark economy. Real income per capita increases in both scenarios: by 1.4 percent in the benchmark economy and by 1.9 percent in the economy without informality.

6.5 Can Lax Labor Market Regulations Reduce Informality?

So far, our results suggest that abolishing informality can have substantial effects on (measured) aggregate productivity and welfare. We now ask the question of how much of a reduction in informality we can achieve by relaxing labor market regulations such as the minimum wage and firing costs. We also conduct experiments reducing the fixed costs of operation in the formal sector, under the assumption that part of these costs (if not all) are driven by institutions leading to a cost of doing business.

Table 26 simulates a removal of the minimum wage. The effect of this removal is imperceptible. Although the minimum wage is binding in equilibrium, if minimum wages are removed, the smallest wage paid by formal firms is not too far from the institutional minimum wage. This conclusion should be taken with a pinch of salt: (1) our estimation does not target the fraction of workers who earn a minimum wage; and (2) our model does not allow for heterogeneous workers. In an economy with unskilled and less productive workers, a larger fraction of firms would bunch at the minimum wage, so that its removal could have a much larger impact on the labor market.

Next, Table 26 gradually reduces firing costs. Perhaps counterintuitively, a full removal of firing costs leads to an increase in informality to 51.6 percent (from 48.5 percent), a decline in aggregate productivity and a decline in real income per capita. Note that the decline in firing costs sharply increases the masses of formal firms in both sectors, but it also strongly increases the masses of informal-sector firms. Firing costs make formal sector operation more attractive, leading some informal firms to formalize, but they also make entry in the informal sector more attractive, as some of these firms can eventually become informal. In practice, this reduction in firing costs leads to a strong entry of less productive firms, driving aggregate productivity down. Although unemployment is reduced by this policy, the aggregate productivity effect dominates, leading to a decline in aggregate welfare.

Table 26: Labor Market Regulations and Informality

Variable	Benchmark	No Min. Wage	Firing Costs $\kappa' = \frac{3}{4}\kappa$	$\kappa' = \frac{1}{2}\kappa$	$\kappa' = \frac{1}{4}\kappa$	$\kappa' = 0$
μ_v	0.883	0.883	0.779	0.72	0.663	0.617
$d_{H,C}$	7.794	7.794	7.84	7.873	7.909	7.943
$d_{H,S}$	8.047	8.047	8.053	8.054	8.055	8.056
Exchange Rate ϵ	20.318	20.318	21.379	22.257	23.244	24.219
Share Emp. iC	0.045	0.045	0.034	0.034	0.035	0.035
Share Emp. fC	0.105	0.105	0.111	0.107	0.104	0.101
Share Emp. iS	0.355	0.355	0.367	0.382	0.394	0.403
Share Emp. fS	0.319	0.319	0.32	0.314	0.31	0.31
Share Unemp.	0.177	0.177	0.169	0.162	0.157	0.151
Share Informality	0.485	0.485	0.482	0.497	0.508	0.516
C-sector Sh. Exporters	0.07	0.07	0.046	0.046	0.046	0.047
Ratio Exports Rev. fC	0.131	0.131	0.124	0.125	0.126	0.127
Avg. Turnover fC	0.343	0.343	0.28	0.278	0.268	0.263
Avg. Turnover fS	0.437	0.437	0.454	0.449	0.446	0.447
N_{fC}	105,734	105,734	170,862	173,452	175,700	177,008
N_{iC}	534,779	534,779	533,242	541,324	548,340	552,423
N_{fS}	835,176	835,176	861,562	897,930	925,244	937,150
N_{iS}	4,922,101	4,922,102	4,954,824	5,163,975	5,321,060	5,389,533
Std Dev log Wages fC	0.263	0.263	0.285	0.293	0.302	0.311
Std Dev log Wages iC	0.573	0.573	0.568	0.575	0.582	0.588
Agg. Productivity C	33.237	33.237	32.982	31.812	30.513	29.524
Std Dev log Wages fS	0.16	0.16	0.156	0.161	0.163	0.164
Std Dev log Wages iS	0.523	0.523	0.54	0.539	0.54	0.545
Agg. Productivity S	9.085	9.085	9.122	8.831	8.668	8.53
Real Income p/c	123.76	123.76	125.549	124.235	123.201	122.105

Notes: Benchmark economy features the parameters in Tables 7 and 8. Aggregate productivity is the employment-weighted average of the productivities z. Real Income per capita $=\frac{I/\bar{L}}{P}$, where $P = \frac{P_C^{\varsigma} P_S^{1-\varsigma}}{\varsigma^{\varsigma}(1-\varsigma)^{(1-\varsigma)}}$. "Ratio Exports Rev. fC" is the ratio between total exports and total revenues among formal C-sector firms. κ is given in Table 7.

If at least part of the fixed costs of production $\bar{c}_{kf}$ are driven by bureaucracy and the cost of doing business in the country, then an institutional reform targeting a reduction in red tape can significantly mitigate these costs. Table 27 simulates two scenarios: (1) fixed costs of production $\bar{c}_{kf}$ in Table 8 are reduced by half; (2) fixed costs of production $\bar{c}_{kf}$ are reduced to match those in the informal sector ($\bar{c}'_{kf} = \bar{c}_k$. Table 27 tells a similar story to the one told by Table 26. A reduction in fixed costs of production leads to increased entry, lower unemployment, but higher informality rates. They also lead to a decline in aggregate productivity and welfare.

Table 27: Fixed Costs of Operation in the Formal Sector and Informality

Variable	Benchmark	$\bar{c}'_{kf} = 0.5 \times \bar{c}_{kf}$	$\bar{c}'_{kf} = \bar{c}_{ki}$
μ_v	0.883	0.705	0.658
$d_{H,C}$	7.794	7.871	7.884
$d_{H,S}$	8.047	8.051	8.051
Exchange Rate ϵ	20.318	22.294	22.752
Share Emp. iC	0.045	0.035	0.029
Share Emp. fC	0.105	0.108	0.112
Share Emp. iS	0.355	0.397	0.415
Share Emp. fS	0.319	0.304	0.293
Share Unemp.	0.177	0.157	0.151
Share Informality	0.485	0.512	0.523
C-sector Sh. Exporters	0.07	0.045	0.031
Ratio Exports Rev. fC	0.131	0.124	0.119
Avg. Turnover fC	0.343	0.268	0.213
Avg. Turnover fS	0.437	0.412	0.396
N_{fC}	105,734	176,632	264,681
N_{iC}	534,779	551,250	455,746
N_{fS}	835,176	971,490	1,018,429
N_{iS}	4,922,101	5,426,290	5,688,447
Std Dev log Wages fC	0.263	0.289	0.292
Std Dev log Wages iC	0.573	0.574	0.514
Agg. Productivity C	33.237	31.099	30.168
Std Dev log Wages fS	0.16	0.158	0.16
Std Dev log Wages iS	0.523	0.527	0.526
Agg. Productivity S	9.085	8.41	8.06
Real Income p/c	123.76	123.741	122.99

Notes: Benchmark economy features the parameters in Tables 7 and 8. Aggregate productivity is the employment-weighted average of the productivities z. Real Income per capita $= \frac{I/\bar{L}}{P}$, where $P = \frac{P_C^{\zeta} P_S^{1-\zeta}}{\zeta^{\zeta}(1-\zeta)^{(1-\zeta)}}$. "Ratio Exports Rev. fC" is the ratio between total exports and total revenues among formal C-sector firms. $\bar{c}_{kf}$ and $\bar{c}_{ki}$ are given in Table 7.

7 Main Takeaways

The main conclusions of the counterfactual experiments we have conducted are as follows:

- Under balanced trade, import tariff movements have negligible effects on welfare, unemployment and informality.

- These effects are amplified if the exchange rate is fixed and deficits are modeled as income transfers from the rest of the world.

- The effects of iceberg trade costs on welfare, unemployment and informality are larger than those from import tariffs. *Reasonable* reductions in trade costs have substantial effects reducing informality (and raising productivity) within the tradable sector, consistent with McCaig and Pavcnik (2018). However, these reductions also lead to a relative increase in the non-tradable sector, an informality-intensive sector. The net effect of lowering trade costs on *overall* informality is small.

- Overall, the (relative) effects of trade liberalization on welfare and labor market outcomes are not very different in an economy with a large informal sector compared to an economy without an informal sector.

- **Most interestingly**, eradicating informality has very strong positive effects on welfare, much larger than (an unrealistic) reduction in iceberg trade costs of over 50 percent.

- Consistent with Dix-Carneiro and Kovak (2019), the informal sector works as a buffer when the economy is hit with negative shocks. The effects of a negative aggregate productivity shock on unemployment and welfare are larger in an economy without an informal sector. However, the welfare gains from eradicating informality are so large that it is hard to justify lenience toward the informal sector on the basis that it works as a buffer in bad economic times.

- A positive productivity shock does not lead to less informality. Our results actually suggest otherwise, as technological progress increases aggregate incomes, leading to a reduction in the productivity threshold of entry in the informal sector.

- Increasing trade openness, reducing the burden of labor market regulations and growth are not enough (by themselves) to offset the misallocation caused by taxes and a large informal sector. Therefore, it seems that increasing enforcement of taxes and regulations is the most effective way to reduce informality. How governments should implement this increase in enforcement is an exciting topic for future research.

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Appendix

A Model Appendix

A.1 Equilibrium Conditions

A.1.1 Steady-state distribution of states

Let $\tilde{\tilde{\psi}}_{ki}(z',\ell)$ be the **mass** distribution of informal firms across the (z',ℓ) spectrum.

$$\tilde{\tilde{\psi}}_{ki}(z',\ell) = \mathbf{1}[\ell=1] \times M_{ki} \times \psi^e_{ki}(z') + \mathbf{1}[\ell \geq 1] \times (1-\alpha_{ki}) N_{ki} \times \left(\int_z \psi_{ki}(z,\ell) I^{stay}_k(z,\ell,i)\, g_k(z'|z) dz \right) \tag{A.1}$$

where $\psi^e_{ki}(z') = \frac{\int_\nu I^{informal}_k(\nu) g_k(z'|\nu) g^e_k(\nu) d\nu}{\int_{z'}\int_\nu I^{informal}_k(\nu) g_k(z'|\nu) g^e_k(\nu) d\nu dz'}$ is the density of z' among entrants in the k informal sector; M_{ki} is the mass of entrants into the k informal sector; N_{ki} is the mass of sector k informal incumbents who started the period and $g^e_k(.)$ is the ergodic distribution of z. M_{ki} and N_{ki} are also equilibrium objects.

For expositional purposes, it is useful to define an interim period, which corresponds to the intra-period adjustment window, when the firm has already drawn its new productivity, z', but has not yet adjusted its labor force. In steady state, the total mass of firms is equal to N_{ki} in the beginning of the period as well as in the interim stage and end of period, so the integral of $\tilde{\tilde{\psi}}_{ki}$ sums to N_{ki}. The interim distribution is given by:

$$\tilde{\psi}_{ki}(z',\ell) = \frac{\tilde{\tilde{\psi}}_{ki}(z',\ell)}{\int_{z'}\int_\ell \tilde{\tilde{\psi}}_{ki}(z',\ell)\, dz' d\ell} = \frac{\tilde{\tilde{\psi}}_{ki}(z',\ell)}{N_{ki}} \tag{A.2}$$

$$= \mathbf{1}[\ell=1] \times \frac{M_{ki}}{N_{ki}} \times \psi^e_{ki}(z') + \mathbf{1}[\ell \geq 1] \times (1-\alpha_{ki}) \times \left(\int_z \psi_{ki}(z,\ell) I^{stay}_k(z,\ell,i)\, g_k(z'|z) dz \right)$$

The end-of-period distribution reproduces the start-of-period distribution:

$$\psi_{ki}(z',\ell') = \frac{\int_\ell \tilde{\psi}_{ki}(z',\ell) I(L_k(z',\ell,i)=\ell') d\ell}{\int_{z'}\int_\ell \tilde{\psi}_{ki}(z',\ell) I(L_k(z',\ell,i)=\ell') d\ell dz'} \tag{A.3}$$

$$= \int_\ell \tilde{\psi}_{ki}(z',\ell) I(L_k(z',\ell,i)=\ell') d\ell$$

where the last equality follows from $\int_{z'}\int_\ell \tilde{\psi}_{ki}(z',\ell) I(L_k(z',\ell,i)=\ell') d\ell dz' = 1$.

When considering the distribution of states across formal firms, we need to take into account that some informal firms change their status to formal. Let $\tilde{\tilde{\psi}}_{kf}(z',\ell)$ be the **mass** distribution of formal firms across the (z',ℓ) spectrum.

$$\tilde{\tilde{\psi}}_{kf}(z',\ell) = \begin{array}{c} \mathbf{1}[\ell=1] \times M_{kf} \times \psi^e_{kf}(z') + \\ \mathbf{1}[\ell \geq 1] \times \left(\begin{array}{c} (1-\alpha_{kf}) N_{kf} \times \left(\int_z \psi_{kf}(z,\ell) I^{stay}_k(z,\ell,f) g_k(z'|z) dz \right) + \\ (1-\alpha_{ki}) N_{ki} \times \left(\int_z \psi_{ki}(z,\ell) I^{change}_k(z,\ell,i) g_k(z'|z) dz \right) \end{array} \right) \end{array} \tag{A.4}$$

Where $\psi^e_{kf}(z') = \frac{\int_\nu I^{formal}_k(\nu) g_k(z'|\nu) g^e_k(\nu) d\nu}{\int_{z'} \int_\nu I^{formal}_k(\nu) g_k(z'|\nu) g^e_k(\nu) d\nu dz'}$ is the density of z' among entrants in the k formal sector; M_{kf} is the mass of entrants into the k formal sector; and N_{ki} is the mass of incumbents who started the period. M_{kf} and N_{kf} are also equilibrium objects. In steady state, the integral of $\tilde{\tilde{\psi}}_{kf}$ sums to N_{kf}. The interim distribution is given by:

$$\tilde{\psi}_{kf}(z',\ell) = \frac{\tilde{\tilde{\psi}}_{kf}(z',\ell)}{\int_{z'} \int_\ell \tilde{\tilde{\psi}}_{kf}(z',\ell) dz' d\ell} = \frac{\tilde{\tilde{\psi}}_{kf}(z',\ell)}{N_{kf}}$$

$$\tilde{\psi}_{kf}(z',\ell) = \begin{array}{c} \mathbf{1}[\ell=1] \times \frac{M_{kf}}{N_{kf}} \times \psi^e_{kf}(z') + \\ \mathbf{1}[\ell \geq 1] \times \left(\begin{array}{c} (1-\alpha_{kf}) \times \left(\int_z \psi_{kf}(z,\ell) I^{stay}_k(z,\ell,f) g_k(z'|z) dz \right) + \\ (1-\alpha_{ki}) \frac{N_{ki}}{N_{kf}} \times \left(\int_z \psi_{ki}(z,\ell) I^{change}_k(z,\ell,i) g_k(z'|z) dz \right) \end{array} \right) \end{array} \tag{A.5}$$

The end-of-period distribution reproduces the start-of-period distribution:

$$\psi_{kf}(z',\ell') = \frac{\int_\ell \tilde{\psi}_{kf}(z',\ell) I(L_k(z',\ell,f)=\ell') d\ell}{\int_{z'} \int_\ell \tilde{\psi}_{kf}(z',\ell) I(L_k(z',\ell,f)=\ell') d\ell dz'} \tag{A.6}$$

$$= \int_\ell \tilde{\psi}_{kf}(z',\ell) I(L_k(z',\ell,f)=\ell') d\ell,$$

where the last equatlity follows from $\int_{z'} \int_\ell \tilde{\psi}_{kf}(z',\ell) I(L_k(z',\ell,f)=\ell') d\ell dz' = 1$.

A.1.2 Entry

Let M_k denote the mass of entrants in sector $k = C, S$. The fraction of entrants into the formal and informal sectors are given respectively by ω_{kf} and ω_{ki}:

$$\omega_{kf} \equiv \Pr\left(I^{formal}_k(\nu) = 1 \right) = \int g^e_k(\nu) d\nu \tag{A.7}$$

$$\omega_{ki} \equiv \Pr\left(I^{informal}_k(\nu) = 1 \right) = \int g^e_k(\nu) d\nu \tag{A.8}$$

Therefore, the masses of entrants in the formal and informal sectors are given by:

$$M_{ki} = \omega_{ki} M_k \tag{A.9}$$
$$M_{kf} = \omega_{kf} M_k \tag{A.10}$$

The masses of entrants into each sector, M_k, are pinned down by the free entry condition (assuming positive entry in both sectors):

$$c_{e,k} = V_k^e = \int \left[V_k^e(\nu, i) I_k^{informal}(\nu) + V_k^e(\nu, f) I_k^{formal}(\nu) \right] g_k^e(\nu) d\nu \tag{A.11}$$

A.1.3 Flow conditions for workers and firms

In order to write the labor market clearing conditions, we first define the following quantities:

- Number of workers in the **beginning** of the period in sector k, working in formal or informal firms (T stands for "total"):

$$W_{kj}^T = N_{kj} \underbrace{\int_z \int_\ell \ell \psi_{kj}(z, \ell) d\ell dz}_{\text{avg. \# of workers per firm}} = L_{kj} \tag{A.12}$$

  for $j = f, i$ and $k = C, S$.

- Number of workers in sector (k, j) who are fired because their firms receive a death shock:

$$W_{kj}^{DS} = \alpha_{kj} N_{kj} \int_z \int_\ell \ell \psi_{kj}(z, \ell) d\ell dz = \alpha_{kj} L_{kj} \tag{A.13}$$

- Number of workers in sector (k, j) who are fired due to **endogenous** firm exit:

$$W_{kj}^{EE} = (1 - \alpha_{kj}) N_{kj} \times \int_z \int_\ell \ell \psi_{kj}(z, \ell) I_k^{exit}(z, \ell, j) d\ell dz \tag{A.14}$$

  where $(1 - \alpha_{kj}) N_{kj}$ is the mass of firms that survive after the death shock hits.

- Number (mass) of **surviving incumbent** firms in sector (k, j) in the interim period:

$$N'_{kj} = (1 - \alpha_{kj}) N_{kj} \int_z \int_\ell \psi_{kj}(z, \ell) I_k^{stay}(z, \ell, j) d\ell dz \tag{A.15}$$

- Number of workers initially in sector (k, j) who are fired due to downsizing at the interim stage:

$$W_{kj}^D = N'_{kj} \int_{z'} \int_\ell \widetilde{\psi}_{kj}^{incumbent}(z', \ell) \left(1 - I_k^{hire}(z', \ell, j)\right) (\ell - L_k(z', \ell, j)) dz' d\ell \tag{A.16}$$

where $\widetilde{\psi}_{kj}^{incumbent}(z',\ell)$ is the distribution of states in the interim stage among **surviving incumbents**. Note that this is not the same distribution as $\widetilde{\psi}_{kj}(z',\ell)$ as it does not include entrants. It is obtained as follows:

$$\begin{aligned}\widetilde{\psi}_{kj}^{incumbent}(z',\ell) &= \frac{(1-\alpha_{kj})N_{kj}}{N'_{kj}}\int_z \psi_{kj}(z,\ell)\, I_k^{stay}(z,\ell,j)\, g_k(z'|z)\,dz \\ &= \frac{\int_z \psi_{kj}(z,\ell)\, I_k^{stay}(z,\ell,j)\, g_k(z'|z)\,dz}{\int_z\int_\ell \psi_{kj}(z,\ell)\, I_k^{stay}(z,\ell,j)\,d\ell dz}\end{aligned} \tag{A.17}$$

- Total fraction of workers in the formal sector of sector k who are laid off, conditional on starting the period in a formal firm in sector k :

$$\begin{aligned}\chi_{kf}^{layoff} &= \frac{W_{kf}^{DS}+W_{kf}^{EE}+W_{kf}^{D}}{W_{kf}^{T}} \\ &= \alpha_{kf} + \frac{\left(\begin{array}{c}(1-\alpha_{kf})\int_z\int_\ell \ell\psi_{kf}(z,\ell)\, I_k^{exit}(z,\ell,f)\,d\ell dz \\ (1-\alpha_{kf})\left(\int_z\int_\ell \psi_{kf}(z,\ell)\, I_k^{stay}(z,\ell,f)\,d\ell dz\right)\times \\ \int_{z'}\int_\ell \widetilde{\psi}_{kf}^{incumbent}(z',\ell)\left(1-I_k^{hire}(z',\ell,f)\right)\left(\ell - L_k(z',\ell,f)\right)dz'd\ell\end{array}\right)}{\int_z\int_\ell \ell\psi_{kf}(z,\ell)\,d\ell d}\end{aligned} \tag{A.18}$$

- Number of firms that start the period as informal firms, but end the period as formal firms (because they formalized).

$$N'_{ki\to f} = (1-\alpha_{ki})N_{ki}\int_z\int_\ell \psi_{ki}(z,\ell)\, I_k^{change}(z,\ell,i)\,d\ell dz \tag{A.19}$$

where $(1-\alpha_{ki})N_{ki}$ is the mass of firms that survive after the death shock hits.

- Distribution of states among firms that switched from informal to formal, in the interim period

$$\begin{aligned}\widetilde{\psi}_{ki\to f}(z',\ell) &= \frac{(1-\alpha_{ki})N_{ki}}{N'_{ki\to f}}\int_z \psi_{ki}(z,\ell)\, I_k^{change}(z,\ell,i)\, g_k(z'|z)\,dz \\ &= \frac{\int_z \psi_{ki}(z,\ell)\, I_k^{change}(z,\ell,i)\, g_k(z'|z)\,dz}{\int_z\int_\ell \psi_{ki}(z,\ell)\, I_k^{change}(z,\ell,i)\,d\ell dz}\end{aligned} \tag{A.20}$$

- Number of workers who started the period in informal firms, but end the period in formal firms (their employers switched to formal, and they were not fired after the interim productivity was realized):

$$W_{k,i\to f} = N'_{ki\to f}\int_{z'}\int_\ell \widetilde{\psi}_{ki\to f}(z',\ell)\left(\begin{array}{c}\ell\times I^{hire}(z',\ell,f)+ \\ L_k(z',\ell,f)\times\left(1-I^{hire}(z',\ell,f)\right)\end{array}\right)d\ell dz' \tag{A.21}$$

- Fraction of workers who start the period in informal firms, but end the period in formal firms:

$$\chi_{ki\to f}^{change} = \frac{W_{k,i\to f}}{W_{ki}^{T}} = \frac{\begin{pmatrix} (1-\alpha_{ki})\left(\int_z\int_\ell \psi_{ki}(z,\ell)\, I_k^{change}(z,\ell,i)\, d\ell dz\right)\times \\ \int_{z'}\int_\ell \widetilde{\psi}_{ki\to f}(z',\ell)\begin{pmatrix} \ell\times I^{hire}(z',\ell,f)+ \\ L_k(z',\ell,f)\times\left(1-I^{hire}(z',\ell,f)\right)\end{pmatrix} d\ell dz' \end{pmatrix}}{\int_z\int_\ell \ell\psi_{ki}(z,\ell)\, d\ell dz} \tag{A.22}$$

- Number of workers who start the period in informal firms, but their employers switched to formal status:

$$W_{ki}^{SF} = (1-\alpha_{ki})\, N_{ki}\int_z\int_\ell \ell\psi_{ki}(z,\ell)\, I_k^{change}(z,\ell,i)\, d\ell dz \tag{A.23}$$

- Fraction of workers who start employed in the informal sector and leave it in the interim period (became unemployed or employer switched to formal):

$$\chi_{ki}^{leave} = \frac{W_{ki}^{DS}+W_{ki}^{EE}+W_{ki}^{SF}+W_{ki}^{D}}{W_{ki}^{T}} = \alpha_{ki} + \frac{\begin{pmatrix} (1-\alpha_{ki})\int_z\int_\ell \ell\psi_{ki}(z,\ell)\, I_k^{exit}(z,\ell,i)\, d\ell dz+ \\ (1-\alpha_{ki})\int_z\int_\ell \ell\psi_{ki}(z,\ell)\, I_k^{change}(z,\ell,i)\, d\ell dz+ \\ (1-\alpha_{ki})\left(\int_z\int_\ell \psi_{ki}(z,\ell)\, I_k^{stay}(z,\ell,i)\, d\ell dz\right)\times \\ \int_{z'}\int_\ell \widetilde{\psi}_{ki}^{incumbent}(z',\ell)\left(1-I_k^{hire}(z',\ell,i)\right)\left(\ell-L_k(z',\ell,i)\right) dz' d\ell \end{pmatrix}}{\int_z\int_\ell \ell\psi_{ki}(z,\ell)\, d\ell dz} \tag{A.24}$$

With these objects, we can define the equilibrium conditions that refer to labor market flows:

$$\chi_{ki}^{leave} L_{ki} = L_u \mu_{ki}^{e} \tag{A.25}$$

$$\chi_{kf}^{layoff} L_{kf} = L_u \mu_{kf}^{e} + L_{ki}\chi_{ki\to f}^{change}. \tag{A.26}$$

These conditions state that the mass of workers in each sector (k, j) cannot be contracting or expanding in equilibrium (expressions (A.25) and (A.26)). Finally, the sum of unemployment and employment levels across sectors equals the total labor force $\overline{L}$:

$$L_{Cf} + L_{Ci} + L_{Sf} + L_{Si} + L_u = \overline{L}. \tag{A.27}$$

We can proceed in a similar way to define the equilibrium flow conditions for firms. The relevant objects are the following:

- Fraction of formal firms exiting sector k:

$$\varrho_{kf}^{exit} = \alpha_{kf} + (1 - \alpha_{kf}) \int_z \int_\ell I_k^{exit}(z, \ell, f)\, \psi_{kf}(z, \ell)\, d\ell dz \tag{A.28}$$

- Fraction of informal firms exiting sector k:

$$\varrho_{ki}^{exit} = \alpha_{ki} + (1 - \alpha_{ki}) \int_z \int_\ell \Big(I_k^{exit}(z, \ell, i) + I_k^{change}(z, \ell, i) \Big)\, \psi_{ki}(z, \ell)\, d\ell dz \tag{A.29}$$

- Fraction of informal firms changing status in sector k:

$$\varrho_{ki}^{change} = (1 - \alpha_{ki}) \int_z \int_\ell I_k^{change}(z, \ell, i)\, \psi_{ki}(z, \ell)\, d\ell dz \tag{A.30}$$

Similarly to workers, the mass of firms in each sector (k, j) must be constant in steady state. This means that the inflow of firms must equal the outflow, which can be written as:

$$\varrho_{kf}^{exit} N_{kf} = M_{kf} + \varrho_{ki}^{change} N_{ki} \tag{A.31}$$

$$\varrho_{ki}^{exit} N_{ki} = M_{ki}. \tag{A.32}$$

A.1.4 Vacancies

Aggregate vacancies in sector kj are given by:

$$V_{kj} = N_{kj} \int_{z'} \int_\ell v_{kj}(z', \ell)\, \widetilde{\psi}_{kj}(z', \ell)\, d\ell dz' + \frac{M_{kj}}{\mu_{kj}^v}, \tag{A.33}$$

where $v_{kj}(z', \ell)$ is the number of vacancies a firm with shock z' and labor force ℓ posts.

A.1.5 Unemployment Benefits / Tax Collection / Transfers

- We assume that all government revenue G (taxes and firing costs) that are not spent in unemployment benefits – T – are rebated to consumers.

$$G - \underbrace{\left(b^u \times \underbrace{\sum_k \left(W_{kf}^{DS} + W_{kf}^{EE} + W_{kf}^{D} \right)}_{\text{mass of } \textbf{formal} \text{ workers who transition to unemployment}} \right)}_{\text{Total Expenditure with Unemployment Benefits}} = T \tag{A.34}$$

$$
\begin{aligned}
G = & \sum_k N_{kf}\tau_y \int_z \int_\ell R_k(z,\ell)\,\psi_{kf}(z,\ell)\,d\ell dz \\
& + \sum_k N_{kf}\tau_w \int_z \int_\ell \max\left\{w_{kf}(z,\ell), \underline{w}\right\} \ell\psi_{kf}(z,\ell)\,d\ell dz \\
& + \sum_k N_{ki} \int_z \int_\ell p_{ki}(\ell)\, R_k(z,\ell)\,\psi_{ki}(z,\ell)\,d\ell dz \\
& + \sum_k N_{kf}\kappa \int_{z'} \int_\ell \widetilde{\psi}_{kf}(z',\ell)\left(\ell - L_k(z',\ell,f)\right)\left(1 - I^{hire}(z',\ell,f)\right) d\ell dz' \\
& + (\tau_a - 1)\,\frac{D_{H,C}\left(\epsilon\tau_a\tau_c\right)^{1-\sigma}}{\tau_a}
\end{aligned} \tag{A.35}
$$

A.1.6 Aggregate Income

Aggregate income is given by total wages, government transfers and total profits:

$$
\begin{aligned}
I = & \sum_k N_{ki} \int_z \int_\ell w_{ki}(z,\ell)\,\ell\psi_{ki}(z,\ell)\,d\ell dz \\
& + \sum_k N_{kf} \int_z \int_\ell \max\left\{w_{kf}(z,\ell), \underline{w}\right\} \ell\psi_{kf}(z,\ell)\,d\ell dz \\
& + G \\
& + \sum_k N_{ki} \int_z \int_\ell \widetilde{\pi}_{ki}(z,\ell)\,\psi_{ki}(z,\ell)\,d\ell dz \\
& + \sum_k N_{kf} \int_z \int_\ell \widetilde{\pi}_{kf}(z,\ell)\,\psi_{kf}(z,\ell)\,d\ell dz \\
& - \sum_k N_{kf}\kappa \int_{z'} \int_\ell \widetilde{\psi}_{kf}(z',\ell)\left(\ell - L_k(z',\ell,f)\right)\left(1 - I^{hire}(z',\ell,f)\right) d\ell dz' \\
& - \sum_{k=C,S\,;\,j=i,f} \left(N_{kj}\overline{H}_{kj} + M_{kj}K_{kj}\right) - \sum_{k=C,S} \varrho_{ki}^{change} N_{ki}\left(K_{kf} - K_{ki}\right) \\
& - \sum_{k=C,S} M_k c_{e,k},
\end{aligned} \tag{A.36}
$$

where profits $\widetilde{\pi}$ are computed before subtracting hiring costs. For the equations above, note that the number of firms in sector (k,j) in the interim stage is given by $N'_{ki} + M_{ki} = N_{ki}$ and $N'_{kf} + M_{kf} + \varrho_{ki}^{change} N_{ki} = N_{kf}$ (in steady state).

A.1.7 Service Sector Market Clearing

Service sector goods are used for final consumption (consumers spend $(1-\zeta)\,I$ on it), and as inputs for hiring costs, fixed costs and entry costs (and fixed costs of exporting).

The average hiring costs in sector (k, j):

$$\overline{H}_{kj} = \int_{z'} \int_{\ell} H_{kj} \left(\ell, L_k \left(z', \ell, j\right)\right) I^{hire} \left(z', \ell, j\right) \widetilde{\psi}_{kj} \left(z', \ell\right) d\ell dz', \tag{A.37}$$

and the fraction of tradable-sector goods firms that export is given by

$$\mu_x = \int_z \int_{\ell} \psi \left(z, \ell\right) I^x \left(z, \ell\right) d\ell dz \tag{A.38}$$

Therefore, we can write the total expenditure on service sector goods as follows:

$$X_S = \left(1 - \zeta\right) I + \sum_{k=C,S\ ;\ j=i,f} \left(N_{kj} \left(\overline{H}_{kj} + \bar{c}_{kj}\right) + M_{kj} K_{kj}\right) + \sum_{k=C,S} \varrho_{ki}^{change} N_{ki} \left(K_{kf} - K_{ki}\right) + N_{Cf} \mu_x f_x + \sum_{k=C,S} M_k c_{e,k} \tag{A.39}$$

A.1.8 Trade Balance

Trade balance implies that total imports must equal total exports, which is given by:

$$\frac{D_{H,C} \left(\epsilon \tau_a \tau_c\right)^{1-\sigma}}{\tau_a} = Exports \tag{A.40}$$

B Estimation Appendix

B.1 Estimation Algorithm

In this section we describe the estimation algorithm in detail, which we break down into several steps for expositional clarity.

$d_{H,C}$, $d_{H,S}$, and μ^v_{Cf}, μ^v_{Ci}, μ^v_{Sf}, and μ^v_{Si} are treated as parameters to be estimated along with the remaining ones, but these are endogenous variables. The procedure makes sure that the value guessed for $d_{H,C}$ is the outcome of the equilibrium (see Step 7 for details). Deviations from remaining equilibrium conditions will be penalized, forcing the outcome of the optimization algorithm to choose values of $d_{H,S}$, and μ^v_{Cf}, μ^v_{Ci}, μ^v_{Sf}, and μ^v_{Si} consistent with the equilibrium. We will treat μ^v_{Cf}, μ^v_{Ci}, μ^v_{Sf}, and μ^v_{Si} as separate parameters to be estimated, but they are all tied by $\mu^v_{kj} = \xi_{kj}\mu^v$. This approach guarantees that we will never end up with a probability of filling a vacancy μ^v_{kj} that is larger than 1.

Step 1: Start with a parameter guess Θ, including values for $d_{H,C}$, $d_{H,S}$, μ^v_{Cf}, μ^v_{Ci}, μ^v_{Sf} and μ^v_{Si}, with $0 \leq \mu^v_{kj} \leq 1$. Obtain d_F using

$$\frac{R^{exports}(z,\ell)}{R(z,\ell)} = (1 - \exp(-\sigma d_F))$$

$$\Rightarrow d_F = \frac{1}{\sigma_C} \log\left(1 - E\left(\frac{R^{exports}}{R}\right)_{Data}\right)$$

$E\left(\frac{R^{exports}}{R}\right)_{Data}$ is the average fraction of revenues coming from exports in the data, at the firm level. In other words, we match that moment exactly.

Step 2: Given that $\xi_{Cf} = 1$, set

$$\mu^v = \mu^v_{Cf},$$

and recover the matching function parameter

$$\xi_{kj} = \frac{\mu^v_{kj}}{\mu^v}$$

for $kj \in \{Ci, Sf, Si\}$.

Step 3: Compute revenue functions $R_k(z,\ell)$, and compute wage schedules.

$$w_{kf}(z,\ell) = \frac{(1-\beta_f)(b+b^u)}{1+\beta_f\tau_w} + \frac{\beta_f(1-\tau_y)}{1+\beta_f\tau_w}\frac{R_k(z,\ell)}{\ell} - \frac{\beta_f}{1+\beta_f\tau_w}\frac{\overline{c}_{kf}}{\ell}$$

$$w_{kf}\left(z,\ell;\underline{w}_f\right) = \max\left\{w_{kf}(z,\ell), \underline{w}_f\right\}$$

$$w_{ki}(z,\ell) = (1-\beta_i)\,b + \beta_i(1-p_{ki}(\ell))\frac{R_k(z,\ell)}{\ell} - \beta_i\frac{\overline{c}_{ki}}{\ell}$$

$$w_{ki}(z,\ell;\underline{w}_i) = \max\{w_{ki}(z,\ell), \underline{w}_i\}$$

Where $\underline{w}_f$ is the minimum wage in the formal sector (which is observed and fixed throughout estimation). $\underline{w}_i$ is the first percentile of the distribution of informal wages in PME and fixed throughout the estimation procedure. This is to avoid zero or negative informal wages.

Step 4: Compute firms' value functions. Obtain firms' policy functions. Solve for firms' entry decisions. Compute the fraction of entrants in the formal and informal sectors as follows:

$$\omega_{kf} \equiv \Pr\left(I_k^{formal}(\nu) = 1\right) = \int I_k^{formal}(\nu)\, g_k^e(\nu)\, d\nu$$
$$\omega_{ki} \equiv \Pr\left(I_k^{informal}(\nu) = 1\right) = \int I_k^{informal}(\nu)\, g_k^e(\nu)\, d\nu$$

Therefore, if M_k is the mass of entrants in sector k, the masses of formal and informal entrants in sector k are given by:

$$M_{ki} = \omega_{ki} M_k$$
$$M_{kf} = \omega_{kf} M_k$$

Step 5: Obtain the entry costs $c_{e,k}$ $(k = C, S)$:

$$c_{e,k} = V_k^e = \int \left[\left(V_k^e(\nu, i) - K_{Ci}\right) I_k^{informal}(\nu) + \left(V_k^e(\nu, f) - K_{Cf}\right) I_k^{formal}(\nu)\right] g_k^e(\nu)\, d\nu$$

These costs will be subtracted from aggregate income, and will be added to the expenditure on S-sector goods.

Step 6: Compute the steady state distribution of states. For informal firms, start with a guess for ψ_{ki}. Then, compute

$$\psi_{ki}^e(z') = \frac{\int g_k(z'|\nu)\, g_k^e(\nu)\, I_k^{informal}(\nu)\, d\nu}{\int_{\widetilde{z}} \int_{\nu} g_k(\widetilde{z}|\nu)\, g_k^e(\nu)\, I_k^{informal}(\nu)\, d\nu d\widetilde{z}}$$

$$\varrho_{ki}^{exit} = \alpha_{ki} + (1 - \alpha_{ki}) \int_z \int_{\ell} \left(I_k^{exit}(z, \ell, i) + I_k^{change}(z, \ell, i)\right) \psi_{ki}(z, \ell)\, d\ell dz$$

In steady state $N_{ki} = (1 - \varrho_{ki}^{exit})\, N_{ki} + M_{ki}$. Therefore, set $\frac{M_{ki}}{N_{ki}}$, the fraction of sector k informal firms that are entrants, to:

$$\boxed{\frac{M_{ki}}{N_{ki}} = \varrho_{ki}^{exit} = \frac{\omega_{ki} M_k}{N_{ki}}.}$$

Now, compute $\widetilde{\psi}_{ki}$:

$$\widetilde{\psi}_{ki}\left(z',\ell\right) = \mathbf{1}\left[\ell = 1\right] \times \varrho_{ki}^{exit} \times \psi_{ki}^{e}\left(z'\right) + \mathbf{1}\left[\ell \geq 1\right] \times \left(1-\alpha_{ki}\right) \times \left(\int_{z} \psi_{ki}\left(z,\ell\right) I_{k}^{stay}\left(z,\ell,i\right) g_{k}(z'|z)dz\right)$$

Update ψ_{ki} with

$$\psi_{ki}\left(z',\ell'\right) = \frac{\int_{\ell} \widetilde{\psi}_{ki}\left(z',\ell\right) I\left(L_{k}\left(z',\ell,i\right)=\ell'\right) d\ell}{\int_{\widetilde{z}} \int_{\ell} \widetilde{\psi}_{ki}\left(\widetilde{z},\ell\right) I\left(L_{k}\left(\widetilde{z},\ell,i\right)=\ell'\right) d\ell d\widetilde{z}}$$

And repeat until convergence of ψ_{ki}. This converged value of ψ_{ki} will be used directly in the computation of ψ_{kf} below.

For formal firms, start with guess for ψ_{kf} and compute

$$\psi_{kf}^{e}\left(z'\right) = \frac{\int g_{k}\left(z'|\nu\right) g_{k}^{e}\left(\nu\right) I_{k}^{formal}\left(\nu\right) d\nu}{\int_{\widetilde{z}} \int_{\nu} g_{k}\left(\widetilde{z}|\nu\right) g_{k}^{e}\left(\nu\right) I_{k}^{formal}\left(\nu\right) d\nu d\widetilde{z}}$$

$$\varrho_{kf}^{exit} = \alpha_{kf} + \left(1-\alpha_{kf}\right) \int_{z} \int_{\ell} I_{k}^{exit}\left(z,\ell,f\right) \psi_{kf}\left(z,\ell\right) d\ell dz$$

$$\varrho_{ki}^{change} = \left(1-\alpha_{ki}\right) \int_{z} \int_{\ell} I_{k}^{change}\left(z,\ell,i\right) \psi_{ki}\left(z,\ell\right) d\ell dz$$

In steady state

$$\begin{aligned} \varrho_{kf}^{exit} N_{kf} &= \varrho_{ki}^{change} \underbrace{N_{ki}}_{\frac{\omega_{ki} M_{k}}{\varrho_{ki}^{exit}}} + \omega_{kf} M_{k} \\ &= M_{k} \left(\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf}\right) \end{aligned}$$

So that:

$$\boxed{\frac{M_{kf}}{N_{kf}} = \frac{M_{k}\omega_{kf}}{N_{kf}} = \frac{\varrho_{kf}^{exit}\omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}}\omega_{ki} + \omega_{kf}}}$$

Also, note that

$$\frac{M_{kf}}{N_{kf}} \times \frac{N_{ki}}{M_{ki}} = \frac{\varrho_{kf}^{exit}\omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}}\omega_{ki} + \omega_{kf}} \frac{1}{\varrho_{ki}^{exit}} = \frac{\varrho_{kf}^{exit}\omega_{kf}}{\varrho_{ki}^{change}\omega_{ki} + \varrho_{ki}^{exit}\omega_{kf}}$$

and

$$\frac{M_{kf}}{N_{kf}} \times \frac{N_{ki}}{M_{ki}} = \frac{\omega_{kf}}{\omega_{ki}} \frac{N_{ki}}{N_{kf}}$$

Therefore,

$$\boxed{\frac{N_{ki}}{N_{kf}} = \frac{\varrho_{kf}^{exit}\omega_{ki}}{\varrho_{ki}^{change}\omega_{ki} + \varrho_{ki}^{exit}\omega_{kf}}}$$

Compute $\widetilde{\psi}_{kf}$ as:

$$\boxed{\widetilde{\psi}_{kf}(z',\ell) = \begin{array}{r} \mathbf{1}[\ell=1] \times \underbrace{\frac{\varrho_{kf}^{exit}\omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}}\omega_{ki} + \omega_{kf}}}_{\frac{M_{kf}}{N_{kf}}} \times \psi_{kf}^{e}(z') + \\ \mathbf{1}[\ell \geq 1] \times \left(\begin{array}{c} (1-\alpha_{kf}) \times \left(\int_z \psi_{kf}(z,\ell) I_k^{stay}(z,\ell,f) g_k(z'|z) dz\right) + \\ (1-\alpha_{ki}) \underbrace{\frac{\varrho_{kf}^{exit}\omega_{ki}}{\varrho_{ki}^{change}\omega_{ki} + \varrho_{ki}^{exit}\omega_{kf}}}_{\frac{N_{ki}}{N_{kf}}} \times \\ \left(\int_z \psi_{ki}(z,\ell) I_k^{change}(z,\ell,i) g_k(z'|z) dz\right) \end{array} \right) \end{array}}$$

Update ψ_{kf} with:

$$\psi_{kf}(z',\ell') = \frac{\int_\ell \widetilde{\psi}_{kf}(z',\ell) I(L_k(z',\ell,f) = \ell') d\ell}{\int_{\widetilde{z}} \int_\ell \widetilde{\psi}_{kf}(\widetilde{z},\ell) I(L_k(\widetilde{z},\ell,f) = \ell') d\ell d\widetilde{z}}$$

$$= \int_\ell \widetilde{\psi}_{kf}(z',\ell) I(L_k(z',\ell,f) = \ell') d\ell$$

And repeat until convergence of ψ_{kf}.
At this point we have the following objects: ψ_{kj}, $\widetilde{\psi}_{kj}$, ϱ_{ki}^{exit}, ϱ_{ki}^{change}, ϱ_{kf}^{exit}, $\chi_{ki\to f}^{change}$, χ_{kf}^{layoff}, and χ_{ki}^{leave} (see equations (A.18), (A.22) and (A.24)).

Step 7: This step solves for masses of entrants M_k's, masses of firms N_{kj}'s, aggregate vacancies V_{kj}'s and mass of unemployment L_u consistent with $d_{H,C}$, $d_{H,S}$, d_F and μ^v.
Step 7a: Write aggregate income I, price indices $P_C^{1-\sigma}$and $P_S^{1-\sigma}$, and expenditure with sector-S intermediates R as functions of masses of entrants M_C and M_S.
Step 7b: Solve for $\frac{M_S}{M_C}$ that matches $d_{H,C}$.
Step 7c: Separately pin down M_C and M_S using the labor market clearing equation $\overline{L} - L_u = \sum_{k=C,S,j=i,f} L_{kj}$. Express M_C and M_S as functions of L_u.
Step 7d: Express masses of firms N_{kj} as functions of L_u.
Step 7e: Express aggregate posted vacancies V_{kj} as functions of L_u.
Step 7f: Use equation for μ^v (and the value initially guessed in Step 1 for μ^v) to obtain L_u consistent with $d_{H,C}$, $d_{H,S}$, d_F and μ^v.
Step 7g: Go back and obtain masses of entrants M_k's, masses of firms N_{kj}'s, and aggregate vacancies V_{kj}'s.

Step 8: Obtain job finding rates μ^e_{kj} using aggregate vacancies V_{kj}'s and mass of unemployment L_u obtained in Step 7.

$$\mu^e_{kj} = \frac{m_{kj}}{L_u} = \xi_{kj}\phi\frac{V_{kj}}{\widetilde{V}}\phi\left(\frac{\widetilde{V}}{L_u}\right)^{\eta}$$

Step 9: Use equations (A.25)-(A.26) to obtain allocations L_{Cf}, L_{Ci}, L_{Sf}, L_{Si}.

$$\begin{aligned}
L_{Ci} &= \frac{\mu^e_{Ci}L_u}{\chi^{leave}_{Ci}} \\
L_{Si} &= \frac{\mu^e_{Si}L_u}{\chi^{leave}_{Si}} \\
L_{Cf} &= \frac{\mu^e_{Cf}L_u + \chi^{change}_{Ci\to f}L_{Ci}}{\chi^{layoff}_{Cf}} \\
L_{Sf} &= \frac{\mu^e_{Sf}L_u + \chi^{change}_{Si\to f}L_{Si}}{\chi^{layoff}_{Sf}}
\end{aligned}$$

Step 10: Compute deviation from the labor market clearing equation:

$$Dev_L = abs\left(\frac{\overline{L} - (L_{Cf} + L_{Ci} + L_{Sf} + L_{Si})}{\overline{L}}\right) ,$$

Step 11: Compute deviations from the C- and S-sector market clearing equations:

$$Dev_C = abs\left(\frac{\zeta I - (Revenue_C + (\tau_a - 1)\,Exports)}{(Revenue_C + (\tau_a - 1)\,Exports)}\right)$$

$$Dev_S = abs\left(\frac{[(1-\zeta)\,I + R] - Revenue_S}{Revenue_S}\right)$$

Where

$$Revenue_k = N_{kf}\int_\ell\int_z R_k(z,\ell)\,\psi_{kf}(z,\ell)\,dzd\ell + N_{ki}\int_\ell\int_z R_k(z,\ell)\,\psi_{ki}(z,\ell)\,dzd\ell$$

And $Revenue_C + (\tau_a - 1)\,Exports$ is total expenditure with goods from the C-sector (after imposing trade balance). In equilibrium:

$$\begin{aligned}
\zeta I &= Revenue_C + \tau_a Imports - Exports \\
&= Revenue_C + (\tau_a - 1)\,Exports
\end{aligned}$$

Note: Given the procedure outlined above, Dev_C should always be zero, unless we are not able to match $d_{H,C}$ with a positive value of $\frac{M_S}{M_C}$.

Step 12: Compute all moments to be matched with those in the data.

Step 13: Compute Loss Function. Add Model/Data deviations to equilibrium penalty $EQ\_Penalty$. The objective function is therefore given by

$$L = L_{mom} + EQ\_Penalty$$

Where L_{mom} penalizes deviations between moments in the data and $EQ\_Penalty$ penalizes deviations from equilibrium restrictions:

$$EQ\_Penalty = W_1 Dev_L + W_2 Dev_C + W_3 Dev_S,$$

With W_1, W_2 and W_3 denoting large weights.

Step 15: Optimization routine picks new parameter vector Θ. Go back to Step 2 until convergence.

Step 16 (Post-estimation): Obtain D_F^* (this is the deep parameter that we need for the counterfactuals as d_F is endogenous):

$$D_F^* = \frac{\left(\exp\left(\sigma_C d_F\right) - 1\right) D_H}{\epsilon^{\sigma_C} \tau_c^{1-\sigma_C}}$$

Where ϵ is the exchange rate value that balances trade:

$$\epsilon = \frac{1}{\tau_a \tau_c} \frac{\left(\tau_a Exports\right)^{\frac{1}{1-\sigma_C}}}{\exp\left(\frac{\sigma_C}{1-\sigma_C} d_{H,C}\right)}$$

B.2 Estimation Algorithm – Details

This document details the steps within Step 7 of the estimation procedure.

Step 7: This step solves for masses of entrants M_k's, masses of firms N_{kj}'s, aggregate vacancies V_{kj}'s and mass of unemployment L_u consistent with $d_{H,C}$, $d_{H,S}$, d_F and μ^v.

We start with some definitions... Averages "per firm". All these quantities can be computed after Step 5, that is, after solving for the steady state distribution of states.

$$Avg\_wbill_{ki} = \int_z \int_\ell \left[w_{ki}\left(z,\ell\right)\ell\right] \psi_{ki}\left(z,\ell\right) d\ell dz \text{ for } k = C, S$$

$$Avg\_wbill_{kf} = \int_z \int_\ell \left[\max\left\{w_{kf}\left(z,\ell\right), \underline{w}\right\}\ell\right] \psi_{kf}\left(z,\ell\right) d\ell dz \text{ for } k = C, S$$

$$Avg\_profit\_tilda_{ki} = \int_z \int_\ell \widetilde{\pi}_{ki}\left(z,\ell\right) \psi_{ki}\left(z,\ell\right) d\ell dz \text{ for } k = C, S$$

$$Avg\_profit\_tilda_{kf} = \int_z \int_\ell \widetilde{\pi}_{kf}\left(z,\ell\right) \psi_{kf}\left(z,\ell\right) d\ell dz \text{ for } k = C, S$$

$$Avg\_Firing\_Costs_{kf} = \kappa \int_{z'} \int_\ell \left[\left(\ell - L_k\left(z',\ell,f\right)\right)\left(1 - I^{hire}\left(z',\ell,f\right)\right)\right] \widetilde{\psi}_{kf}\left(z',\ell\right) d\ell dz' \; k = C, S$$

$$Avg\_Hiring\_Costs_{kj} = \int_{z'} \int_{\ell} \left[H_{kj}\left(\ell, L_k\left(z', \ell, j\right)\right) I^{hire}\left(z', \ell, j\right) \right] \widetilde{\psi}_{kj}\left(z', \ell\right) d\ell dz' \;\; k = C, S;\; j = i, f$$

$$Avg\_Revenue_{kf} = \int_{z} \int_{\ell} R_k\left(z, \ell\right) \psi_{kf}\left(z, \ell\right) d\ell dz \text{ for } k = C, S$$

$$Avg\_InfPenalty_{ki} = \int_{z} \int_{\ell} \left[p_{ki}\left(\ell\right) R_k\left(z, \ell\right) \right] \psi_{ki}\left(z, \ell\right) d\ell dz \text{ for } k = C, S$$

$$Avg\_Vacancies_{kj} = \int_{z'} \int_{\ell} v_{kj}\left(z', \ell\right) \widetilde{\psi}_{kj}\left(z', \ell\right) d\ell dz' \text{ for } k = C, S;\; j = i, f$$

$$Avg\_Exports_{Cf} = \left(1 - \exp\left(-\sigma d_F\right)\right) \int_{z} \int_{\ell} \left[R_C\left(z, \ell\right) I^x\left(z, \ell\right) \right] \psi_{kf}\left(z, \ell\right) d\ell dz$$

$$Fraction\_Export_{Cf} = \int_{z} \int_{\ell} I^x\left(z, \ell\right) \psi_{Cf}\left(z, \ell\right) d\ell dz$$

$$\begin{aligned} Avg\_Price_{kj} &= \int_{z} \int_{\ell} p_{kj}\left(z, \ell\right)^{1-\sigma} \psi_{kj}\left(z, \ell\right) dz d\ell \\ &= \int_{z} \int_{\ell} \left(\frac{R_k\left(z, \ell\right)}{z\ell} \right)^{1-\sigma} \psi_{kj}\left(z, \ell\right) dz d\ell \text{ for } k = C, S;\; j = i, f \end{aligned}$$

$$Avg\_size_{kj} = \int_{z} \int_{\ell} \ell \psi_{kj}\left(z, \ell\right) d\ell dz \text{ for } k = C, S;\; j = i, f$$

Step 7a: Write aggregate income I, price indices $P_C^{1-\sigma}$ and $P_S^{1-\sigma}$, and expenditure with sector-S intermediates R as functions of masses of entrants M_C and M_S.

At this point we have (for $k = C, S$):

$$M_{ki} = \omega_{ki} M_k$$

$$M_{kf} = \omega_{kf} M_k$$

$$N_{ki} = \frac{\omega_{ki}}{\varrho_{ki}^{exit}} M_k$$

$$N_{kf} = \frac{\varrho_{ki}^{change} \omega_{ki} + \varrho_{ki}^{exit} \omega_{kf}}{\varrho_{kf}^{exit} \varrho_{ki}^{exit}} M_k$$

Write Government Revenue as:

$$\begin{aligned}
G = & N_{Cf}\tau_y Avg_Revenue_{Cf} \\
& + N_{Sf}\tau_y Avg_Revenue_{Sf} \\
& + N_{Cf}\tau_w Avg_wbill_{Cf} \\
& + N_{Sf}\tau_w Avg_wbill_{Sf} \\
& + N_{Ci} Avg_InfPenalty_{Ci} \\
& + N_{Si} Avg_InfPenalty_{Si} \\
& + N_{Cf} Avg_Firing_Costs_{Cf} \\
& + N_{Sf} Avg_Firing_Costs_{Sf} \\
& + (\tau_a - 1)\, N_{Cf} Avg_Exports_{Cf},
\end{aligned}$$

Where the last term imposes balanced trade ($Exports = Imports$).

Write Aggregate Income as:

$$
\begin{aligned}
I = & N_{Ci} Avg\_wbill_{Ci} \\
& + N_{Si} Avg\_wbill_{Si} \\
& + N_{Cf} Avg\_wbill_{Cf} \\
& + N_{Sf} Avg\_wbill_{Sf} \\
& + N_{Ci} Avg\_profit\_tilda_{Ci} \\
& + N_{Si} Avg\_profit\_tilda_{Si} \\
& + N_{Cf} Avg\_profit\_tilda_{Cf} \\
& + N_{Sf} Avg\_profit\_tilda_{Sf} \\
& - N_{Cf} Avg\_Firing\_Costs_{Cf} \\
& - N_{Sf} Avg\_Firing\_Costs_{Sf} \\
& - N_{Cf} Avg\_Hiring\_Costs_{Cf} \\
& - N_{Ci} Avg\_Hiring\_Costs_{Ci} \\
& - N_{Sf} Avg\_Hiring\_Costs_{Sf} \\
& - N_{Si} Avg\_Hiring\_Costs_{Si} \\
& - M_{Cf} K_{Cf} \\
& - M_{Ci} K_{Ci} \\
& - M_{Sf} K_{Sf} \\
& - M_{Si} K_{Si} \\
& - N_{Ci} \varrho_{Ci}^{change} (K_{Cf} - K_{Ci}) \\
& - N_{Si} \varrho_{Si}^{change} (K_{Sf} - K_{Si}) \\
& - M_C c_{e,C} \\
& - M_S c_{e,S} \\
& + G
\end{aligned}
$$

Use the following equations to write aggregate income as a function of M_C and M_S:

$$N_{Ci} = \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C \tag{A.41}$$

$$N_{Si} = \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S \tag{A.42}$$

$$N_{Cf} = \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \tag{A.43}$$

$$N_{Sf} = \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \tag{A.44}$$

$$M_{Ci} = \omega_{Ci} M_C \tag{A.45}$$

$$M_{Si} = \omega_{Si} M_S \tag{A.46}$$

$$M_{Cf} = \omega_{Cf} M_C \tag{A.47}$$

$$M_{Sf} = \omega_{Sf} M_S \tag{A.48}$$

We can rewrite government revenue:

$$
\begin{aligned}
G = & \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \tau_y Avg\_Revenue_{Cf} \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \tau_y Avg\_Revenue_{Sf} \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \tau_w Avg\_wbill_{Cf} \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \tau_w Avg\_wbill_{Sf} \\
& + \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} Avg\_InfPenalty_{Ci} \\
& + \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} Avg\_InfPenalty_{Si} \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Avg\_Firing\_Costs_{Cf} \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S Avg\_Firing\_Costs_{Sf} \\
& + (\tau_a - 1) \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Avg\_Exports_{Cf}
\end{aligned}
$$

And aggregate income:

$$
\begin{aligned}
I = & \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} Avg\_wbill_{Ci} \\
& + \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} Avg\_wbill_{Si} \\
& + \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C Avg\_wbill_{Cf} \\
& + \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S Avg\_wbill_{Sf} \\
& + \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} Avg\_profit\_tilda_{Ci} \\
& + \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} Avg\_profit\_tilda_{Si} \\
& + \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C Avg\_profit\_tilda_{Cf} \\
& + \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S Avg\_profit\_tilda_{Sf} \\
& - \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C Avg\_Firing\_Costs_{Cf} \\
& - \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S Avg\_Firing\_Costs_{Sf} \\
& - \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C Avg\_Hiring\_Costs_{Cf} \\
& - \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} Avg\_Hiring\_Costs_{Ci} \\
& - \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S Avg\_Hiring\_Costs_{Sf} \\
& - \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} Avg\_Hiring\_Costs_{Si} \\
& - \omega_{Cf} M_C K_{Cf} \\
& - \omega_{Ci} M_C K_{Ci} \\
& - \omega_{Sf} M_S K_{Sf} \\
& - \omega_{Si} M_S K_{Si} \\
& - \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} \varrho_{Ci}^{change} \left(K_{Cf} - K_{Ci}\right) \\
& - \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} \varrho_{Si}^{change} \left(K_{Sf} - K_{Si}\right) \\
& - M_C c_{e,C} \\
& - M_S c_{e,S} \\
& + \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C \tau_y Avg\_Revenue_{Cf} \\
& + \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S \tau_y Avg\_Revenue_{Sf} \\
& + \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C \tau_w Avg\_wbill_{Cf} \\
& + \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S \tau_w Avg\_wbill_{Sf} \\
& + \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} Avg\_InfPenalty_{Ci} \\
& + \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} Avg\_InfPenalty_{Si} \\
& + \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C Avg\_Firing\_Costs_{Cf} \\
& + \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S Avg\_Firing\_Costs_{Sf} + (\tau_a - 1) \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C Avg\_Exports_{Cf}
\end{aligned}
$$

Define a_C:

$$
\begin{aligned}
a_C \equiv \frac{I_C}{M_C} = {} & \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg\_wbill_{Cf} \\
& + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg\_wbill_{Ci} \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg\_profit\_tilda_{Cf} \\
& + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg\_profit\_tilda_{Ci} \\
& - \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg\_Hiring\_Costs_{Cf} \\
& - \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg\_Hiring\_Costs_{Ci} \\
& - \omega_{Cf} K_{Cf} \\
& - \omega_{Ci} K_{Ci} \\
& - \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \varrho_{Ci}^{change} \left(K_{Cf} - K_{Ci}\right) \\
& - c_{e,C} \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \tau_y Avg\_Revenue_{Cf} \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \tau_w Avg\_wbill_{Cf} \\
& + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg\_InfPenalty_{Ci} \\
& + \left(\tau_a - 1\right) \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg\_Exports_{Cf},
\end{aligned}
\tag{A.49}
$$

Where I_C is income coming from C-sector activity.

Define a_S:

$$
\begin{aligned}
a_S \equiv \frac{I_S}{M_S} = & \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg\_wbill_{Sf} \\
& + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg\_wbill_{Si} \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg\_profit\_tilda_{Sf} \\
& + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg\_profit\_tilda_{Si} \\
& - \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg\_Hiring\_Costs_{Sf} \\
& - \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg\_Hiring\_Costs_{Si} \\
& - \omega_{Sf} K_{Sf} \\
& - \omega_{Si} K_{Si} \\
& - \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \varrho_{Si}^{change} \left(K_{Sf} - K_{Si}\right) \\
& - c_{e,S} \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \tau_y Avg\_Revenue_{Sf} \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \tau_w Avg\_wbill_{Sf} \\
& + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg\_InfPenalty_{Si},
\end{aligned}
\tag{A.50}
$$

Where I_S is income coming from S-sector activity.

We therefore write aggregate income as:

$$
I = a_C M_C + a_S M_S \tag{A.51}
$$

Price Indices
C-sector

$$\begin{aligned} P_{C,H}^{1-\sigma} &= N_{Cf}Avg\_Price_{Cf} + N_{Ci}Avg\_Price_{Ci} \\ &= \underbrace{\begin{pmatrix} \frac{\varrho_{Ci}^{change}\omega_{Ci}+\varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg\_Price_{Cf} \\ +\frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg\_Price_{Ci} \end{pmatrix}}_{b_C^1} M_C \\ &= b_C^1 M_C \end{aligned}$$

$$P_F^{1-\sigma} = (\epsilon\tau_a\tau_c)^{1-\sigma}$$

Under Trade Balance:

$$Exports = \frac{D_{H,C}\,(\epsilon\tau_a\tau_c)^{1-\sigma}}{\tau_a}$$

$$\begin{aligned} &\Rightarrow (\epsilon\tau_a\tau_c)^{1-\sigma} = \frac{\tau_a \times Exports}{D_{H,C}} \\ &= \frac{\tau_a \times N_{Cf}Avg\_Exports_{Cf}}{D_{H,C}} \\ &= \underbrace{\frac{\tau_a \times Avg\_Exports_{Cf}}{\exp(\sigma \times d_{H,C})} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}}}_{b_C^2} M_C \\ &= b_C^2 M_C \end{aligned}$$

$$\begin{aligned} P_C^{1-\sigma} &= b_C^1 M_C + b_C^2 M_C \\ P_C^{1-\sigma} &= b_C M_C \end{aligned} \tag{A.52}$$

$$
\begin{aligned}
b_C^1 &= \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg\_Price_{Cf} \\
&\quad + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg\_Price_{Ci} \\
b_C^2 &= \frac{\tau_a \times Avg\_Exports_{Cf}}{\exp\left(\sigma \times d_{H,C}\right)} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \\
b_C &= b_C^1 + b_C^2 \qquad \text{(A.53)}
\end{aligned}
$$

S-sector

$$
\begin{aligned}
P_S^{1-\sigma} &= N_{Sf} Avg\_Price_{Sf} + N_{Si} Avg\_Price_{Si} \\
&= \underbrace{\begin{pmatrix} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg\_Price_{Sf} \\ + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg\_Price_{Si} \end{pmatrix}}_{b_S} M_S
\end{aligned}
$$

$$
P_S^{1-\sigma} = b_S M_S \qquad \text{(A.54)}
$$

$$
\begin{aligned}
b_S &= \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg\_Price_{Sf} \\
&\quad + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg\_Price_{Si} \qquad \text{(A.55)}
\end{aligned}
$$

Intermediate expenditures with Services

$$
\begin{aligned}
R = {} & \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \left(Avg\_Hiring\_Costs_{Cf} + \bar{c}_{Cf}\right) \\
& + \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} \left(Avg\_Hiring\_Costs_{Ci} + \bar{c}_{Ci}\right) \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \left(Avg\_Hiring\_Costs_{Sf} + \bar{c}_{Sf}\right) \\
& + \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} \left(Avg\_Hiring\_Costs_{Si} + \bar{c}_{Si}\right) \\
& + \omega_{Cf} M_C K_{Cf} \\
& + \omega_{Ci} M_C K_{Ci} \\
& + \omega_{Sf} M_S K_{Sf} \\
& + \omega_{Si} M_S K_{Si} \\
& + \varrho_{Ci}^{change} \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} \left(K_{Cf} - K_{Ci}\right) \\
& + \varrho_{Si}^{change} \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} \left(K_{Sf} - K_{Si}\right) \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Fraction\_Export_{Cf} f_x \\
& + M_C c_{e,C} \\
& + M_S c_{e,S}
\end{aligned}
$$

Define c_C:

$$
\begin{aligned}
c_C \equiv \frac{R_C}{M_C} = & \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \left(Avg\_Hiring\_Costs_{Cf} + \overline{c}_{Cf}\right) \\
& + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \left(Avg\_Hiring\_Costs_{Ci} + \overline{c}_{Ci}\right) \\
& + \omega_{Cf} K_{Cf} \\
& + \omega_{Ci} K_{Ci} \\
& + \varrho_{Ci}^{change} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \left(K_{Cf} - K_{Ci}\right) \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Fraction\_Export_{Cf} f_x \\
& + c_{e,C},
\end{aligned}
\tag{A.56}
$$

Where R_C is intermediate expenditure with services coming from C-sector activity.

Define c_S:

$$\begin{aligned} c_S \equiv \frac{R_S}{M_S} &= \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \left(Avg\_Hiring\_Costs_{Sf} + \overline{c}_{Sf}\right) \\ &+ \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \left(Avg\_Hiring\_Costs_{Si} + \overline{c}_{Si}\right) \\ &+ \omega_{Sf} K_{Sf} \\ &+ \omega_{Si} K_{Si} \\ &+ \varrho_{Si}^{change} \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \left(K_{Sf} - K_{Si}\right) \\ &+ c_{e,S}, \end{aligned} \tag{A.57}$$

Where R_S is intermediate expenditure with services coming from S-sector activity.

We can therefore write:

$$R = c_C M_C + c_S M_S \tag{A.58}$$

Recapping...

$$\begin{cases} I = a_C M_C + a_S M_S \\ P_C^{1-\sigma} = b_C M_C \\ P_S^{1-\sigma} = b_S M_S \\ R = c_C M_C + c_S M_S \end{cases}$$

Step 7b: Solve for $\frac{M_S}{M_C}$ that matches $d_{H,C}$.

Substituting (A.51) and (A.52) into:

$$d_{H,C} = \log\left(\left(\frac{\zeta I}{P_C^{1-\sigma}}\right)^{\frac{1}{\sigma}}\right)$$

We obtain:

$$\begin{aligned}
\exp\left(\sigma \times d_{H,C}\right) &= \frac{\zeta I}{P_C^{1-\sigma}} \\
&= \frac{\zeta\left(a_C M_C + a_S M_S\right)}{b_C M_C} \\
&= \frac{\zeta a_C}{b_C} + \frac{\zeta a_S}{b_C}\frac{M_S}{M_C}
\end{aligned}$$

Find $\frac{M_S}{M_C}$ that perfectly rationalizes the "guess" value of $d_{H,C}$.

$$\begin{aligned}
\exp\left(\sigma \times d_{H,C}\right) &= \frac{\zeta a_C}{b_C} + \frac{\zeta a_S}{b_C}\frac{M_S}{M_C} \\
\Rightarrow \left(\frac{M_S}{M_C}\right)^{*} &= \frac{b_C}{\zeta a_S}\left(\exp\left(\sigma \times d_{H,C}\right) - \frac{\zeta a_C}{b_C}\right) \qquad \text{(A.59)}
\end{aligned}$$

Step 7c: Separately pin down M_C and M_S using the labor market clearing equation $\overline{L} - L_u = \sum_{k=C,S,j=i,f} L_{kj}$. Express M_C and M_S as functions of L_u.

To separately pin down M_C and M_S, use the labor market clearig equation.

$$\begin{aligned}
\overline{L} - L_u &= N_{Cf} Avg_Size_{Cf} + N_{Ci} Avg_Size_{Ci} + N_{Sf} Avg_Size_{Sf} + N_{Si} Avg_Size_{Si} \\
&= \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C Avg_Size_{Cf} + \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} Avg_Size_{Ci} + \\
&\frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S Avg_Size_{Sf} + \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} Avg_Size_{Si} \\
&= \left(\frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} Avg_Size_{Cf} + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg_Size_{Ci} \right) M_C + \\
&\left(\frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} Avg_Size_{Sf} + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg_Size_{Si} \right) M_S
\end{aligned}$$

At this point, we can only express M_C and M_S as functions of L_u.
From now on write

$$\left(\frac{M_S}{M_C} \right)^* = \frac{b_C}{\zeta a_S} \left(\frac{\zeta a_C}{b_C} - \exp\left(\sigma \times d_{H,C}\right) \right)$$

$$\Rightarrow M_S = \underbrace{\frac{b_C}{\zeta a_S} \left(\exp\left(\sigma \times d_{H,C}\right) - \frac{\zeta a_C}{b_C} \right)}_{A} M_C$$

Therefore:

$$\begin{aligned}
M_S &= A M_C \\
A &= \frac{b_C}{\zeta a_S} \left(\exp\left(\sigma \times d_{H,C}\right) - \frac{\zeta a_C}{b_C} \right)
\end{aligned} \tag{A.60}$$

So that:

$$\begin{aligned}
\overline{L} - L_u = &\left(\frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} Avg_Size_{Cf} + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg_Size_{Ci} \right) M_C + \\
&\left(\frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} Avg_Size_{Sf} + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg_Size_{Si} \right) A M_C \\
&= B \times M_C
\end{aligned}$$

$$B = \left(\frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg\_Size_{Cf} + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg\_Size_{Ci} \right) + \left(\frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg\_Size_{Sf} + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg\_Size_{Si} \right) A \tag{A.61}$$

Finally:

$$M_C = \frac{\overline{L} - L_u}{B} \tag{A.62}$$

$$M_S = \frac{A}{B} \left(\overline{L} - L_u \right) \tag{A.63}$$

Step 7d: Express masses of firms N_{kj} as functions of L_u.

Substituting (A.62) and (A.63) into (A.41)-(A.44) to obtain the masses of firms:

$$N_{Ci} = \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C = \underbrace{\frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \frac{1}{B}}_{E_C} \left(\overline{L} - L_u\right) = E_C \left(\overline{L} - L_u\right)$$

$$N_{Si} = \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S = \underbrace{\frac{\omega_{Si}}{\varrho_{Si}^{exit}} \frac{A}{B}}_{E_S} \left(\overline{L} - L_u\right) = E_S \left(\overline{L} - L_u\right)$$

$$N_{Cf} = \underbrace{\frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} \frac{1}{B}}_{D_C} \left(\overline{L} - L_u\right)$$

$$N_{Sf} = \underbrace{\frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} \frac{A}{B}}_{D_S} \left(\overline{L} - L_u\right)$$

Step 7e: Express aggregate posted vacancies V_{kj} as functions of L_u.

Now, substituting the expressions for the N_{kj}'s to obtain the number of vacancies in each sector as a function of L_u:

$$\begin{aligned} V_{Cf} &= N_{Cf} Avg\_Vacancies_{Cf} + \frac{\omega_{Cf} M_C}{\mu^v_{Cf}} \\ &= Avg\_Vacancies_{Cf} \times D_C \left(\overline{L} - L_u\right) + \frac{\omega_{Cf}}{\mu^v_{Cf}} \frac{1}{B} \left(\overline{L} - L_u\right) \\ &= \underbrace{\left(Avg\_Vacancies_{Cf} \times D_C + \frac{\omega_{Cf}}{\mu^v_{Cf}} \frac{1}{B} \right)}_{F_C} \left(\overline{L} - L_u\right) \\ &= F_C \times \left(\overline{L} - L_u\right) \end{aligned} \tag{A.64}$$

$$\begin{aligned} V_{Ci} &= N_{Ci} Avg\_Vacancies_{Ci} + \frac{\omega_{Ci} M_C}{\mu^v_{Ci}} \\ &= Avg\_Vacancies_{Ci} \times E_C \left(\overline{L} - L_u\right) + \frac{\omega_{Ci}}{\mu^v_{Ci}} \frac{1}{B} \left(\overline{L} - L_u\right) \\ &= \underbrace{\left(Avg\_Vacancies_{Ci} \times E_C + \frac{\omega_{Ci}}{\mu^v_{Ci}} \frac{1}{B} \right)}_{G_C} \left(\overline{L} - L_u\right) \\ &= G_C \times \left(\overline{L} - L_u\right) \end{aligned} \tag{A.65}$$

$$\begin{aligned} V_{Sf} &= N_{Sf} Avg\_Vacancies_{Sf} + \frac{\omega_{Sf} M_S}{\mu^v_{Sf}} \\ &= \underbrace{\left(Avg\_Vacancies_{Sf} \times D_S + \frac{\omega_{Sf}}{\mu^v_{Sf}} \frac{A}{B} \right)}_{F_S} \left(\overline{L} - L_u\right) \\ &= F_S \times \left(\overline{L} - L_u\right) \end{aligned} \tag{A.66}$$

$$\begin{aligned} V_{Si} &= N_{Si} Avg\_Vacancies_{Si} + \frac{\omega_{Si} M_S}{\mu^v_{Si}} \\ &= \underbrace{\left(Avg\_Vacancies_{Si} \times E_S + \frac{\omega_{Si}}{\mu^v_{Si}} \frac{A}{B} \right)}_{G_S} \left(\overline{L} - L_u\right) \\ &= G_S \times \left(\overline{L} - L_u\right) \end{aligned} \tag{A.67}$$

Step 7f: Use equation for μ^v (and the value guessed for μ^v) to obtain L_u consistent with $d_{H,C}$, $d_{H,S}$, d_F and μ^v.

We have written each V_{kj} in terms of L_u. Now, note that

$$\mu^v = \phi \left(\frac{L_u}{\widetilde{V}} \right)^{1-\eta}$$

We can invert this equation to obtain L_u.

$$\mu^v = \phi \left(\frac{L_u}{J\left(\overline{L} - L_u\right)} \right)^{1-\eta}$$

$$\Rightarrow L_u^* = \frac{\left(\mu^v\right)^{\frac{1}{1-\eta}} J\overline{L}}{\phi^{\frac{1}{1-\eta}} + \left(\mu^v\right)^{\frac{1}{1-\eta}} J}$$

Step 7g: Go back and obtain masses of entrants M_k's (equations (A.62) and (A.63)), masses of firms N_{kj}'s (equations (A.41)-(A.44)), and aggregate vacancies V_{kj}'s (equations (A.64)-(A.67)). We are now able to compute transitions out of unemployment μ^e_{kj} (Step 7).

Additional Material

We compute the share of sales from exports among exporters. We omit the C subscript, as only C-sector firms can export.
Remember that the optimal share of output exported is given by equation (35):

$$\begin{aligned}\eta &= \left(1+\frac{\tau_c^{\sigma-1}}{\epsilon^\sigma}\frac{D_H}{D_F^*}\right)^{-1}\\ &= \left(1+\frac{D_H}{D_F^*}\frac{1}{\epsilon^\sigma\tau_c^{1-\sigma}}\right)^{-1}\end{aligned}$$

Manipulating the equation for d_F we obtain:

$$\begin{aligned}\exp\left(d_F\right) &= \left(\frac{D_F^*}{D_H}\epsilon^\sigma\tau_c+\tau_c^\sigma\right)^{\frac{1-\sigma}{\sigma}}\left[\tau_c^{\sigma-1}+\frac{D_F^*}{D_H}\epsilon^\sigma\right]\\ &= \left(\frac{D_F^*}{D_H}\epsilon^\sigma\tau_c^{1-\sigma}+1\right)^{\frac{1-\sigma}{\sigma}}\left(\tau_c^\sigma\right)^{\frac{1-\sigma}{\sigma}}\left[\tau_c^{\sigma-1}+\frac{D_F^*}{D_H}\epsilon^\sigma\right]\\ &= \left(1+\frac{D_F^*}{D_H}\epsilon^\sigma\tau_c^{1-\sigma}\right)^{\frac{1-\sigma}{\sigma}}\left[1+\frac{D_F^*}{D_H}\epsilon^\sigma\tau_c^{1-\sigma}\right]\\ &= \left(1+\frac{D_F^*}{D_H}\epsilon^\sigma\tau_c^{1-\sigma}\right)^{\frac{1}{\sigma}}\end{aligned}$$

So that

$$\exp\left(\sigma d_F\right) = \left(1+\frac{D_F^*}{D_H}\epsilon^\sigma\tau_c^{1-\sigma}\right)$$

Domestic Sales of exporters are given by:

$$R^{dom}\left(z,\ell\right) = \exp\left(d_H\right)\left(z\ell\right)^{\frac{\sigma-1}{\sigma}}\left(1-\eta\right)^{\frac{\sigma-1}{\sigma}}$$

We now compute $(1-\eta)^{\frac{\sigma-1}{\sigma}}$:

$$\begin{aligned}1-\eta &= 1-\left(1+\frac{D_H}{D_F^*}\frac{1}{\epsilon^\sigma\tau_c^{1-\sigma}}\right)^{-1}\\ &= 1-\frac{1}{1+\frac{D_H}{D_F^*}\frac{1}{\epsilon^\sigma\tau_c^{1-\sigma}}}\\ &= \frac{D_H}{D_F^*\epsilon^\sigma\tau_c^{1-\sigma}+D_H}\\ &= \exp\left(-\sigma d_F\right)\\ &\Rightarrow \left(1-\eta\right)^{\frac{\sigma-1}{\sigma}} = \exp\left(\left(1-\sigma\right)d_F\right)\end{aligned}$$

So domestic sales are given by:

$$R^{dom}(z,\ell) = \exp\left(d_H + (1-\sigma)\, d_F\right)(z\ell)^{\frac{\sigma-1}{\sigma}}$$

So that the fraction of domestic sales among exporters is given by:

$$\begin{aligned}
\frac{R^{dom}(z,\ell)}{R(z,\ell)} &= \frac{\exp\left(d_H + (1-\sigma)\, d_F\right)(z\ell)^{\frac{\sigma-1}{\sigma}}}{\exp\left(d_H + d_F\right)(z\ell)^{\frac{\sigma-1}{\sigma}}} \\
&= \frac{\exp\left((1-\sigma)\, d_F\right)}{\exp\left(d_F\right)} \\
&= \exp\left(-\sigma d_F\right)
\end{aligned}$$

So that the fraction of export sales among exporters is given by:

$$\frac{R^{exports}(z,\ell)}{R(z,\ell)} = \left(1 - \exp\left(-\sigma d_F\right)\right)$$

Therefore, aggregate exports are given by:

$$\begin{aligned}
Exports &= N_{Cf} \int\int R^{exports}(z,\ell)\, I^x(z,\ell)\, \psi_{Cf}(z,\ell)\, dz d\ell \\
&= N_{Cf}\left(1 - \exp\left(-\sigma d_F\right)\right) \int\int R(z,\ell)\, I^x(z,\ell)\, \psi_{Cf}(z,\ell)\, dz d\ell \\
&= N_{Cf}\left(1 - \exp\left(-\sigma d_F\right)\right) \int\int \exp\left(d_H + d_F\right)(z\ell)^{\frac{\sigma-1}{\sigma}}\, I^x(z,\ell)\, \psi_{Cf}(z,\ell)\, dz d\ell
\end{aligned}$$

C Simulation Appendix

C.1 Simulation Algorithm

In this section we describe the simulation algorithm in detail, which we break down into several steps for expositional clarity. The algorithm allows for the home country to hold a trade imbalance, given by the difference between aggregate imports and aggregate exports. The trade imbalance is a deficit if it is positive and a surplus if it is negative. The trade deficit is modeled as an income transfer from Foreign to Home. We can impose an exogenous trade deficit (such as zero – trade balance), in which case the exchange rate will adjust to meet it exactly. Otherwise, we can impose an exogenous exchange rate, in which case a non-zero imbalance may arise in equilibrium. We denote the trade deficit by

$$TradeDeficit = Imports - Exports.$$

Step 1: Start with guesses of $d_{H,C}$, $d_{H,S}$, μ^v, and ϵ.

Step 2: Compute d_F implied by the guesses of $d_{H,C}$ and ϵ.

$$d_F = \log\left(\left(1 + \frac{D_F^*}{\exp\left(\sigma \times d_{H,C}\right)}\epsilon^{\sigma}\tau_c^{1-\sigma}\right)^{\frac{1}{\sigma}}\right)$$

Step 3: Compute revenue functions $R_k\left(z, \ell\right)$, and compute wage schedules.

$$w_{kf}\left(z,\ell\right) = \frac{\left(1-\beta_f\right)\left(b+b^u\right)}{1+\beta_f\tau_w} + \frac{\beta_f\left(1-\tau_y\right)}{1+\beta_f\tau_w}\frac{R_k\left(z,\ell\right)}{\ell} - \frac{\beta_f}{1+\beta_f\tau_w}\frac{\overline{c}_{kf}}{\ell}$$

$$w_{kf}\left(z,\ell;\underline{w}_f\right) = \max\left\{w_{kf}\left(z,\ell\right),\underline{w}_f\right\}$$

$$w_{ki}\left(z,\ell\right) = \left(1-\beta_i\right)b + \beta_i\left(1-p_{ki}\left(\ell\right)\right)\frac{R_k\left(z,\ell\right)}{\ell} - \beta_i\frac{\overline{c}_{ki}}{\ell}$$

$$w_{ki}\left(z,\ell;\underline{w}_i\right) = \max\left\{w_{ki}\left(z,\ell\right),\underline{w}_i\right\}$$

Where $\underline{w}_f$ is the minimum wage in the formal sector (which is observed and fixed throughout estimation). $\underline{w}_i$ is the first percentile of the distribution of informal wages in PME and fixed throughout the estimation procedure. This is to avoid zero or negative informal wages.

Step 4: Compute firms' value functions. Obtain firms' policy functions. Solve for firms' entry decisions. Compute the fraction of entrants in the formal and informal sectors as follows:

$$\omega_{kf} \equiv \Pr\left(I_k^{formal}\left(\nu\right)=1\right) = \int I_k^{formal}\left(\nu\right)g_k^e\left(\nu\right)d\nu$$

$$\omega_{ki} \equiv \Pr\left(I_k^{informal}\left(\nu\right)=1\right) = \int I_k^{informal}\left(\nu\right)g_k^e\left(\nu\right)d\nu$$

Therefore, if M_k is the mass of entrants in sector k, the masses of formal and informal entrants in sector k are given by:

$$M_{ki} = \omega_{ki} M_k$$
$$M_{kf} = \omega_{kf} M_k$$

Step 5: Compute the expected value of entry in each sector $k = C, S$.

$$V_k^e = \int \left[\left(V_k^e(\nu, i) - K_{Ci}\right) I_k^{informal}(\nu) + \left(V_k^e(\nu, f) - K_{Cf}\right) I_k^{formal}(\nu) \right] g_k^e(\nu)\, d\nu$$

Step 6: Compute the steady state distribution of states. For informal firms, start with a guess for ψ_{ki}. Then, compute

$$\psi_{ki}^e(z') = \frac{\int g_k(z'|\nu)\, g_k^e(\nu)\, I_k^{informal}(\nu)\, d\nu}{\int_{\widetilde{z}} \int_\nu g_k(\widetilde{z}|\nu)\, g_k^e(\nu)\, I_k^{informal}(\nu)\, d\nu d\widetilde{z}}$$

$$\varrho_{ki}^{exit} = \alpha_{ki} + (1 - \alpha_{ki}) \int_z \int_\ell \left(I_k^{exit}(z, \ell, i) + I_k^{change}(z, \ell, i) \right) \psi_{ki}(z, \ell)\, d\ell dz$$

In steady state $N_{ki} = (1 - \varrho_{ki}^{exit})\, N_{ki} + M_{ki}$. Therefore, set $\frac{M_{ki}}{N_{ki}}$, the fraction of sector k informal firms that are entrants, to:

$$\boxed{\frac{M_{ki}}{N_{ki}} = \varrho_{ki}^{exit} = \frac{\omega_{ki} M_k}{N_{ki}}.}$$

Now, compute $\widetilde{\psi}_{ki}$:

$$\begin{aligned} \widetilde{\psi}_{ki}(z', \ell) = {} & \mathbf{1}[\ell = 1] \times \varrho_{ki}^{exit} \times \psi_{ki}^e(z') \\ & + \mathbf{1}[\ell \geq 1] \times (1 - \alpha_{ki}) \times \left(\int_z \psi_{ki}(z, \ell)\, I_k^{stay}(z, \ell, i)\, g_k(z'|z) dz \right) \end{aligned}$$

Update ψ_{ki} with

$$\psi_{ki}(z', \ell') = \frac{\int_\ell \widetilde{\psi}_{ki}(z', \ell)\, I\left(L_k(z', \ell, i) = \ell'\right) d\ell}{\int_{\widetilde{z}} \int_\ell \widetilde{\psi}_{ki}(\widetilde{z}, \ell)\, I\left(L_k(\widetilde{z}, \ell, i) = \ell'\right) d\ell d\widetilde{z}}$$

And repeat until convergence of ψ_{ki}. This converged value of ψ_{ki} will be used directly in the computation of ψ_{kf} below.

For formal firms, start with guess for ψ_{kf} and compute

$$\psi_{kf}^e(z') = \frac{\int g_k(z'|\nu)\, g_k^e(\nu)\, I_k^{formal}(\nu)\, d\nu}{\int_{\widetilde{z}} \int_\nu g_k(\widetilde{z}|\nu)\, g_k^e(\nu)\, I_k^{formal}(\nu)\, d\nu d\widetilde{z}}$$

$$\varrho_{kf}^{exit} = \alpha_{kf} + (1 - \alpha_{kf}) \int_z \int_\ell I_k^{exit}(z, \ell, f)\, \psi_{kf}(z, \ell)\, d\ell dz$$

$$\varrho_{ki}^{change} = (1-\alpha_{ki}) \int_z \int_\ell I_k^{change}(z,\ell,i)\,\psi_{ki}(z,\ell)\,d\ell dz$$

In steady state

$$\begin{aligned}
\varrho_{kf}^{exit} N_{kf} &= \varrho_{ki}^{change} \underbrace{N_{ki}}_{\frac{\omega_{ki} M_k}{\varrho_{ki}^{exit}}} + \omega_{kf} M_k \\
&= M_k \left(\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf} \right)
\end{aligned}$$

So that:

$$\boxed{\frac{M_{kf}}{N_{kf}} = \frac{M_k \omega_{kf}}{N_{kf}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf}}}$$

Also, note that

$$\frac{M_{kf}}{N_{kf}} \times \frac{N_{ki}}{M_{ki}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf}} \frac{1}{\varrho_{ki}^{exit}} = \frac{\varrho_{kf}^{exit} \omega_{kf}}{\varrho_{ki}^{change} \omega_{ki} + \varrho_{ki}^{exit} \omega_{kf}}$$

and

$$\frac{M_{kf}}{N_{kf}} \times \frac{N_{ki}}{M_{ki}} = \frac{\omega_{kf}}{\omega_{ki}} \frac{N_{ki}}{N_{kf}}$$

Therefore,

$$\boxed{\frac{N_{ki}}{N_{kf}} = \frac{\varrho_{kf}^{exit} \omega_{ki}}{\varrho_{ki}^{change} \omega_{ki} + \varrho_{ki}^{exit} \omega_{kf}}}$$

Compute $\widetilde{\psi}_{kf}$ as:

$$\boxed{\widetilde{\psi}_{kf}(z',\ell) = \begin{array}{c}
\mathbf{1}[\ell = 1] \times \underbrace{\frac{\varrho_{kf}^{exit} \omega_{kf}}{\frac{\varrho_{ki}^{change}}{\varrho_{ki}^{exit}} \omega_{ki} + \omega_{kf}}}_{\frac{M_{kf}}{N_{kf}}} \times \psi_{kf}^e(z') + \\
\mathbf{1}[\ell \geq 1] \times \left(\begin{array}{c}
(1-\alpha_{kf}) \times \left(\int_z \psi_{kf}(z,\ell)\, I_k^{stay}(z,\ell,f)\, g_k(z'|z) dz \right) + \\
(1-\alpha_{ki}) \underbrace{\frac{\varrho_{kf}^{exit} \omega_{ki}}{\varrho_{ki}^{change} \omega_{ki} + \varrho_{ki}^{exit} \omega_{kf}}}_{\frac{N_{ki}}{N_{kf}}} \times \\
\left(\int_z \psi_{ki}(z,\ell)\, I_k^{change}(z,\ell,i)\, g_k(z'|z) dz \right)
\end{array} \right)
\end{array}}$$

Update ψ_{kf} with:

$$\psi_{kf}\left(z', \ell'\right) = \frac{\int_{\ell} \widetilde{\psi}_{kf}\left(z', \ell\right) I\left(L_k\left(z', \ell, f\right) = \ell'\right) d\ell}{\int_{\widetilde{z}} \int_{\ell} \widetilde{\psi}_{kf}\left(\widetilde{z}, \ell\right) I\left(L_k\left(\widetilde{z}, \ell, f\right) = \ell'\right) d\ell d\widetilde{z}}$$

$$= \int_{\ell} \widetilde{\psi}_{kf}\left(z', \ell\right) I\left(L_k\left(z', \ell, f\right) = \ell'\right) d\ell$$

And repeat until convergence of ψ_{kf}.
At this point we have the following objects: ψ_{kj}, $\widetilde{\psi}_{kj}$, ϱ_{ki}^{exit}, ϱ_{ki}^{change}, ϱ_{kf}^{exit}, $\chi_{ki\rightarrow f}^{change}$, χ_{kf}^{layoff}, and χ_{ki}^{leave} (see equations (A.18), (A.22) and (A.24)).

Step 7: This step solves for masses of entrants M_k's, masses of firms N_{kj}'s, aggregate vacancies V_{kj}'s consistent with $d_{H,C}$, $d_{H,S}$, μ^v, ϵ and $\overline{P}_S$.
Step 7a: The price index for the service sector is fixed at the value implied in estimation. $P_S = \overline{P}_S$.
Step 7b: Write aggregate income I, price indices $P_C^{1-\sigma}$and $P_S^{1-\sigma}$, and expenditure with sector-S intermediates R as functions of masses of entrants M_C, M_S and exchange rate ϵ.
Step 7c: Solve for M_S given $\overline{P}_S$ and solve for M_C that perfectly matches the guess for $d_{H,C}$.
Step 7d: Obtain masses of firms N_{kj}.
Step 7e: Obtain aggregate vacancies V_{kj}.

Step 8: Obtain the value of L_u consitent with the guesses for μ^v, $d_{H,C}$, $d_{H,S}$, and ϵ:

$$L_u = \frac{\mu^v \widetilde{V}}{\left(1 - \left(\mu^v\right)^{\theta}\right)^{1/\theta}}$$

Step 9: Obtain job finding rates μ_{kj}^e. Note that:

$$\mu_{kj}^e = \xi_{kj} \frac{V_{kj}}{\widetilde{V}} \left(1 - \left(\mu^v\right)^{\theta}\right)^{1/\theta}$$

Step 10: Use equations (A.25)-(A.26) to obtain allocations L_{Cf}, L_{Ci}, L_{Sf}, L_{Si}.

$$
\begin{aligned}
L_{Ci} &= \frac{\mu^e_{Ci} L_u}{\chi^{leave}_{Ci}} \\
L_{Si} &= \frac{\mu^e_{Si} L_u}{\chi^{leave}_{Si}} \\
L_{Cf} &= \frac{\mu^e_{Cf} L_u + \chi^{change}_{Ci \to f} L_{Ci}}{\chi^{layoff}_{Cf}} \\
L_{Sf} &= \frac{\mu^e_{Sf} L_u + \chi^{change}_{Si \to f} L_{Si}}{\chi^{layoff}_{Sf}}
\end{aligned}
$$

Step 11: Compute Imports and Exports

$$
Exports = N_{Cf} \times Avg\_Exports_{Cf}
$$

$$
Imports = \frac{\exp\left(\sigma \times d_{H,C}\right)\left(\epsilon \tau_a \tau_c\right)^{1-\sigma}}{\tau_a}
$$

Step 12: Compute deviations

$$
Dev_L = abs\left(\left(L_{Cf} + L_{Ci} + L_{Sf} + L_{Si} + L_u\right) - \overline{L}\right)/\overline{L}
$$

$$
\begin{aligned}
Dev_{Tr} &= abs\left(\left(Imports - Exports\right) - TradeDeficit\right)/Imports \\
Dev_{FE,C} &= abs\left(V^e_C - c^e_C\right)/c^e_C \\
Dev_{FE,S} &= abs\left(V^e_S - c^e_S\right)/c^e_S
\end{aligned}
$$

Step 13: Compute Loss Function.

$$
L = norm\left(\mathbf{Dev}\right)
$$

$$
\mathbf{Dev} = \begin{pmatrix} Dev_L \\ Dev_{Tr} \\ Dev_{FE,C} \\ Dev_{FE,S} \end{pmatrix}
$$

If $TradeDeficit$ is exogenously imposed (e.g., to zero), then the exchange rate ϵ must adjust to meet the Trade Balance condition. Instead, if the exchange rate ϵ is exogenously imposed, then the Trade Balance condition is vacuous (it is always equal to zero).

C.2 Simulation Algorithm – Details

This document details the steps within Step 7 of the simulation procedure.

Step 7: This step solves for masses of entrants M_k's, masses of firms N_{kj}'s, and aggregate vacancies V_{kj}'s consistent with $d_{H,C}$, $d_{H,S}$, D^*_F, ϵ and μ^v.

We start with some definitions... Averages "per firm". All these quantities can be computed after Step 5, that is, after solving for the steady state distribution of states.

$$Avg\_wbill_{ki} = \int_z \int_\ell \left[w_{ki}(z,\ell)\,\ell\right] \psi_{ki}(z,\ell)\, d\ell dz \text{ for } k = C, S$$

$$Avg\_wbill_{kf} = \int_z \int_\ell \left[\max\left\{w_{kf}(z,\ell), \underline{w}\right\} \ell\right] \psi_{kf}(z,\ell)\, d\ell dz \text{ for } k = C, S$$

$$Avg\_profit\_tilda_{ki} = \int_z \int_\ell \widetilde{\pi}_{ki}(z,\ell)\, \psi_{ki}(z,\ell)\, d\ell dz \text{ for } k = C, S$$

$$Avg\_profit\_tilda_{kf} = \int_z \int_\ell \widetilde{\pi}_{kf}(z,\ell)\, \psi_{kf}(z,\ell)\, d\ell dz \text{ for } k = C, S$$

$$Avg\_Firing\_Costs_{kf} = \kappa \int_{z'} \int_\ell \left[(\ell - L_k(z',\ell,f))\left(1 - I^{hire}(z',\ell,f)\right)\right] \widetilde{\psi}_{kf}(z',\ell)\, d\ell dz' \;\; k = C, S$$

$$Avg\_Hiring\_Costs_{kj} = \int_{z'} \int_\ell \left[H_{kj}(\ell, L_k(z',\ell,j))\, I^{hire}(z',\ell,j)\right] \widetilde{\psi}_{kj}(z',\ell)\, d\ell dz' \;\; k = C, S;\; j = i, f$$

$$Avg\_Revenue_{kf} = \int_z \int_\ell R_k(z,\ell)\, \psi_{kf}(z,\ell)\, d\ell dz \text{ for } k = C, S$$

$$Avg\_InfPenalty_{ki} = \int_z \int_\ell \left[p_{ki}(\ell)\, R_k(z,\ell)\right] \psi_{ki}(z,\ell)\, d\ell dz \text{ for } k = C, S$$

$$Avg\_Vacancies_{kj} = \int_{z'} \int_\ell v_{kj}(z',\ell)\, \widetilde{\psi}_{kj}(z',\ell)\, d\ell dz' \text{ for } k = C, S;\; j = i, f$$

$$Avg\_Exports_{Cf} = (1 - \exp(-\sigma d_F)) \int_z \int_\ell \left[R_C(z,\ell)\, I^x(z,\ell)\right] \psi_{kf}(z,\ell)\, d\ell dz$$

$$Fraction\_Export_{Cf} = \int_z \int_\ell I^x(z,\ell)\, \psi_{Cf}(z,\ell)\, d\ell dz$$

$$\begin{aligned} Avg\_Price_{kj} &= \int_z \int_\ell p_{kj}(z,\ell)^{1-\sigma}\, \psi_{kj}(z,\ell)\, dz d\ell \\ &= \int_z \int_\ell \left(\frac{R_k(z,\ell)}{z\ell}\right)^{1-\sigma} \psi_{kj}(z,\ell)\, dz d\ell \text{ for } k = C, S;\; j = i, f \end{aligned}$$

$$Avg\_size_{kj} = \int_z \int_\ell \ell \psi_{kj}(z,\ell)\, d\ell dz \text{ for } k = C, S;\; j = i, f$$

Step 7a: The price index for the service sector is fixed at the value implied in estimation. In estimation, nominal variables such as income and wages were pinned down by the nominal moments in the data, so we allowed all prices to adjust. Here, one price must be fixed. Therefore, the S-sector price index is given by:

$$P_S = \overline{P}_S$$

Step 7b: Write aggregate income I, price indices $P_C^{1-\sigma}$ and $P_S^{1-\sigma}$, and expenditure with sector-S intermediates R as functions of masses of entrants M_C and M_S.

At this point we have (for $k = C, S$):

$$M_{ki} = \omega_{ki} M_k$$

$$M_{kf} = \omega_{kf} M_k$$

$$N_{ki} = \frac{\omega_{ki}}{\varrho_{ki}^{exit}} M_k$$

$$N_{kf} = \frac{\varrho_{ki}^{change} \omega_{ki} + \varrho_{ki}^{exit} \omega_{kf}}{\varrho_{kf}^{exit} \varrho_{ki}^{exit}} M_k$$

Write Government Revenue as:

$$
\begin{aligned}
G = {} & N_{Cf}\tau_y Avg\_Revenue_{Cf} \\
& + N_{Sf}\tau_y Avg\_Revenue_{Sf} \\
& + N_{Cf}\tau_w Avg\_wbill_{Cf} \\
& + N_{Sf}\tau_w Avg\_wbill_{Sf} \\
& + N_{Ci} Avg\_InfPenalty_{Ci} \\
& + N_{Si} Avg\_InfPenalty_{Si} \\
& + N_{Cf} Avg\_Firing\_Costs_{Cf} \\
& + N_{Sf} Avg\_Firing\_Costs_{Sf} \\
& + \frac{\tau_a - 1}{\tau_a} \exp\left(\sigma \times d_{H,C}\right) \left(\epsilon \tau_a \tau_c\right)^{1-\sigma},
\end{aligned}
$$

Write Aggregate Income as:

$$
\begin{aligned}
I = & N_{Ci} Avg\_wbill_{Ci} \\
& + N_{Si} Avg\_wbill_{Si} \\
& + N_{Cf} Avg\_wbill_{Cf} \\
& + N_{Sf} Avg\_wbill_{Sf} \\
& + N_{Ci} Avg\_profit\_tilda_{Ci} \\
& + N_{Si} Avg\_profit\_tilda_{Si} \\
& + N_{Cf} Avg\_profit\_tilda_{Cf} \\
& + N_{Sf} Avg\_profit\_tilda_{Sf} \\
& - N_{Cf} Avg\_Firing\_Costs_{Cf} \\
& - N_{Sf} Avg\_Firing\_Costs_{Sf} \\
& - N_{Cf} Avg\_Hiring\_Costs_{Cf} \\
& - N_{Ci} Avg\_Hiring\_Costs_{Ci} \\
& - N_{Sf} Avg\_Hiring\_Costs_{Sf} \\
& - N_{Si} Avg\_Hiring\_Costs_{Si} \\
& - M_{Cf} K_{Cf} \\
& - M_{Ci} K_{Ci} \\
& - M_{Sf} K_{Sf} \\
& - M_{Si} K_{Si} \\
& - N_{Ci} \varrho_{Ci}^{change} \left(K_{Cf} - K_{Ci} \right) \\
& - N_{Si} \varrho_{Si}^{change} \left(K_{Sf} - K_{Si} \right) \\
& - M_C c_{e,C} \\
& - M_S c_{e,S} \\
& + G \\
& + TradeDeficit
\end{aligned}
$$

$TradeDefici$t is interpreted as an income transfer from Foreign to Home.

Use the following equations to write aggregate income as a function of M_C and M_S:

$$N_{Ci} = \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C \tag{A.68}$$

$$N_{Si} = \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S \tag{A.69}$$

$$N_{Cf} = \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \tag{A.70}$$

$$N_{Sf} = \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \tag{A.71}$$

$$M_{Ci} = \omega_{Ci} M_C \tag{A.72}$$

$$M_{Si} = \omega_{Si} M_S \tag{A.73}$$

$$M_{Cf} = \omega_{Cf} M_C \tag{A.74}$$

$$M_{Sf} = \omega_{Sf} M_S \tag{A.75}$$

We can rewrite government revenue:

$$
\begin{aligned}
G = & \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \tau_y Avg\_Revenue_{Cf} \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \tau_y Avg\_Revenue_{Sf} \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \tau_w Avg\_wbill_{Cf} \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \tau_w Avg\_wbill_{Sf} \\
& + \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} Avg\_InfPenalty_{Ci} \\
& + \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} Avg\_InfPenalty_{Si} \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Avg\_Firing\_Costs_{Cf} \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S Avg\_Firing\_Costs_{Sf} \\
& + \frac{\tau_a - 1}{\tau_a} \exp\left(\sigma \times d_{H,C}\right) \left(\epsilon \tau_a \tau_c\right)^{1-\sigma}
\end{aligned}
$$

And aggregate income:

$$
\begin{aligned}
I = & \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} Avg\_wbill_{Ci} \\
& + \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} Avg\_wbill_{Si} \\
& + \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C Avg\_wbill_{Cf} \\
& + \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S Avg\_wbill_{Sf} \\
& + \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} Avg\_profit\_tilda_{Ci} \\
& + \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} Avg\_profit\_tilda_{Si} \\
& + \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C Avg\_profit\_tilda_{Cf} \\
& + \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S Avg\_profit\_tilda_{Sf} \\
& - \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C Avg\_Firing\_Costs_{Cf} \\
& - \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S Avg\_Firing\_Costs_{Sf} \\
& - \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C Avg\_Hiring\_Costs_{Cf} \\
& - \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} Avg\_Hiring\_Costs_{Ci} \\
& - \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S Avg\_Hiring\_Costs_{Sf} \\
& - \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} Avg\_Hiring\_Costs_{Si} \\
& - \omega_{Cf} M_C K_{Cf} \\
& - \omega_{Ci} M_C K_{Ci} \\
& - \omega_{Sf} M_S K_{Sf} \\
& - \omega_{Si} M_S K_{Si} \\
& - \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} \varrho_{Ci}^{change} \left(K_{Cf} - K_{Ci}\right) \\
& - \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} \varrho_{Si}^{change} \left(K_{Sf} - K_{Si}\right) \\
& - M_C c_{e,C} \\
& - M_S c_{e,S} \\
& + \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C \tau_y Avg\_Revenue_{Cf} \\
& + \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S \tau_y Avg\_Revenue_{Sf} \\
& + \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C \tau_w Avg\_wbill_{Cf} \\
& + \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S \tau_w Avg\_wbill_{Sf} \\
& + \frac{\omega_{Ci} M_C}{\varrho_{Ci}^{exit}} Avg\_InfPenalty_{Ci} \\
& + \frac{\omega_{Si} M_S}{\varrho_{Si}^{exit}} Avg\_InfPenalty_{Si} \\
& + \frac{\varrho_{Ci}^{change} \omega_{Ci} + \varrho_{Ci}^{exit} \omega_{Cf}}{\varrho_{Cf}^{exit} \varrho_{Ci}^{exit}} M_C Avg\_Firing\_Costs_{Cf} \\
& + \frac{\varrho_{Si}^{change} \omega_{Si} + \varrho_{Si}^{exit} \omega_{Sf}}{\varrho_{Sf}^{exit} \varrho_{Si}^{exit}} M_S Avg\_Firing\_Costs_{Sf} + \frac{\tau_a - 1}{\tau_a} \exp\left(\sigma \times d_{H,C}\right) \left(\epsilon \tau_a \tau_c\right)^{1-\sigma} + TradeDeficit
\end{aligned}
$$

Define a_C:

$$
\begin{aligned}
a_C \equiv\ & \frac{I_C}{M_C} \\
=\ & \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg\_wbill_{Cf} \\
& + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg\_wbill_{Ci} \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg\_profit\_tilda_{Cf} \\
& + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg\_profit\_tilda_{Ci} \\
& - \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg\_Hiring\_Costs_{Cf} \\
& - \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg\_Hiring\_Costs_{Ci} \\
& - \omega_{Cf} K_{Cf} \\
& - \omega_{Ci} K_{Ci} \\
& - \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \varrho_{Ci}^{change} \left(K_{Cf} - K_{Ci} \right) \\
& - c_{e,C} \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \tau_y Avg\_Revenue_{Cf} \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \tau_w Avg\_wbill_{Cf} \\
& + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg\_InfPenalty_{Ci}
\end{aligned}
$$

Where I_C is income coming from C-sector activity.

Define a_S:

$$
\begin{aligned}
a_S \equiv & \frac{I_S}{M_S} \\
= & \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg\_wbill_{Sf} \\
& + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg\_wbill_{Si} \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg\_profit\_tilda_{Sf} \\
& + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg\_profit\_tilda_{Si} \\
& - \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg\_Hiring\_Costs_{Sf} \\
& - \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg\_Hiring\_Costs_{Si} \\
& - \omega_{Sf}K_{Sf} \\
& - \omega_{Si}K_{Si} \\
& - \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \varrho_{Si}^{change}\left(K_{Sf} - K_{Si}\right) \\
& - c_{e,S} \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \tau_y Avg\_Revenue_{Sf} \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \tau_w Avg\_wbill_{Sf} \\
& + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg\_InfPenalty_{Si},
\end{aligned}
\tag{A.76}
$$

Where I_S is income coming from S-sector activity.

We therefore write aggregate income as:

$$
\begin{aligned}
I &= a_C M_C + a_S M_S + (\tau_a - 1)\, Imports + TradeDeficit \\
&= a_C M_C + a_S M_S + (\tau_a - 1)\, Imports + (Imports - Exports) \\
&= a_C M_C + a_S M_S + \tau_a Imports - Exports \\
&= a_C M_C + a_S M_S + \exp\left(\sigma \times d_{H,C}\right)\left(\epsilon \tau_a \tau_c\right)^{1-\sigma} - N_{Cf} Avg\_Exports_{Cf} \\
&= a_C M_C + a_S M_S + \exp\left(\sigma \times d_{H,C}\right)\left(\epsilon \tau_a \tau_c\right)^{1-\sigma} - \\
&\frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Avg\_Exports_{Cf}
\end{aligned}
\tag{A.77}
$$

Price Indices
C-sector

$$\begin{aligned} P_{C,H}^{1-\sigma} &= N_{Cf} Avg\_Price_{Cf} + N_{Ci} Avg\_Price_{Ci} \\ &= \underbrace{\left(\begin{array}{c} \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg\_Price_{Cf} \\ + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg\_Price_{Ci} \end{array} \right)}_{b_C} M_C \\ &= b_C M_C \end{aligned}$$

$$P_F^{1-\sigma} = (\epsilon \tau_a \tau_c)^{1-\sigma}$$

$$P_C^{1-\sigma} = b_C M_C + (\epsilon \tau_a \tau_c)^{1-\sigma} \tag{A.78}$$

$$\begin{aligned} b_C = & \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg\_Price_{Cf} \\ & + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} Avg\_Price_{Ci} \end{aligned}$$

S-sector

$$\begin{aligned} P_S^{1-\sigma} &= N_{Sf} Avg\_Price_{Sf} + N_{Si} Avg\_Price_{Si} \\ &= \underbrace{\left(\begin{array}{c} \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg\_Price_{Sf} \\ + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg\_Price_{Si} \end{array} \right)}_{b_S} M_S \end{aligned}$$

$$P_S^{1-\sigma} = b_S M_S \tag{A.79}$$

$$\begin{aligned} b_S = & \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} Avg\_Price_{Sf} \\ & + \frac{\omega_{Si}}{\varrho_{Si}^{exit}} Avg\_Price_{Si} \end{aligned} \tag{A.80}$$

Intermediate expenditures with Services

$$
\begin{aligned}
R = & \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C \left(Avg\_Hiring\_Costs_{Cf} + \bar{c}_{Cf}\right) \\
& + \frac{\omega_{Ci}M_C}{\varrho_{Ci}^{exit}} \left(Avg\_Hiring\_Costs_{Ci} + \bar{c}_{Ci}\right) \\
& + \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S \left(Avg\_Hiring\_Costs_{Sf} + \bar{c}_{Sf}\right) \\
& + \frac{\omega_{Si}M_S}{\varrho_{Si}^{exit}} \left(Avg\_Hiring\_Costs_{Si} + \bar{c}_{Si}\right) \\
& + \omega_{Cf}M_C K_{Cf} \\
& + \omega_{Ci}M_C K_{Ci} \\
& + \omega_{Sf}M_S K_{Sf} \\
& + \omega_{Si}M_S K_{Si} \\
& + \varrho_{Ci}^{change}\frac{\omega_{Ci}M_C}{\varrho_{Ci}^{exit}} \left(K_{Cf} - K_{Ci}\right) \\
& + \varrho_{Si}^{change}\frac{\omega_{Si}M_S}{\varrho_{Si}^{exit}} \left(K_{Sf} - K_{Si}\right) \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Fraction\_Export_{Cf} f_x \\
& + M_C c_{e,C} \\
& + M_S c_{e,S}
\end{aligned}
$$

Define c_C:

$$
\begin{aligned}
c_C \equiv \frac{R_C}{M_C} = {} & \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} \left(Avg\_Hiring\_Costs_{Cf} + \overline{c}_{Cf}\right) \\
& + \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \left(Avg\_Hiring\_Costs_{Ci} + \overline{c}_{Ci}\right) \\
& + \omega_{Cf} K_{Cf} \\
& + \omega_{Ci} K_{Ci} \\
& + \varrho_{Ci}^{change} \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} \left(K_{Cf} - K_{Ci}\right) \\
& + \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Fraction\_Export_{Cf} f_x \\
& + c_{e,C},
\end{aligned} \tag{A.81}
$$

Where R_C is intermediate expenditure with services coming from C-sector activity.

Define c_S:

$$\begin{aligned} c_S \equiv \frac{R_S}{M_S} &= \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} \left(Avg\_Hiring\_Costs_{Sf} + \bar{c}_{Sf}\right) \\ &+ \frac{\omega_{Si}}{\varrho_{Si}^{exit}} \left(Avg\_Hiring\_Costs_{Si} + \bar{c}_{Si}\right) \\ &+ \omega_{Sf}K_{Sf} \\ &+ \omega_{Si}K_{Si} \\ &+ \varrho_{Si}^{change}\frac{\omega_{Si}}{\varrho_{Si}^{exit}} \left(K_{Sf} - K_{Si}\right) \\ &+ c_{e,S}, \end{aligned} \tag{A.82}$$

Where R_S is intermediate expenditure with services coming from S-sector activity.

We can therefore write:

$$R = c_C M_C + c_S M_S \tag{A.83}$$

Recapping...

$$\begin{cases} I = a_C M_C + a_S M_S + \exp\left(\sigma \times d_{H,C}\right)\left(\epsilon\tau_a\tau_c\right)^{1-\sigma} - \\ \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Avg\_Exports_{Cf} \\ P_C^{1-\sigma} = b_C M_C + \left(\epsilon\tau_a\tau_c\right)^{1-\sigma} \\ P_S^{1-\sigma} = b_S M_S \\ R = c_C M_C + c_S M_S \end{cases}$$

Step 7c: Given that sector-S price index is fixed at $P_S = \overline{P}_S$, we can back out M_S.

$$M_S = \frac{\overline{P}_S^{1-\sigma}}{b_S} \tag{A.84}$$

Now, we find M_C that matches the guessed value for $d_{H,C}$.

Substituting (A.77) and (A.78) into:

$$d_{H,C} = \log\left(\left(\frac{\zeta I}{P_C^{1-\sigma}}\right)^{\frac{1}{\sigma}}\right)$$

Leads to

$$\begin{aligned}
\exp\left(\sigma \times d_{H,C}\right) &= \frac{\zeta I}{P_C^{1-\sigma}} \\
&= \frac{\left(\begin{array}{c} \zeta a_C M_C + \zeta a_S M_S + \zeta \exp\left(\sigma \times d_{H,C}\right)\left(\epsilon \tau_a \tau_c\right)^{1-\sigma} \\ -\zeta \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C Avg_Exports_{Cf} \end{array}\right)}{b_C M_C + \left(\epsilon \tau_a \tau_c\right)^{1-\sigma}}
\end{aligned}$$

$$\Rightarrow M_C = \frac{\zeta a_S M_S + \exp\left(\sigma \times d_{H,C}\right)\left(\epsilon \tau_a \tau_c\right)^{1-\sigma}\left(\zeta - 1\right)}{\exp\left(\sigma \times d_{H,C}\right) b_C - \zeta a_C + \zeta \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} Avg\_Exports_{Cf}} \tag{A.85}$$

Step 7d: Now that we have values of M_C and M_S, we obtain masses of firms N_{kj}.

Substituting (A.85) and (A.84) into (A.68)-(A.71) to obtain the masses of firms:

$$N_{Ci} = \frac{\omega_{Ci}}{\varrho_{Ci}^{exit}} M_C$$

$$N_{Si} = \frac{\omega_{Si}}{\varrho_{Si}^{exit}} M_S$$

$$N_{Cf} = \frac{\varrho_{Ci}^{change}\omega_{Ci} + \varrho_{Ci}^{exit}\omega_{Cf}}{\varrho_{Cf}^{exit}\varrho_{Ci}^{exit}} M_C$$

$$N_{Cf} = \frac{\varrho_{Si}^{change}\omega_{Si} + \varrho_{Si}^{exit}\omega_{Sf}}{\varrho_{Sf}^{exit}\varrho_{Si}^{exit}} M_S$$

Step 7e: Obtain aggregate posted vacancies V_{kj}.
Now, substituting the expressions for the N_{kj}'s to obtain the number of vacancies in each sector as a function of L_u:

$$V_{Cf} = N_{Cf} Avg\_Vacancies_{Cf} + \frac{\omega_{Cf} M_C}{\mu^v_{Cf}}$$

$$V_{Ci} = N_{Ci} Avg\_Vacancies_{Ci} + \frac{\omega_{Ci} M_C}{\mu^v_{Ci}}$$

$$V_{Sf} = N_{Sf} Avg\_Vacancies_{Sf} + \frac{\omega_{Sf} M_S}{\mu^v_{Sf}}$$

$$V_{Si} = N_{Si} Avg\_Vacancies_{Si} + \frac{\omega_{Si} M_S}{\mu^v_{Si}}$$