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July, 2022

Working Paper No. wp2017

NBER WORKING PAPER SERIES

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Working Paper 30270
http://www.nber.org/papers/w30270

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
July 2022

We thank V. V. Chari, Matt Elliott, Ben Golub, Matthew Jackson, Ben Moll, Stephen Morris, Juan Passadore, and Alireza Tahbaz-Salehi for helpful discussions as well as seminar participants at Minnesota, MIT, NYU, NY Federal Reserve, Princeton, SAET Faro, and various other seminars and conferences. Chandrasekhar is grateful for support from the Alfred P. Sloan Foundation and the National Science Foundation GFRP. Townsend is grateful for support for research and surveys from the NICHD, UTCC, the Bank of Thailand, and the Thailand Research Fund. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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JEL No. D14,E44,G01,L14,O16

ABSTRACT

Consider an economy in which agents face income risk but interact in a stochastic financial network where the randomness is dictated by both chance and choice. We study the financial centrality of an agent defined as the ex-ante marginal social value of providing a small liquid asset to that agent. We show financially central agents are not only those who are linked often, but are more likely to be linked when (i) the realized network is fragmented, (ii) income risk is high, (iii) shocks are positively correlated,(iv) attitudes toward risk are more sensitive in the aggregate, and (v) there are tail risks. We apply our framework to models of financial markets with participation shocks, supply chains subject to disruptions, and village risk-sharing networks. We also study how the stochastic financial network structure influences bargaining, thereby endogenizing Pareto weights in the planner's problem. Evidence from Thai villages is consistent with these bargaining foundations, showing that agents who are more central indeed receive greater Pareto weight. We conclude by examining the welfare consequences of targeting larger liquid assets to key traders in markets, and to the most liquidity-sensitive links in supply chains.

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1. Introduction

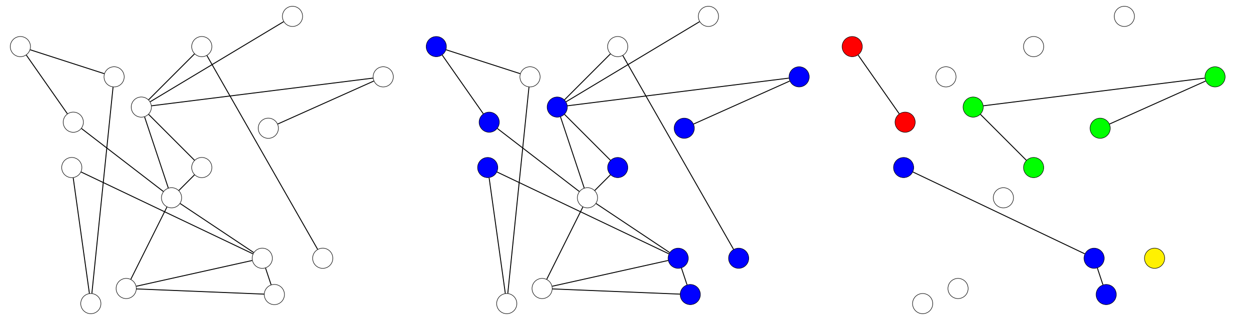
Delirium: *You use that word so much. Responsibilities. Do you ever think about what that means?*
Dream: *Well, I use it to refer to that area of existence over which I exert a certain amount of ... influence.*
Delirium: *It's more than that. The things we do make echoes.*
–Neil Gaiman, *The Sandman, Vol. 9: The Kindly Ones*

Networks mediate the function of financial markets in myriad contexts, including but not limited to banking systems, over-the-counter bond markets, supply chains, financial traders, and informal financial relationships to abate risk in villages (Bramoulle and Kranton, 2007; Bloch, Genicot, and Ray, 2008; Ambrus, Mobius, and Szeidl, 2014; Jackson, Barraquer, and Tan, 2012; Ambrus, Gao, and Milan, 2021; Ambrus and Elliott, 2021). Both developing a positive description of how network shape and position relates to function of the market and understanding normative implications such as how a policymaker may want to intervene—e.g., provide liquidity—in the market necessitates the study of which agents are of particular value in a network of financial transactions.

We study the value of agents to a policymaker in a *stochastic financial network* (SFN). In our model, agents interact in a stochastic environment. Whether i is able to exchange with j—either directly (bilateral transfers) or indirectly (transfers mediated by bilateral transfers through a possible chain of intermediate agents)—in a given state of the world can depend both on exogenous factors and endogenous factors such as agents' decisions to participate. This distribution over who can interact with whom in each state of the world constitutes the SFN, which is a distribution over all partitions of the population into various transaction markets. As will become clear, this is a generalization of a fixed, state-invariant financial network which is a nested special case.

An agent's centrality in the SFN is defined by the marginal value to the policymaker of providing a small liquid asset to the agent prior to the state of the world being realized. This is analogous to the diffusion literature where the unit of information replaces the liquid asset. Agent i's position in the SFN determines the other agents with whom i indirectly or directly transacts with in each state of the world and therefore the amount of social value providing the asset to i delivers.

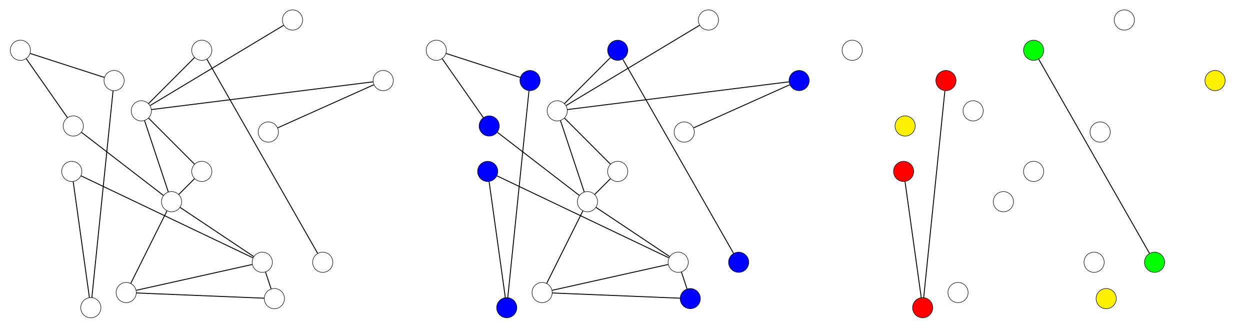
To better understand the SFN, let us define first a *social graph* (SG)—a binary, undirected graph denoted by G—which encodes whether or not i and j are able to directly make transfers ($G_{ij} \in \{0, 1\}$). The SFN will be a distribution over sub-graphs of G, capturing two facts. First, not all agents are active in each state of the world possibly due to exogenous shocks or endogenous decisions or both. Let A denote the set of *active agents* in some state. Second, given the restriction that an agent i can only directly make transfers to agents j who are both active ($i, j \in A$) and also linked to i in SG ($G_{ij} = 1$), for a given realization of the SFN, G may be split into disconnected components. These are the components of the induced subgraph G_A which is the restriction of G to the set of active agents. Figure 1 presents an example to clarify the concept. The special case in which SFN is the degenerate distribution in which every agent is always active ($A = \{1, \ldots, n\}$) returns the SG as the deterministic transaction network.



(A) Social Graph G dictating potential bilateral transfers

(B) Active agent set A determined exogenously or endogenously

(C) Components of G_A: feasible trading "markets" in realized network



(D) Social Graph G dictating potential bilateral transfers

(E) Active agent set A' determined exogenously or endogenously

(F) Components of $G_{A'}$: feasible trading "markets" in realized network

Figure 1. Panel A presents the social graph G which determines allowable bilateral transfers. Panel B presents the set of active agents A in the realized state, which can be due to exogenous or endogenous reasons. Panel C presents the induced realization of SFN, the induced graph G_A, as well as the components (or "markets"), on which full risk-sharing can be achieved, in different colors. All isolated nodes are in autarky in this state and consume their income. Panels D-F present the analogous sequence for a set of active agents A' in a different realization and the induced graph $G_{A'}$.

So the social graph determines feasible direct transfers. If SG is the complete graph, then all agents in principle can transact with all others, barring exogenous shocks or endogenous decisions to not participate. But if i and j are in separate components of the SG to begin with, and therefore there are no paths from i to j in G, then in all states of the world they will be unable to transact directly as well as indirectly. Notice, agents in one component are unable to indirectly transact with those in another component and cannot mutually insure each other.

For standard reasons, we do not focus on how the details of the SG impacts the quality of risk sharing beyond how it influences the distribution of components. Specifically, we follow Bramoulle and Kranton (2007) and progeniture work wherein agents make repeated bilateral transactions with their SG links (where $G_{ij} = 1$) and thereby can implement any risk sharing arrangement at the component level.

Instead, we concern our selves with the structure and centrality of agents in the SFN—that is, the distribution of who interacts with whom in a given state of the world. We are interested in the probability distribution over partitions of the social network into distinct components (including isolated nodes). At times we call these distinct components "markets" (or trading rooms) since each captures an independent connected set of agents who can implement any arbitrary consumption arrangement at the component (market) level (Bramoulle and Kranton, 2007).

In Section 3 we define our notion of *financial centrality.* We ask which individuals are the most financially central in the sense of being valued through a policymaker's intervention, how this centrality relates to the SFN structure and economic fundamentals, what economic foundations give rise to such a pattern of network centrality, and what this says about normative policy (e.g., allocation of liquid assets).

The centralized planning problem that delivers Pareto optimal allocations is the problem of maximizing a Pareto weighted sum of ex-ante expected utilities of agents subject to shock contingent resource and to participation constraints. So, the *financial centrality of an agent i* is then the increment in ex-ante social value, a marginal increase in the objective function of the planner, derived from providing an infinitesimal liquid asset to i ex-ante—that is,

$$FC_i := \text{Marginal Social Value of giving } \epsilon > 0 \text{ to } i \text{ whenever they can trade.}$$

The first order conditions with respect to this liquidity ϵ, when ϵ is driven to zero, are then the value of liquidity and the correct measure of financial centrality of each trader i. It is the expectation of the joint product of the value of liquidity as the shadow price in the resource constraint and the participation indicator of that player i. In fact, we show that even for a small (but non-infinitesimal) amount of liquidity assets to be provided, if FC_i has a unique maximum, then all of the liquid assets will be provided to only a single agent.

Note that this has an analog in optimal seeding in a diffusion process, where one asks to which node should a policymaker provide a marginal unit of information in order to get the most wide-spread diffusion (Erdos and Renyi, 1959; Friedkin and Johnsen, 1997; Bollobas, 1998; DeMarzo et al., 2003; Durrett, 2007; Jackson, 2008; Golub and Jackson, 2010). However, a key difference with our resulting notion of centrality comes from the fact that these extra consumption goods are scarse, and unlike information, are rival in consumption. Therefore, is not only the number of the agents we can give this extra liquidity, but also their marginal utilitites of consumption. Therefore, central agents are not those who get to spread the extra resources to a large number of people, but rather those who get them to the ones that most need it.

An important consideration that emerges from our perspective is whether the liquid asset itself changes the endogenous stochastic financial network structure. So, we introduce the concept of being *inert* or *responsive to provision of an infinitesimal liquid asset.* When the distribution of participation in exchange does not respond to provision of the asset, which can happen both in exogenous but also endogenous participation models, we say it is inert. When the distribution of interaction itself changes, we say it is responsive. We study both.

In Section 4 we present the main results of our analysis. First, in Section 4.1 taking our perspective on financial centrality, we characterize the financial centrality of agents in terms of the structure of the stochastic network and economic fundamentals. The result is intuitive, but distinct in an important way, from standard analyses of network centrality in financial market. Typical analyses follow an intuition that agents who are highly linked, either directly or indirectly, are very central: shocks propagate further. Our results highlights an important other feature which can dominate the analysis. Central agents are those who are not simply ever-present. Rather, they are those who have numerous links with other agents who themselves have few alternative transaction partners. Said differently, let $d_{G_A,i}$ be the degree of i when A is the set of active agents in the induced graph G_A. Then i is particularly central when neighbors j ($G_{A,ij} = 1$) have $d_{G_A,j}$ small for typical realizations of active agents A.

That is, agents are more central if they link with less, rather than more, central agents and are present exactly when markets are thin. This is because when agents exhibit prudence, the expected marginal utility of consumption is an increasing function of average consumption volatility, which decreases when markets are bigger. When there is heterogeneity among agents—for instance they vary in variability of shocks, or agents who are connected have correlated shocks— central agents are those who are linked to sets of individuals with variable or correlated financials (e.g., similar occupations, similar portfolio holdings). This characterization of financial centrality stands in stark contrast to typical ideas of central nodes in the financial network and also contrasts with more broad notions of centralities in many disciplines. Those perspectives say i is more central if those around you are more central. Our result proves exactly the opposite is true: i is particularly valuable if i links with the "weak".

Second, we then consider foundations for centrality in Section 5. From the perspective of an Arrow Debreu economy, we show that financial centrality is the price of a personalized debt asset that implements the planner's optimal allocation as a Walrasian equilibrium with transfers. We also consider bargaining foundations. We show that when agents bargain to establish rules as to how value is split in the network, financial centrality precisely determines how these shares are determined. Under Nash bargaining, there is a positive linear relationship between the representing Pareto weight on the bargaining solution allocation and the financial centrality measure. The analysis suggests a unique pattern to look for in the data, not predicted by other standard models: agents who transact with others who themselves have fewer transaction partners, as well as transaction partners who have clustered shocks and more variable shocks, will have higher centrality and therefore receive greater average consumption in a risk-sharing environment. Risk-sharing data from Thai villages are consistent with these bargaining foundations.

Third, as shown in Section 6.1, our results are generalizable to settings in which agents can endogenously choose to enter the market. The SFN can respond to incentives as well as the very provision of the liquid asset.

Fourth, in Section 6.2, we generalize the model to include larger discrete liquid assets, turn to normative policy considerations, and confirm the earlier measures of valuable traders should be used to direct the provision of these assets. Thus, just as the earlier notions of financial centrality consistent with ex-post shocks and contagion have influenced the way policy makers think about prudential regulation, here our ex-ante measure of financial centrality could be used to think about monetary policy. We are reminded of Jeremy Stein's 2013 discussion[1] of how central bank liquidity should be priced ex-ante in an auction—a price which in turn could serve as a guide to policy makers concerning market conditions, a feedback loop to policy. Our contribution would be a measure of which traders or institutions have a key value in channeling the incremental liquidity to the market.

Section 7 is a conclusion. All proofs unless otherwise noted are contained in the appendix. Further, the appendix contains considerable generalizations of our analysis. Specifically, we (a) provide a generalization to markets with heterogenous fundamentals (e.g., Pareto weights, risk-preferences, dependent endowment processes); (b) provide generalized foundations for market participation; (c) extend the analysis to multiple segmented markets; (d) analyze several contrasting examples of endogenous participation.

2. Related Literature

Our work speaks to various seemingly distinct branches of the literature: risk sharing in networks, financial market risk, and supply chain disruptions. We provide a unifying framework and, to the best of our knowledge, unique contributions.

One branch of the literature studies risk-sharing networks (see e.g., Bramoulle and Kranton (2007); Bloch, Genicot, and Ray (2008); Ambrus, Mobius, and Szeidl (2014); Jackson, Barraquer, and Tan (2012); Ambrus, Gao, and Milan (2021); Ambrus and Elliott (2021)) from a variety of angles: e.g., how repeated bilateral transfers may facilitate perfect risk-sharing at the component level irrespective of network topology, how the topological structure affects the extent of insurance sustained, the role of capacity constraints, endogenous formation with concerns of stability. We build on the assumption in the literature that agents can only make bilateral transfers to those with whom they share a link in the social network and maintain that with repeated bilateral transfers agents can always perfectly smooth risk at the component level of the realization of the stochastic financial network (absent other frictions). The core innovation is to allow for arbitrary exogenous and endogenous activation of nodes into the risk-sharing environment and study financial centrality as per our concept in this case. In contrast to much of the above literature where central agents are often characterized by their value in shock dissipation (comparable to Bonacich centrality and other eigenvector-like centralities), our work distinctly focuses on what makes individuals central, in the sense of the planner's objective function when considering the provision of a liquid asset, when participation is exogenous or endogenous. So, while our results will of course involve eigenvector-like centrality components, a new, dominant piece will factor into the measure of centrality which is captured in our model.

A second branch focuses on financial markets and liquidity risk. For instance, in Duffie et al. (2005) there is a single underlying consumption good and two types of assets, a safe liquid asset such as a bank account which can be traded instantaneously, and a consol that requires finding a trading parter, in a search environment. Traders buy and sell these assets among themselves and with market makers. Search frictions make the markets imperfect. For us here in this paper, we feature risk averse traders who would like to hedge the income risk from the portfolio they hold, but who suffer from market participation risk. Relatedly, Longstaff (1992) studies the value

[1] https://www.federalreserve.gov/newsevents/speech/stein20130419a.htm

of liquidity and the distinction between on-the-run vs off-the run treasuries. Liquidity can vary across assets. However we do shift the emphasis and language somewhat from limited asset trade, in which some assets provide liquidity in disruptions, to limited market participation, with a focus on traders, specifically which key traders can provide liquidity to those who remain. Further, Weill (2007) studies the role of market makers in providing liquidity when there is large and temporary pressure as well as order execution delays. He refers to market makers as leaning against the wind. The paper studies optimal dynamic liquidity provision in a theoretical market. In our paper, key traders look like market makers in the sense that they provide liquidity to a subset of traders. However, unlike Weill (2007) we focus on quantifying the value of such market makers and potential heterogeneity among them. Like Weill (2007) we also move beyond marginal movements in liquidity and study optimal central bank provision liquidity, which should identify key traders as those to whom liquidity should be targeted ex ante. Lagos and Zhang (2020) also feature the role of Central Banks in the provision of liquidity in wholesale markets. A monetary authority injects or withdraws money via lump-sum transfers or taxes to investors in the second sub-period, in the Walrasian market. We adopt an extreme version of this; liquidity can only be injected via traders carrying it into markets and not when agents are in autarky.

A third branch focuses on contagion in financial networks. Much of this literature focuses on a kind of non-linearity, whereby positive and negative shocks propagate asymmetrically through a network. For instance, with solvency constraints, a positive shock may leave the network intact whereas a large enough negative shock may have a large adverse impact on welfare. Intriguingly, agents may vary in whether they are central for positive versus negative shocks and further, the optimal network structure may vary in the size of the shock (e.g., for small shocks the complete graph but for large shocks the empty graph).[2] Our notion of centrality is different.

3. Model

We begin by introducing the general model of the stochastic financial network, providing some examples of special cases, and establishing the planner's problem.[3]

3.1. **Setup.**

3.1.1. *Preferences.* Consider an economy with a set $I = \{1, ..., n\}$ of agents, one good, and one period. This can easily be generalized to multiple goods and periods.

Agents face idiosyncratic income risk, where $y = (y_1, ..., y_n)$ denotes the vector of income realizations for all agents in the economy, which we assume are drawn from some distribution $F(y)$. Let $\mu_i = \mathbb{E}(y_i)$ denote the mean, $\sigma_i^2 = \mathbb{E}(y_i - \mu_i)^2$ the variance, and Σ the variance-covariance matrix.

[2] The theory is developed in, among others, Allen and Gale (2000), Freixas et al. (2000) and Eisenberg and Noe (2001), Gai and Kapadia (2010), Blume, Easley, Kleinberg, Kleinberg, and Tardos (2011), Battiston et al. (2012), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Elliott, Golub, and Jackson (2014), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015), Farboodi (2021), Babus (2016), Elliott and Hazell (2016), Cabrales, Gottardi, and Vega-Redondo (2017), and Farboodi et al. (2022). Empirical work includes Upper and Worms (2004), Bech and Atalay (2010), Boss et al. (2004), Cohen-Cole et al. (2013), Craig and Von Peter (2014), Cont et al. (2013), Langfield et al. (2014), Jaramillo (2012), and others. See Allen and Babus (2009), Summer (2013), and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2016) for surveys.

[3] In the Online Appendix B, we provide other interpretations of the notation: financial markets with limited participation and supply chain economies with production shocks and limited networks.

Agents have expected utility preferences, with utility function $u_i(c_i)$, which we assume to be strictly increasing, strictly concave, and sufficiently smooth. Agents exhibit prudence: $u_i'''(c) > 0$.

3.1.2. *Social Graph (SG).* Individuals reside in an undirected, unweighted *social graph* (both of these can be relaxed) G, with $G_{ij} \in \{0,1\}$ denoting whether there is a social link between agents i and j. As a shorthand we call them friends, though in practice agents may be banks, firms, and so on and G represnts arbitrary constraints. We assume that G is connected meaning there exists some path between every pair of nodes, so it has just a single component.

The function of SG is that i can directly transact with j if and only if $G_{ij} = 1$. As is well-known in the literature, the results of Bramoulle and Kranton (2007) show that with repeated biliateral transfers among friends any connected component in a graph is able to fully smooth risk within the component, irrespective of the specific details of the network topology (under the assumptions maintained in this paper). All that matters is the component is connected so eventually any amount required can be indirectly transferred between any two members of that component to facilitate full risk-sharing.

3.2. **Stochastic Financial Network (SFN).**

3.2.1. *Active Agents.* To understand the SFN, we first define *active agents*. These are individuals who in a given state actively participate in interpersonal transactions due to either chance or endogenous choice. Let $A \subset I$ be the set of active agents. These are the agents who are able to make transfers to other active agents; the inactive agents only consume their endowments. Let G_A be the subgraph of G induced by A, with $G_{A,ij} = 1$ if and only if $i, j \in A$ and $G_{ij} = 1$. Thus, G_A is the restriction of G to the nodes in A.

Notice that just because G is fully connected does not mean that G_A is fully connected as depicted in Figure 1. The network of feasible transactions given active agent set A may have several connected components. While members of each of these components can transact with each other indirectly, no cross-component transfers in G_A is possible.

The distribution over A, the set of active agents, determined both due to exogenous shocks and endogenous responses, therefore determines a distribution over G_A. This distribution, $\mathrm{P}(G_A)$ can depend on (a) exogenous random factors such as opportunities to transact and (b) endogenous responses to the realization of the random endowment y. It is the relevant object for our analysis since it describes which individuals are able to transact with which and in what state.

3.2.2. *The Fully General Stochastic Financial Network.* There is nothing special about having one set of active agents all of whom who can mutually transact as long as they are connected in the graph. We generalize this.

Let $\mathcal{P}$ be the set of all partitions of the set of nodes I, Let $\pi \in \mathcal{P}$ denote a partition and $p^\pi \in \pi$ denote a part of this partition. If $i, j \in p^\pi$ are members of the same part, we say there is an equivalence relation $i \sim_\pi j$.

Given a partition π, there is an induced subgraph G_π where

$$G_{\pi,ij} = \begin{cases} 1 & \text{if } G_{ij} = 1 \text{ and } i \sim_\pi j \\ 0 & \text{otherwise.} \end{cases}$$

Agents are connected in the subgraph if and only if (a) they are members of the same part of the partition and (b) they have a social relationship in the base social graph (so $G_{ij} = 1$). It is immediate that this is a generalization of the above example of a single set of active agents.

We are interested in a probability distribution $\mathrm{P}(G_\pi)$ of subgraphs of G induced by a probability distribution over the set of all partitions $\mathcal{P}$. It is useful to track the components induced by the partition. G_π may contain a collection of r disjoint connected components. We let $M_\pi = \{m_1, m_2, \ldots m_r\}$ be the list of these components, which we interchangeably call "markets" or "trading rooms" which provide more familiar nomenclature for various applications.

We are now ready to define the stochastic financial network. Let the state be a pair (π, y) of a partition and a realized income endowment, denoted $s \in S := \mathcal{P} \times \mathbb{R}^n$. The *stochastic financial network* is a probability distribution $\mathrm{P}(s)$ over S which may arise due to exogenous or endogenous factors. We describe examples below.

3.2.3. *Feasible Transactions in the Financial Network.* Consider some realized state s. Define a variable that captures if an agent is active in a given component for each partition π: $\zeta_i^\pi = k$ if $i \in m_k$ and $|m_k| > 1$ and $\zeta_i^\pi = 0$ if $i \in m_k = \{i\}$. Thus, ζ^π denotes which part of the partition i is in, and sets all isolates to 0 for convenience.

Isolated individuals are not active and therefore $c_i = y_i$ when $\zeta_i^\pi = 0$. Meanwhile, by assumption as in Bramoulle and Kranton (2007), since agents can indirectly transact with any other in the same connected component, through repeated bilateral transfers with their friends, we can write

$$\sum_{i \in m_k} c_i \leq \sum_{i \in m_k} y_i \text{ for every } m_k \in M_\pi. \tag{3.1}$$

Note that this includes the isolates since in that case $m_k = \{i\}$. This expression says that for every component or market, the total amount of consumption may not exceed the total endowment among the members of this component.

3.2.4. *Examples of Stochastic Financial Networks.* We provide two simple examples of stochastic financial networks,

Example 3.1 (Percolation/Contagion Process)**.** In this example, prior to income realization, a set of individuals are seeded with some information that increases their odds of actively participating in financial exchange. This information rapidly diffuses through the network generating some set of active agents prior to the realization of incomes. For example, this could simply be that some people hear that there will likely be some need to engage in risk sharing this season and then word spreads quickly. Or, in contrast, it could be that some households hear about an opportunity to temporarily migrate to the city from their village and this information diffuses—active risk-sharing agents back home are those who stayed, having never received the diffused information.

More formally, consider a communication process that occurs (instantenously in the model from communication rounds $t = 0, \ldots, T$) prior to the one-shot risk-sharing phase. At $t = 0$, a set of seeds $J \subset I$ is shocked to to be active, $\zeta_i^J = 1$ for every $i \in J$. Then the process diffuses through the network through discrete time where in each period t, every node that has been activated $\zeta_i^J = 1$ infroms and activates each of its neighbors i.i.d. with probability α. When made active, nodes stay active. The process terminates after T periods. After T periods, the active agents are able to engage in financial exchange.

This microfounds the SFN, a resulting distribution over which nodes are active through T periods, which is a distribution over π and therefore M_π. Here M_π would denote all nodes that are active and part of the same component in the induced subgraph among active agents. This contagion process is a stylized example for a number of things. This could include hearing about an opportunity or a new need, being informed that financial exchange is happening (as in through an informal financial group such as a RoSCA), having a correlated outside option such as hearing about temporary migration and exiting the village, and so on. The general point is an exogenous percolation process of any kind, for any reason, naturally generates a distribution over components of active agents as well as isolates and therefore determines the SFN $\mathrm{P}(s)$.

Unlike the previous example, the next example looks at an endogenous determination of $\mathrm{P}(s)$.

Example 3.2 (Endogenous Participation)**.** In this example G is a complete graph, so $G_{ij} = 1$ for all $i \neq j$ meaning that all bilateral transfers are possible throughout any component realized. However, agents face some private entry cost k_i to be an active participant in the financial transaction. For instance, there may be other obligations or outside options such as temporary migration in the season. There is a Bayes Nash Equilibrium with cutoffs where the probability distribution of the stochastic financial network $\mathrm{P}(s)$ is driven by the distribution over costs and the resulting equilibrium entry decision (who chooses to be active given their costs and their beliefs about others' costs determines π). We study this in Section 6.1.

3.3. **The Planner's Problem.** Going forward, unless otherwise noted, we simplify exposition by focusing on the case where only one "market" is formed. That is, the distribution is such that there is exactly one non-trivial component and the remainder of nodes are isolated. In Online Appendix F we explore a generalization with several components (simultaneous segmented markets), and show how to map all of the results of this special model to the general case. We focus on the single market case for expositional simplicity since the multiple component case is conceptually identical but notationally cumbersome.

Formally, let $\zeta \in \{0,1\}^n$ be the participation vector (which formally we can model as a shock to the consumption set of agents). We can take ζ_i as binary precisely because we focus on the "single market" case. As previously noted, participation includes both exogenous as well as a wide class of endogenous participation models, described below. Here if $\zeta_i = 0$ then $c_i = y_i$. However, if $\zeta_i = 1$, then consumption and income do not have to coincide as agents can make transfers in such states. The relevant state, in the Arrow and Debreu sense of enumerating all shocks and indexing the commodity space by them, is then $s = (y, \zeta) \in S := \mathbb{R}^n_+ \times \{0,1\}^n$. A *feasible consumption allocation* is a function $c(s) = (c_i(s))_{i \in I}$ such that, for every $s = (y, \zeta)$, $c_i(s) = y_i$ whenever $\zeta_i = 0$ and it is resource feasible: i.e., $\sum_i \zeta_i c_i \leq \sum_i \zeta_i y_i$ for all s.

State s is drawn from a probability distribution $\mathrm{P}(y, \zeta)$ which is common knowledge among agents, and we assume the support of the distribution, S, to be discrete for most proofs, for expositional simplicity. This is a primitive of our baseline environment.

The timing of the realization of income and market shocks matters and will give rise to different measures of centrality. The baseline assumption in most of the paper considers income and market participation shocks as independent random variables. One can think about this as a case where market participation, ζ, is assigned first and then, independent of this, income shocks y are drawn. This may describe exogenous settings in which transaction opportunities arise, to first order, from a set of pre-determined agents (e.g., relatives or individuals with whom trust has been established over many years, or as in supply chains) and where shocks to availability, awareness, or costs further affect participation. It also describes some endogenous settings more generally, described below.

Suppose the allocation can be determined as if there were a planner who tries to choose among resource feasible allocations to maximize a linear welfare functional,[4] with Pareto weights vector $\lambda \in \mathbb{R}^n_+$, effectively choosing $c(s)$ to solve:

$$V(\lambda) := \max_{(c_i(\cdot))_{i=1,\ldots,n}} \mathbb{E}_s \left\{ \sum_{i=1}^n \lambda_i u_i \left[c_i(s) \right] \right\} \tag{3.2}$$

subject to

$$\sum_{i=1}^n \zeta_i c_i(s) \leq \sum_{i=1}^n \zeta_i y_i(s) \text{ for all } (y, \zeta) \tag{3.3}$$

and

$$c_i(s) = y_i \text{ for all } s = (y, \zeta) : \zeta_i = 0. \tag{3.4}$$

We therefore consider a setting where a set of n agents who may have heterogeneous preferences, heterogeneous income processes, endogenous participation decisions, and for whom the planner has heterogeneous Pareto weights, are assigned consumption allocations that maximize the planner's objective function, as a way of generating and characterizing constrained Pareto optimal allocations.

It should be clear that our environment covers numerous applications such as an economy with liquid assets, production networks, income as portfolio returns, among others. Next we provide an example to illustrate this with other examples in Online Appendix B.

Example 3.3 (Policy Experiment: An Economy with Liquid Assets)**.** Consider an extension where agents face income risk, and have investment opportunities. Namely, let $\tilde{s} \in \tilde{S}$ denote the underlying aggregate state of nature. Agents have underlying income streams $e_i(\tilde{s})$, and let $e(\tilde{s}) = (e_1(\tilde{s}), \ldots, e_n(\tilde{s})) \in \mathbb{R}^n$ endowment realizations for all agents in the economy. There is a tradable, consumable asset available for purchase/sale or reallocation by the planner, with a gross return of 1, which we refer to as a "liquid asset". Thus here "market access" is not only the ability to trade with other agents, but also access to an external market, where the liquid asset can be traded for consumption goods. We regard this asset as providing liquidity, effectively a bank account with zero net interest. With more than two states s, the asset market structure here is exogenously incomplete, allowing some but not all transformation and smoothing of intertemporal consumption. Let A_i for agent i be her endowment of liquid assets in the economy. Let $t_i(\tilde{s}) \in \mathbb{R}$ be the ex-post consumption good net transfers that the social planner chooses to smooth consumption

[4] We also consider the case where the planner cannot choose $c(\cdot)$ (Online Appendix F.2).

risk. Consumption for each agent is then $c_i(\tilde{s}) = e_i(\tilde{s})$ if $\zeta_i(\tilde{s}) = 0$ and

$$c_i(\tilde{s}) = e_i(\tilde{s}) + A_i + t_i(\tilde{s}) \tag{3.5}$$

if $\zeta_i(\tilde{s}) = 1$. Transfers need to net out among trading agents; i.e.

$$\sum_{i=1}^{n} \zeta_i(\tilde{s})\, t_i(\tilde{s}) = 0 \text{ for all } \tilde{s} \in \tilde{S}.$$

To map this environment into the general, reduced form model, given the initial allocation of liquid assets A, we define $s = \left(y^A(\tilde{s}), \zeta(\tilde{s})\right)$ where

$$y_i^A(s) = e_i(s) + A_i \text{ if } \zeta_i(\tilde{s}) = 1 \text{ and } y^A(s) = e_i(s) \text{ otherwise.}$$

3.4. **Financial Centrality.** Our definition of financial centrality is motivated by the policy experiment illustrated in Example 3.3. Specifically, we define our measure of *financial centrality of an agent* i as the increment in value for the planner of providing a liquid asset to agent i whenever they can trade. The liquid asset corresponds to giving $\epsilon > 0$ to agent i each time they are in the market, so

$$\forall (\zeta, y): \; \zeta_i = 1 \implies y_i' = y_i + \epsilon.$$

This is an increase in the expectation of y_i conditional on the agent having market access ($\zeta_i = 1$). Let $V_{i,\epsilon}(\lambda)$ be the maximum value of program (3.2) given such an asset provision to agent i.

Definition 3.1. We define *financial centrality* of agent $i \in I$ as

$$FC_i := \frac{\partial V_{i,\epsilon}(\lambda)}{\partial \epsilon}\Big|_{\epsilon=0}.$$

One justification for this is that it is analogous to the diffusion case where the goal is to maximize take-up and the policymaker is interested in seeding some node with information, which plays the role of the asset. A related justification is as follows. From the planner's perspective, the agents in the economy can be thought of as assets, in the sense of Lucas Jr (1978). When a planner considers providing liquidity, the role an agent plays is to be available to trade: the agent only fulfills the role when they are available of course. This corresponds precisely to the idea of an asset that pays only in certain states—in this case being present. Consequently, the fundamental value of the asset corresponds precisely to integrating over the marginal increments in social welfare, given by the equilibrium pricing kernel, over all states where the asset pays (again, here being present).

Example (Liquid Assests (cont.): Must Be Allocated to One Agent)**.** Returning to Example 3.3 with initial endowment of liquid assets $A = \{A_i\}_{i \in I}$ and an income distribution defined as $y_i(s) = e_i(s) + \zeta_i A_i$, so agents can utilize extra liquid assets when active. Imagine now that the social planner can increase the total supply of liquid assets by $\epsilon > 0$. We show that financial centrality not only gives the marginal social value of liquidity, but also that for any total supply of liquid assets below some threshold $\bar{\epsilon}$, all the excess supply liquidity should be allocated to the agents with highest financial centrality. When FC_i is uniquely maximized, then even a non-infinitesimal amount will be allocated to only one agent.

Proposition 3.1. *Suppose $V(A)$ is differentiable and let $\phi^* = \max_j FC_j$. Let $\Delta A^*_{i,\epsilon}$ denote the optimal increase in liquid assets for agent i. Then, there exist $\bar{\epsilon} > 0$ such that if $\epsilon < \bar{\epsilon}$ then $\Delta A^*_{i,\epsilon} > 0$ if and only if $FC_i = \phi^*$.*

The intuition behind the second part of Proposition 3.1 relies on the fact that if V is differentiable at $t = 0$, then it is approximately a linear function, and hence it is locally maximized by allocating all the resources to the agent with highest marginal value, given by our notion of financial centrality. We study the large transfer case, beyond $\bar{\epsilon}$, in Section 6.2.

3.5. **Inert and Responsive to Liquid Asset Provision.** We say that an environment is *inert to the provision of infinitesimal liquid asset* if $\mathrm{P}(s)$ is constant under changes in $\mathbb{E}(y_i \mid \zeta_i = 1)$, for all $i \in I, y \in Y$ and $\zeta : \zeta_i = 1$. This would be the case if the market formation process (either exogenously determined or endogenously determined) is completely independent from the income distribution, and would have the feature that a marginal liquid asset has no effect on the market participation distribution. Consider an example wherein individuals decide whether or not to enter a market knowing the set of others who have the opportunity to participate in this state of the world; participation has some known fixed cost. In (the pure strategy maximal entry) equilibrium, all or no such agents choose to participate and in the homogenous parameter case agents' decisions purely depend on the number of other agents who have the opportunity: there is a threshold participation opportunity size above which all agents will participate. An infinitesimal liquid asset clearly cannot change this endogenous distribution of participation decisions. Further, if the equilibrium with maximal entry is inert, then so is a mixed strategy equilibrium with independent mixes.

Environments where the above property fails are models that are *responsive*. All exogenous market participation models are inert. Endogenous market participation environments may be inert or responsive, and this depends on the details of the model. An example of such an environment (which we will study) is one where agents have to decide whether to (costly) access the market or not, before observing income draws. In this environment, agents draw fixed market participation costs $k_i \geq 0$ from some distribution $G(k_1, \ldots, k_n)$ which has full support in an interval in $\mathbb{R}^n$, and decide to access the market if the expected utility of having market access (integrating over income draws and market participation decisions of other agents) net of the trading cost k_i exceeds the expected autarky value. In any equilibrium, agents will have a cutoff cost such that they only access the market for low enough k_i. This model will typically display responsiveness to infinitesimal liquidity since the asset would, in particular, increase the expected utility of for agent i from getting market access ($\zeta_i = 1$), therefore changing the equilibrium market participation distribution. In environments where agents endogenously influence the income distribution (through costly production or by choosing an investment portfolio) the liquidity can have effects on changing the income distribution at the margin. We study an environment with endogenous investment decisions in Online Appendix H.

More formally, decompose $\mathrm{P}(s) = \mathrm{P}(y)\,\mathrm{P}(\zeta \mid y)$. We want to understand how the provision of the liquid asset may affect the probability of each original state. For this, we need to write a model, which will give as a result the score of the state with respect to the small asset ϵ_i. Formally, a *model* will be a mapping $\epsilon_i \to \mathrm{P}(s \mid \epsilon_i)$, with the associated score function (with respect to a marginal provision to agent i) defined as

$$\mathbf{S}_i(s) := \frac{\partial \ln\left[\mathrm{P}(s \mid \epsilon_i)\right]}{\partial \epsilon_i} \Big|_{\epsilon_i=0}$$

which (if desired) can be decomposed into the score of the income distribution, and the score of market participation (given income) simply as $\mathbf{S}_i(s) = \mathbf{S}_i(\zeta \mid y) + \mathbf{S}_i(y)$, where the former term is the *market participation effect* and the latter term is the *income effect*.

An environment will be *inert* when $\mathbf{S}_i(s) = 0$ almost surely and *responsive* otherwise. An important thing to note is that when ζ is independent of y, this does *not* mean that the score turns to zero: but rather that it gets simplified to $\mathbf{S}_i(s) = \mathbf{S}_i(\zeta) + \mathbf{S}_i(y)$.

4. Results

4.1. **Financial Centrality and Network Structure.** We begin by demonstrating that financial centrality can be written as two terms composed of the inert and responsive components. All proofs are in Appendix A unless otherwise noted.

Proposition 4.1. *Suppose the environment is responsive to infinitesimal liquidity injection, and that $c(\cdot)$ solves program (3.2). Then financial centrality can be written as:*

$$FC_i := \underbrace{\mathbb{E}_s\left\{\zeta_i q(s)\right\}}_{\textit{inert component}} + \underbrace{\mathbb{E}_s\left\{\sum_{i\in I} \lambda_i u_i(c_i(s)) \times \mathbf{S}_i(s)\right\}}_{\textit{responsive component}}.$$

It is instructive to decompose this into three effects: (i) risk sharing; (ii) participation; (iii) income distribution. Since we can write $\mathbf{S}_i(s) = \mathbf{S}_i(\zeta \mid y) + \mathbf{S}_i(y)$, we can separate the responsive component and write financial centrality as a function of three interpretable quantities.

$$FC_i := \underbrace{\mathbb{E}_s\left\{\zeta_i q(s)\right\}}_{\text{risk sharing effect}} + \underbrace{\mathbb{E}_s\left\{\sum_{i\in I} \lambda_i u_i(c_i(s)) \times \mathbf{S}_i(\zeta \mid y)\right\}}_{\text{participation effect}} + \underbrace{\mathbb{E}_s\left\{\sum_{i\in I} \lambda_i u_i(c_i(s)) \times \mathbf{S}_i(y)\right\}}_{\text{income distribution effect}}.$$

In order to explore this further, let us assume that we are in the simplest possible case where all agents are identical, each have the same Pareto weight, and income is drawn i.i.d. Let $n_\zeta := \sum_i \zeta_i$ be the market size at market ζ, and $h(\zeta) := \mathbb{E}_y\left[q(y,\zeta)\right]$. We characterize financial centrality in this environment and it is easy to see that here we have inertness.

Proposition 4.2. *Suppose $u_i = u$ and $\lambda_i = 1/n$ for all i, and income draws are independent and identically distributed across agents. Then $q(s) = u'\left[\overline{y}(s)\right]$ and $c_i(s) = \zeta_i \overline{y}(s) + (1-\zeta_i) y_i$. Moreover, if $u(\cdot)$ is analytic then we can approximate $h(\zeta) \approx u'(\mu)\left(1 + \gamma\frac{\sigma^2}{n_\zeta}\right)$, where $\gamma := (1/2) u'''(\mu)/u'(\mu)$. Therefore,*

$$FC_i \propto \mathrm{P}(\zeta_i = 1) \times \left[1 + \gamma\sigma^2 \mathbb{E}\left(\frac{1}{n_\zeta} \mid \zeta_i = 1\right)\right]. \tag{4.1}$$

An intuition is as follows. Centrality can be decomposed into two pieces. Financial centrality is higher when (1) the agent has a higher probability of trading ($\mathrm{P}(\zeta_i = 1) \uparrow$) and (2) the market size conditional on the agent entering is smaller. Finally, the degree to which each of these matters can depend on features that characterize the vulnerability of the members of the realized component: the mean income, degree of risk aversion, degree of prudence (convexity of marginal utility of consumption, which governs precautionary savings), and variability of income (measured by the coefficient of variation).

We generalize this considerably in Section A.2, particularly in Proposition A.1 to allow for heterogenous preferences, volatilies of income, correlations in income, and aggregate shocks. Here we show that in the case where $u_i(c) = -r_i^{-1} \exp(-r_i c)$ and $y \sim \mathcal{N}(\mu, \Sigma)$ we can obtain a closed form expression for financial centrality:

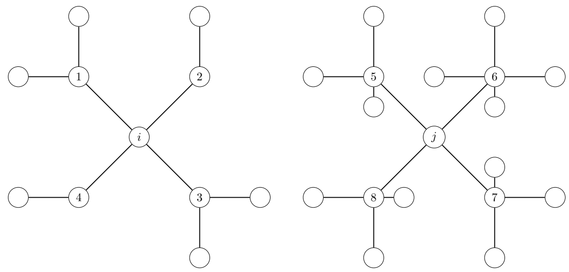
$$FC_i = \mathbb{E}_\zeta \left\{ \zeta_i \exp\left(-\overline{r}_\zeta \overline{\mu}_\zeta\right) \overline{\lambda}_\zeta \exp\left(\frac{\overline{r}_\zeta^2}{2} \times \frac{\overline{\sigma}_\zeta^2}{n_\zeta}\right) \right\},$$

where $\overline{\mu}_\zeta := n_\zeta^{-1} \sum_j \zeta_j \mu_j$ and $\overline{\sigma}_\zeta^2 := n_\zeta^{-1} \sum \zeta_j \zeta_k \sigma_{jk}$ are, respectively, the average mean and variance of income for the agents present at market ζ, $\overline{r}_\zeta = \left(n_\zeta^{-1} \sum \zeta_j r_j^{-1}\right)^{-1}$ is the harmonic mean of the absolute risk aversion of agents present at ζ, and $\overline{\lambda}_\zeta = \exp\left[n_\zeta^{-1} \sum \zeta_i \left(\overline{r}_\zeta / r_j\right) \ln\left(\lambda_j\right)\right]$ is a geometric weighted average of the Pareto weights of agents present at ζ, weighted by how risk averse they are compared with the market average.

The robust implications of this are as follows. First, again we see that agents who tend to participate when the component (or trading room) is has few nodes are more financially central. Second, agents who tend to participate when those whom the planner values more are more central. Third, those who participate when there is greater volatility are more central. This could be because the agents have more volatility themselves or even because agents have more positively correlated incomes so the aggrate exhibits greater variance. Fourth, agents are more central if the average agent in the market has a lower endowment in expectation when the agent in question is in the market. Fifth, agents are more central if the degree of risk aversion when the agent is in the market is higher. Overall, the notion of financial centrality captures a generalized notion of market thinness. The planner values the agents precisely whom are able to provide transfers to others who need it when they are in particular dire need. In doing so, the planner takes into account who will be in the realized component in equilibrium. Though this is intuitive, it provides an economically relevant relationship between our fundamentals and a notion of centrality.

Example 4.1 (Inviting Neighbors in SG (Special Case of Example 3.1))**.** We consider a stylized example of market formation where individuals are called to trade with their neighbors in SG. We provide richer examples in Online Appendix D. Imagine that when an agent i is called to trade in some state, they invite all their friends. That is, the entire neighborhood of agent i in G.[5] In such a case, the resulting SFN is simply a distribution over all neighborhoods of agents $j = 1, ..., n$ in the network with weights given by the probability that j is called to be the market organizer.

[5]To give an example for the case of endogenous participation, imagine this graph to be such that for every node every neighborhood represents the set of individuals who have the opportunity to participate and in equilibrium this neighborhood does attend the market. In such settings, it is without loss to proceed as if participation is exogenous.



(A) (B)

FIGURE 2. Two agents in a large social graph G (not necessarily fully pictured), i and j, in Panels (A) and (B) respectively. Many typical measures of centrality (e.g., eigenvector-like centralities) would treat j as being more central than i, ceteris paribus, since they have the same degree and j's neighbors' degrees are higher. Our notion of financial centrality ranks $FC_j < FC_i$.

More specifically, let $d_i := \sum_j G_{ij}$ denote the degree of node i (here with $G_{ii} = 1$) and let $N_i := \{j \in I : G_{ij} = 1\}$ denote the neighborhood of i. Participation is drawn as follows. With probability $z_i = \frac{1}{n}$, each agent is selected to be the host. Then $\zeta_i = 1$ and also $\zeta_j = 1\{j \in N_i\}$. We can compute financial centrality as

$$FC_i = \frac{1}{n}\left\{d_i + \gamma\sigma^2 \sum_j \frac{G_{ij}}{d_j}\right\}.$$

Agents who have larger neighborhoods in SG are more financially central (from the d_i term) in the usual way, but in particular holding that fixed agents that have neighbors who have smaller neighborhoods are more financially central (from the $\frac{1}{d_j}$ term). The notion of financial centrality derived from our model may be quite different from traditional notions of centrality, such as degree, betweenness, eigenvector-like (e.g., Katz-Bonacich) centralities, among others. To see the contrast, observe that rather than one's centrality increasing in the degree of ones' neighbors, one's centrality declines if ones' neighbors have higher degree.

In Figures 2a and 2b, we compare two agents i and j in different parts of a large network (so n is the same for both of them). Observe that agent j in Figure 2b is more central than the one in Figure 2b, i, according to most commonly used centrality measures, since they can reach more agents in the same number of steps (higher eigenvector centrality, for example). However, the agent j is less financially central in the induced stochastic financial network than i, since (a) it has the same probability of having market access, but (b) the markets they have access to are bigger (in the first order stochastic dominance sense) to those that agent i reaches, and is hence less important. This is because of the logic of consumption variance reduction: a dollar given to the agent i will reduce consumption variance a lot more than agent j.

We conclude this section with an observation on the case of segmented markets where the stochastic financial network generates states wherein by chance or choice there are multiple non-trivial components of active agents in the network. Recall that for some realized partition $\pi \in \mathcal{P}$, the list of components was given by $M_\pi = \{m_1, m_2, \dots m_r\}$. In Proposition F.1 (Online Appendix F.1) we show that the centrality formula for inert environments is equivalent to the centrality measure in an economy where we only consider "active markets" where agent i is present (since they cannot affect the consumption of agents in other markets). In particular, using the CARA-Normal model as before, but now with market segmentation, financial centrality is essentially the same. Let $m_i = m_i(s)$ be a random variable denoting the market that i is active in, in a given state s. Then the financial centrality is

$$FC_i = \mathbb{E}_s \left\{ \zeta_i \exp\left(-\overline{r}_{m_i} \overline{\mu}_{m_i}\right) \overline{\lambda}_{m_i} \exp\left(\frac{\overline{r}^2_{m_i}}{2} \times \frac{\overline{\sigma}^2_{m_i}}{|m_i|} \right) \right\}$$

where m_i is the market where i is participating, and for any market $m \subseteq I$, we define the average volatility as $\overline{\sigma}^2_m := \frac{1}{|m|} \sum_{\{j,k\} \subseteq m} \sigma_{jk}$, $\overline{r}_m = \left(\frac{1}{|m|} \sum_{j \in m} \frac{1}{r_i} \right)^{-1}$ is the (harmonic) average risk aversion in market m, and $\overline{\lambda}_m := \exp\left(\frac{1}{|m|} \sum_{j \in m} \frac{\overline{r}_m}{r_j} \ln\left(\lambda_j\right) \right)$ is the geometric average of the agents Pareto weights, weighted by their relative risk aversion (with respect to the market average $\overline{r}_m$. The key point is that now centrality for an agent can be thought as in reference to only their interactions. The single relevant market in each state when calculating their value to the planner is simply the component they are in and none other.

5. Foundations for Financial Centrality

In this section we provide two foundations. First, we look at an Arrow Debreu economy. We demonstrate that, as defined, financial centrality is the price of personalized debt. Second, we take seriously the view that agents may interact in a cooperative bargaining phase underlying the risk-sharing division of surplus. Asymmetries in the social graph, along with features of the economic environment, may privilege some agents above others. We show that the Pareto weights implied in the planner's problem describing the equilibrium allocations coming from this bargaining process exactly capture financial centrality. That is, if agents bargained over division of surplus and then parttook in risk-sharing through the SFN, exactly those who are more financially central in our sense would be those who receive higher Pareto weight in the planner's problem corresponding to this outcome.

5.1. **An Arrow Debreu Economy: Financial Centrality as the Price of Personalized Debt.** First we study an Arrow Debreu economy. The main assumptions we need for results in this section are inert market participation and that income and market participation shocks are independent; without this assumption, there could be non-pecuniary externalities in the market participation decision which will not be reflected in the equilibrium prices (i.e., we may lose the constrained-efficiency result).

We consider an Arrow Debreu economy, where agents can buy and sell claims on income and consumption, contingent on the configuration of the market and the nature of income shocks. However, agents cannot buy or sell income claims that will pay off in states where they are unable to trade (since there is no physical way to make such transfers), which we formalize as "consumption space shocks" as in Mas-Colell et al. (1995). Formally, let $A_{\hat{s}}$ denote the Arrow Debreu (AD) asset[6] that pays 1 unit of the consumption good if the state is $s = \hat{s}$, nothing if $s \neq \hat{s}$, and $a_i(\hat{s}) \in \mathbb{R}$ the demand of asset $A_{\hat{s}=s}$ by agent i. Indexing on the named player allows those in the market to acquire the liquid asset held by that player.

Consumption for agent i at state $s = (y, \zeta)$ is then $c_i(s) = y_i + a_i(s)$. The market participation constraint can be introduced by imposing a *physical constraint*: whenever $\zeta_i = 0$ we must have $a_i(s) \in \{0\}$ (i.e., agents cannot trade in assets that they will not be able to be present in the market to clear the trades ex-post).

To simplify proofs and exposition, we consider cases where there is only a countable number of possible income shocks, so that $S = \prod_i (Y_i \times \{0,1\})$ is also countable, and where $\mathrm{P}(s \in S) > 0$ for all $s \in S$. Given Arrow Debreu prices $\hat{r}(s)$ for each A_s and a vector of lump sum transfers $\tau = (\tau_i)_{i \in I}$, such that $\sum_{i \in I} \tau_i = 0$, agents choose consumption and asset purchases to maximize expected utility, given her budget constraint:

$$\max_{\{c_i(s), a_i(s)\}} \mathbb{E}_s \{u_i [c_i(s)]\} \tag{5.1}$$

$$s.t: \begin{cases} c_i(s) = y_i(s) + a_i(s) & \text{for all } s \in S \\ a_i(s) = 0 & \text{for all } s \in S : \zeta_i = 0 \\ \sum_{s \in S} a_i(s) \hat{r}(s) \leq \tau_i. \end{cases} \tag{5.2}$$

As we did when defining the Lagrange multipliers for the planning problem, we normalize the price function as $r(s) = \hat{r}(s) / \mathrm{P}(s)$, changing the budget constraint in the consumer problem as

$$\mathbb{E}_s [a_i(s) r(s)] := \sum_{s \in S} a_i(s) r(s) \mathrm{P}(s) \leq \tau_i. \tag{5.3}$$

A *Walrasian Equilibrium with transfers* τ is a triple (c, a, r) $=(\{c_i(s), a_i(s)\}_{i \in I, s \in S}, \{r(s)\}_{s \in S})$ such that

- $\{c_i(s), a_i(s)\}_{s \in S}$ solves (5.1) with budget constraint (5.3) for all $i \in I$, given (normalized) prices $r(s) = \hat{r}(s) / \mathrm{P}(s)$ and $\tau = (\tau_i)_{i \in I}$,
- asset markets clear: $\sum_{i \in I} a_i(s) = 0$ for all $s \in S$,[7] and
- consumption good markets clear: $\sum_{i \in I} \zeta_i c_i(s) \leq \sum_{i \in I} \zeta_i y_i$ for all $s \in S$.

In Proposition 5.1, we show a version of the First and Second Welfare Theorems for this economy, which is an application of the classical welfare theorems to this environment (Mas-Colell et al. (1995)). This can be qualified as a welfare theorem with "constrained efficiency," since the constraint that lack of market access (i.e., $\zeta_i = 0$) implies autarkic consumption is interpreted as a physical constraint (i.e., a social planner could not change an inactive agent's consumption either).

Proposition 5.1 (Welfare Theorems). *Suppose the environment exhibits market participation inert to provision of an infinitesimal liquid asset, with $\zeta \perp y$. Take a planner's problem (3.2) with*

[6]Formally, $A_{\hat{s}}(s) = \begin{cases} 1 & \text{if } s = \hat{s} \\ 0 & \text{otherwise} \end{cases}$ is the return matrix of the AD security paying only at state $\hat{s}$.

[7]If $\exists \tilde{s} \in S : \mathrm{P}(\tilde{s}) = 0$, then we can interpret this condition as imposing the constraint that $a_i(\tilde{s}) = 0$ for all $i \in I$ (i.e., agents cannot trade in probability zero events).

Pareto weights $\lambda \in \Delta^n$, and an optimizing allocation $c = (c_i(s))_{i \in I, s \in S}$, with normalized Lagrange multipliers $q(s)$ (as defined in (A.1)). Then, (c, r) is a Walrasian Equilibrium with transfers τ, where $r(s) = q(s)$ for all $s \in S$ and $\tau_i = \mathbb{E}_s \{[c_i - y_i(s)]\, q(s)\}$. On the other hand, if (c, r) is such an equilibrium with transfers τ, then there exist Pareto weights $\lambda \in \Delta^n$ such that c is the allocation solving planner's problem (3.2) (where we again have $q(s) = r(s)$).

We provide a proof in Online Appendix E. An important Corollary of Proposition 5.1 (and most classical proofs of Second Welfare Theorems in various settings) is that it gives us an explicit formulation for the equilibrium Arrow Debreu security prices at the implementing equilibrium, which coincide with the shadow values $q(s)$ at the resource constraint at each state s.[8] Since we can interpret this economy as one with complete markets (once we interpret market participation shocks as consumption sets shocks) $r(s)\,\mathrm{P}(s)$ is a *pricing kernel*, which greatly simplifies the pricing of additional assets, if available to the market. More explicitly, if we add to this economy, on top of the Arrow Debreu securities offered, an asset with return payoff function $\rho(s) \in \mathbb{R}$, its (no arbitrage) equilibrium price in this economy would be

$$\text{Price} = \mathbb{E}_s [\rho(s) \times r(s)] := \sum_{s \in S} \rho(s)\, r(s)\, \mathrm{P}(s)\,.$$

Using the results from Proposition 5.1, financial centrality can be thought as the equilibrium price of an asset (which we dubbed personalized debt) with return payoff matrix $\rho^i(s) = 1$ if $s : \zeta_i = 1$.

Proposition 5.2. *Suppose $y \perp \zeta$ and let (c, r) be the Walrasian Equilibrium with transfers $\tau = (\tau_i)_{i \in I}$ that implements the planner's problem (3.2) optimal allocation c with Pareto weights $\lambda \in \Delta^n$. Then*

$$FC_i = \sum_{s \in S} \rho^i(s)\, \hat{r}(s) = \sum_{s \in S} \rho^i(s)\, r(s)\, \mathrm{P}(s)\,.$$

That is, financial centrality is the price of a personalized debt asset implementing Walrasian Equilibrium with transfers.

5.2. **Bargaining Foundations.** We next study a foundation where agents engage in ex-ante cooperative (Nash) bargaining. We show that there is a positive linear relationship between the "representing Pareto weight" of an agent—the weight that would be assigned under the planner's problem that implements the outcome—and her financial centrality measure. Suppose agents decide the social contract by bargaining ex-ante among themselves. Agents receive an expected utility $U_i = \mathbb{E}[u_i(c_i(s))]$ in a contract. If they reject the proposed social contract, then agents get their "disagreement point," or autarky value, $U_i^{aut} = \mathbb{E}_{y_i}[u_i(y_i)]$. The social contract is the choice of a feasible consumption allocation $c(s) = \{c_i(s)\}_{s=(y,\zeta)}$. If the bargaining process satisfies Pareto optimality, linearity in utilities, and independence of irrelevant alternatives, then there exist weights $(\alpha_i) \in \Delta^n$ such that the bargaining solution solves

$$c(s) = \arg\max_{\hat{c}(s)} \prod_{i \in I} \{\mathbb{E}_s \{\zeta_i u_i [\hat{c}(s)] + (1 - \zeta_i)\, u_i(y_i)\} - \mathbb{E}_{y_i}[u_i(y_i)]\}^{\alpha_i}$$

[8]If the environment had endogenous participation where agents choose whether or not to trade, as in Section C.2, then there typically will be pecuniary externalities from this choice. This will not be reflected in the equilibrium prices. A richer model where agents could pay others for their market participation (e.g., a Lindahl equilibrium) would restore efficiency.

subject to $\sum_i \zeta_i \hat{c}_i(s) \leq \sum_i \zeta_i y_i$. This is equivalent to solving the following program:

$$\max_{c(s)} \sum_{i \in I} \alpha_i \ln \left\{ \mathbb{E}_s \zeta_i \left[u_i \left(c_i(s) \right) - u_i \left(y_i \right) \right] \right\}$$

$$\text{s.t}: \sum_{i \in I} \zeta_i c_i(s) \leq \sum_{i \in I} \zeta_i y_i \text{ for all } (y, \zeta).$$

Holding everything else fixed, agents with higher financial centrality have also higher Pareto weights. If agents bargain over risk sharing contracts, holding autarky as a threat point of the negotiation, then agents with higher centrality should have higher portions of aggregate income.

Proposition 5.3. *Suppose $u_i(c) = -r_i^{-1} \exp(-r_i c)$ and $y \sim \mathcal{N}(\mu, \Sigma)$. Then, the Pareto weights associated with the asymmetric Nash bargaining solution with bargaining weights $\alpha \in \Delta^n$ satisfy the following fix point equations:*

$$\lambda_i = \frac{\alpha_i r_i + FC_i(\lambda)}{\mathrm{P}(\zeta_i = 1) \cdot \exp\left(-r_i \mu_i + \frac{r_i^2}{2} \sigma_i^2\right)} \text{ for all } i. \tag{5.4}$$

In particular, for the symmetric Nash bargaining solution ($\alpha_i = 1$) with homogeneous preferences and i.i.d. income, we show that the representing Pareto weights need not uniform ($\lambda_i \neq 1/n$) but rather satisfy

$$\ln(\lambda_i) = \kappa + \ln\left[r + FC_i(\lambda)\right] - \ln\left\{\mathrm{P}(\zeta_i = 1)\right\} \tag{5.5}$$

so the heterogeneity in the market participation process has a bite. In Appendix (A.3.2), we study two alternative foundations: an alternative asymmetric Kalai-Smorodinsky solution, which delivers a similar relationship between centrality and bargaining weight (Proposition A.2) and a Walrasian General Equilibrium model (Online Appendix E).

Example 5.1 (Empirics; Village Risk Sharing)**.** Next, we study the empirical content of our theory. To do this, we look at the Townsend Thai village data over 15 years. This data follows 338 households across 16 villages where we have detailed data on consumption, income, and transactions across villagers (Townsend, 2016). In particular, in this setting we have variation in the number of transactions per time period. A complete empirical analysis of the patterns of risk sharing in these villages, motivated by and based on the framework outlined in this paper, can be found in (Kinnan et al., 2019).

Our theory has a unique prediction. Those that provide more value—higher measures of financial centrality—are exactly those that are in the market when the market is thin (in a generalized sense including few active traders and greater per-trader-volatility in income). And those who are more central in this sense claim a greater share of the surplus, in this case higher average consumption.

We proceed in two steps. First, develop a measure that reflects FC_i. As shown above, if Pareto weights are determined by bargaining, then a more financially central individual i has a higher Pareto weight λ_i. Though we do not observe financial centrality, we can use observations on consumption in panel data to obtain an estimate of a function for each agent i which is monotonically increasing in the Pareto weight λ_i. We can estimate a regression of consumption on household

TABLE 1. Do Pareto weights correlate with measures of market thinness when the agent is active?

VARIABLES	(1) α_i	(2) α_i	(3) α_i
ρ_i^ζ	0.095		0.112
	(0.041)		(0.045)
ρ_i^σ		0.103	0.118
		(0.050)	(0.051)
Observations	338	338	338

Robust standard errors in parentheses. The dependent variable is a (mean zero, standardized) Pareto weight estimate of a given household, obtained from using the vectors of household fixed effects from a regression of consumption on household income. Regressors are each standardized as well.

income, using only active periods

$$c_{ivt} = \alpha_i + \beta y_{ivt} + \delta_{tv} + \epsilon_{ivt}$$

where t is time, α_i is a household fixed-effect, and δ_{vt} is a village-by-time fixed-effect. Under CARA utility the α_i is a monotone function of Pareto weights λ_i.

We also know from our theory the crucial component in our financial centrality measure is market thinness. So we compute measures of market thinness for each household. We can observe the number of active agents in a given village in a given period as well as the volatility due to the composition of active agents. As such, we define

$$\rho_i^\zeta := \mathrm{cov}_t\left(\zeta_{it}, \frac{1}{n_{vt}}\right) \text{ and } \rho_i^\sigma := \mathrm{cov}_t\left(\zeta_{it}, \bar{\sigma}_t\right),$$

where n_{vt} is the number of active participants in period t in a village v, computed from the transfers data as mentioned, and where $\bar{\sigma}_t^2 := \frac{1}{n_{\zeta_t}} \sum_{i,j} \zeta_{it}\zeta_{jt}\widehat{\sigma}_{i,j,t}$ is an estimate of the volatility at period t, where $\hat{\sigma}_{i,j,t}$ is the measured covariance between households i and j's income.

To study the prediction of the theory, we study a regression

$$\alpha_i = \beta_0 + \beta_1\rho_i^\zeta + \beta_2\rho_i^\sigma + u_i.$$

Our theory suggests that $\beta_1 > 0$ and $\beta_2 > 0$ as being present in generalized thin markets corresponds to higher endogenously determined Pareto weight and therefore higher average consumption. This is an observational claim, but it is not mechanical: that those who are present exactly when the market is thin tend to receive a greater mean consumption, is consistent with our model.

Table 1 presents the results. Columns 1-2 include each measure of market thinness when the agent enters one-by-one and column 3 includes them together. We see a one-standard deviation increase in the tendency to enter when the market is thin in numbers is associated with a corresponding 0.095 standard deviation increase in mean consumption (column 1, $p = 0.021$). Similarly, a one-standard deviation increase in the tendency to enter when the market is thin in the sense of high volatility is associated with a corresponding 0.103 standard deviation increase in the mean consumption (column 2, $p = 0.04$). These estimates are stable to being jointly included (column 3).

Taken together, the results are consistent with a story where agents have determined Pareto weights through a bargaining process, and those who have higher weights and therefore higher financial centrality are precisely those who tend to be active traders when the market is thin either in terms of numbers of individuals or volatility. This observation is new to the literature and, to our knowledge, unique to our model.

6. Extensions to Endogenous Entry and Large Transfers

6.1. **Choosing to Enter the Market.** While we have focused on environments that were inert to an infinitesimal liquidity injection in the above, we show an example where there is a shift in the participation decision itself. We provide several other examples of endogenous participation, showing when this is responsive or inert to liquidity injection, in Online Appendix C. In this example, the consumption allocation $c = (c_i(s))_{i\in I, s\in S}$ is common knowledge, but agents have random market participation costs, which are privately observed. Formally, agents observe a cost $k_i \in K_i$, and costs are jointly distributed according to distribution $G(k)$ with full support in an interval in $\mathbb{R}^n$, and are independent of the income shocks y.

Given the consumption allocation, an equilibrium market participation is a set of mappings $\zeta_i^* : K_i \to \{0,1\}$ such that if $\zeta_i^*(k_i) = 1$ then

$$\mathbb{E}_{y,k}\left\{u_i\left[c_i\left(y, \zeta_{-i}^*(k_{-i})\right)\right] - u_i(y_i) \mid k_i\right\} \geq k_i. \tag{6.1}$$

That is, it is a Bayesian Nash Equilibrium in an incomplete information game where agents's strategies are their market participation decisions.

In this example, we assume $u_i = u$ for all i, $\lambda_i = 1/n$ for all n, and $y_i \sim \mathcal{N}(\mu, \sigma^2)$ i.i.d., so that $c_i = \overline{y}_\zeta$ whenever $\zeta_i = 1$. We also assume that the conditional distribution of $k_{-i} \mid k_i$ is FOSD increasing in k_i.[9]

Under these assumptions, we show that

(1) $\zeta_i(k_i) = 0$ for all k_i is the lowest participation equilibrium.
(2) There exist thresholds $\overline{k} = \left(\overline{k}_i\right)_{i\in I}$ and an equilibrium $\overline{\zeta}(k)$ such that $\overline{\zeta}_i(k_i) = 1$ if and only if $k_i \leq \overline{k}_i$. Moreover, $\overline{\zeta}_i(k_i) \geq \zeta_i^*(k_i)$ for all $k \in K^n$, all agents $i \in I$, and for any other BNE participation strategy. In what follows, we characterize the market participation equilibrium with highest market participation (i.e., highest n_ζ) for all realizations of private costs $\overline{\zeta}(k)$.

Define k_m^* as the threshold if agents had complete information about the market size: $k_m^* := \mathbb{E}_y\{u(\overline{y}_m) - u(y_i)\} \geq 0$.[10] Also, for $m \leq n$ define $\pi\left(m, \overline{k}\right) := \mathrm{P}\left(\sum_i \overline{\zeta}_i(k_i) = m\right)$ as the probability distribution over market size (m) given the threshold policies $(\bar{k})$, which can be written as a function of the thresholds $\overline{k}$ as $\pi\left(m, \overline{k}\right) = \sum_{J:|J|=m} \mathrm{P}\left(k_j \leq \overline{k}_j \forall j \in J \text{ and } k_h > \overline{k}_h \forall h \notin J\right)$. Equation (6.1)

[9]This is satisfied, for example, if costs are independent, $k_i \sim G_i(k_i)$ for all i. It is also satisfied if $k_i = K + \xi_i$, where K is a common random variable, and $\xi_i \sim_{i.i.d} F(\xi)$ with zero mean.

[10]Because $u(\cdot)$ is strictly increasing and concave, and $\overline{y}_m$ is a mean preserving spread of $\overline{y}_z$ with $z \leq m$ (since $\overline{y}_m \sim \mathcal{N}\left(\mu, \sigma^2/m\right)$).

can be used to obtain a fix point equation for the thresholds $\overline{k}$:

$$\overline{k}_j = \Psi_j\left(\overline{k}, \epsilon_i\right) := \sum_{m\leq n} \mathbb{E}_y \left\{ u\left(\overline{y}_m + \overline{\zeta}_i\left(k_i\right)\frac{\epsilon_i}{m}\right) - u\left(y_i\right)\right\} \times \pi\left(m, \overline{k} \mid \overline{k}_j\right)$$

where $\pi\left(m, \overline{k} \mid k_j\right) = \mathrm{P}\left(\sum_i \overline{\zeta}_i\left(k_i\right) = m \mid k_j\right)$. Finally, let $\mathbf{J}_k = \left[\frac{\partial \Psi_i}{\partial k_j}\right]_{i,j\in I}$ be the Jacobian (with respect to k) of the above vector Ψ, and define the matrix of *distributed cross-centralities* $\mathbf{F}_{n\times n}$ as

$$\mathbf{F}_{ij} = \mathbb{E}_s\left[\zeta_i\zeta_j\frac{q\left(s\right)}{n_\zeta} \mid \overline{k}_j\right] \text{ where } \zeta_j = \overline{\zeta}_j\left(k_j\right).$$

Intuitively, when agent i gets ϵ extra unit of income, then in the optimal equilibrium allocation, if agent j is also present, j obtains ϵ/n. This extra income increases expected utility by $q\left(s\right)$. Proposition 6.1 shows the decomposition of centrality into risk sharing and participation effects.

Proposition 6.1. *Under the above assumptions,*

$$FC_i = \mathbb{E}_s\left[\zeta_i q\left(s\right)\right] + \Lambda'\left(I - \mathbf{J}_k\right)^{-1}\cdot\mathbf{F}^{(i)} = \mathbb{E}_s\left[\zeta_i q\left(s\right)\right] + \Lambda'\sum_{t\in\mathbb{N}}\left[\mathbf{J}_k\right]^t\cdot\mathbf{F}^{(i)}$$

where $\mathbf{F}^{(i)} = \left(\mathbf{F}_{i1}, \mathbf{F}_{i2}, \ldots \mathbf{F}_{in}\right)$ *and* $\Lambda = \left(\Lambda_j\right)_{j\in I}$ *where* $\Lambda_j := \sum_{m=1}^n mk_m^*\frac{\partial\pi\left(m,\overline{k}\right)}{\partial\overline{\overline{k}}_j} \geq 0$.

The risk-sharing component is as usual. The participation effect can be interpreted as follows. Consider a term $\left[\mathbf{J}_k^t\right]_{ij}$. If $t = 1$, this directly encodes the change in the participation of i when j's threshold cost of entry changes infinitesimally. For higher t, as is usual for such positive matrices, this encodes a (weighted) chain of terms. If $t = 2$, it is easy to see it now sums over every chain, $\sum_l \frac{\partial\Psi_i}{\partial k_l}\frac{\partial\Psi_l}{\partial k_j}$, which captures both the change in the participation decision of i due to the increase in cost for l as well as change in participation for l due to an increase in cost for j. This can be thought of as a chain rule, or the indirect effect of distance 2 by increasing the equilibrium threshold cost for j. Now more generally for higher orders of t, this encodes larger chains. This is typical of numerous notions of network centralities in the literature and analogously our (weighted) endogenous network here is $\mathbf{J}_k$.

The more subtle feature here is that not only do chains of participation effects matter, but also these are weighted by the very effect of the liquid asset itself. A typical eigenvector-like centrality for adjacency matrix G would be of the form $x \propto \sum_t \left(\theta G\right)^t\cdot 1$ where $\theta < 1$ and x_i is the centrality. Here all paths from i to js of t lengths are counted and added up. In our case, we do not add up the terms with equal weight, but rather weight by $\frac{\partial\Psi_j}{\partial\epsilon_i}$—the change in j's participation decision due to the injection itself. (In the proof we show that the above term is equivalent, $\mathbf{F}_{ij} = \frac{\partial\Psi_j}{\partial\epsilon_i}$.) So returning to an overall term, we can write the participation effect as

$$\sum_j \Lambda_j\frac{\partial\Psi_j}{\partial\epsilon_i}\left\{\sum_{t\in\mathbb{N}}\left[\mathbf{J}_k\right]^t\right\}_{ji}.$$

The interpretation is clear. It takes the weighted direct and indirect effects of the marginal change in participation due to a cost increase but then weights the effect of i on every other agent in the network by how much their participation is also directly affected by the asset provision itself, holding the entry cost fixed.

To understand the intuition, consider the following simplified case. Imagine that i was the only agent with endogenous entry (i.e., all other agents that may have market access when i enters have costs that are negative so entry is free or above the threshold for entry so they never enter). In this case, numerous terms drop from the above, and so the participation effect of financial centrality immediately becomes

$$\Lambda_i \cdot \mathbb{E}_s \left\{ \zeta_i q\left(s\right) \frac{1}{n_\zeta} \right\}.$$

This is a monotone function of the risk-sharing effect of financial centrality meaning the same agents who are financially central without the endogenous effect will be financially central with such an effect. Of course, the more general case involves the network of effects characterized above.

6.2. **Large Transfers.** We now consider a thought experiment where a larger transfer T can be distributed to a subset of all agents and define financial centrality in this large-transfer setting. We show that the intuition studied in the small-transfer case holds true for non-marginal transfers.

We consider increasing the endowment of a subset of agents $J \subseteq I$ across all values of income, whenever they can trade, by a total amount $T > 0$ to finance this increase. The policy consists of offering a "credit line" but really a transfer, contingent only on participation and without any repayment obligations. Then $t = \left(t_j\right)_{j \in J} \geq 0$ changes the income process for agent $j \in J$ to $\hat{y}_j\left(s\right) = y_j + \zeta_j t_j$ for all $s = \left(\zeta, y\right)$ with $\sum t_j = T$. This is a commitment to a named trader j without knowing what situation the trader will be in.

If $V\left(t\right)$ is the maximization problem's value function, with income process $y_j = \hat{y}_j$, the planner would choose $t = \left(t_j\right)_{j \in J} \geq 0$ to solve

$$\max_{t \in \mathbb{R}_+^{|J|}} V\left(t\right) \text{ s.t } \sum_{j \in J} t_j \leq T. \tag{6.2}$$

Note that $V\left(t\right)$ here is a general value function, which could come from the corresponding solution V of program (3.2), but this not required. This allows us to define financial centrality more generally.

Definition 6.1. We define *financial centrality for total transfers* T of agent $i \in I$, where $t^\star$ is a maximizer of program (3.2), as

$$FC_i^T := V_i\left(t^*\right) = \frac{\partial V}{\partial t_i} \mid_{t=t^*} .$$

The financial centrality for total transfers T is defined relative to a hypothetical transfer of T and computes the relative gain in the value due to rewarding agent i with a transfer t_i for any maximizing transfer vector t^* such that $\sum_j t_j^* = T$.[11]

Any allocation that maximizes the objective function must, when giving a transfer to a set of agents K, not benefit at the margin by providing transfers to a set of agents $J \backslash K$. We show that in fact there will be a cutoff where the (endogenously determined) set of agents who are provided non-zero transfer will be financially more central than all other agents who receive no transfers in equilibrium. Further, if the total to be transferred is small enough, then the unique solution

[11] Program (6.2) will typically have a unique solution in our applications. However, if there is more than one maximizing transfer scheme, the choice of where to evaluate V for the definition of centrality is irrelevant, as long as $V\left(T\right)$ is differentiable (see Corollary 5 in Milgrom and Segal (2002)).

is to provide the entirety of T to a single agent rather than a subset of agents, and in this case $FC_i^T \approx \frac{\partial V}{\partial t_i}\mid_{t=0}$ and the agent has the highest financial centrality, which corresponds to the leading case we have been studying earlier in the paper. When we evaluate centrality at $T=0$ (and hence $t^*=0$ is the only possible solution) we write (with some abuse of notation) simply $FC_i^{T=0} = FC_i$.

Next, we consider the situation where the transfers $T>0$ are non-trivial in size but in the case of the CARA preferences with risk aversion parameter r, normally distributed endowments with variance covariance matrix (σ_{ij}), and heterogenous Pareto weights λ_i. We show that if $t_i^*>0$ then we can calculate the financial centrality for transfer T for agent i as

$$FC_i^T = \sum_{\zeta\in\{0,1\}^n} \mathrm{P}(\zeta) \underbrace{\overline{\lambda}_\zeta \exp(-r\overline{\mu}) \exp\left(r^2 \frac{\overline{\sigma}_\zeta^2}{2n_\zeta}\right)}_{:=h(\zeta)} \exp(-r\bar{t}_\zeta).$$

Here $\overline{\lambda}_\zeta = \exp\left[n_\zeta^{-1}\sum_j \zeta_j \ln(\lambda_j)\right]$ is the simple geometric average of Pareto weights at market ζ, and $\bar{t}_\zeta = n_\zeta^{-1}\sum \zeta_j t_j$ the average liquidity made available at market ζ. We can see that the average income, volatility, and market size when i is present all contribute to financial centrality in the usual way.

For any set of agents $A\subseteq I$, let $\zeta^A\in\{0,1\}^n$ denote the market where only agents belonging to A have market access.

Proposition 6.2. *Take the CARA-normal model with $\zeta \perp y$ and homogeneous preferences ($r_i = r$ for all i) and let $t^*\in\mathbb{R}_+^n$ be a solution to (6.2) with $J = I$.*

(1) *If $t_i^*>0$ then we can calculate the financial centrality for transfer T for agent i as*

$$FC_i^T = \sum_{\zeta\in\{0,1\}^n} \mathrm{P}(\zeta)\, \overline{\lambda}_\zeta \exp(-r\overline{\mu}) \exp\left(r^2 \frac{\overline{\sigma}_\zeta^2}{2n_\zeta}\right) \exp(-r\bar{t}_\zeta).$$

(2) *If i,j are such that such that $t_i^*>0$ and*

$$\mathrm{P}\left(\zeta^{\{i,A\}}\right) h\left(\zeta^{\{i,A\}}\right) \geq \mathrm{P}\left(\zeta^{\{j,A\}}\right) h\left(\zeta^{\{j,A\}}\right) \text{ for all } A\subseteq I\setminus\{i,j\} \tag{6.3}$$

then $t_i^* \geq t_j^*$. If there exist some $A\subseteq I\setminus\{i,j\}$ for which (6.3) is strict, then $t_i^* > t_j^*$.

This says that if i is more central than j in a strong sense, then i will receive higher transfers than j. Of course, the condition for Proposition 6.3, part 2 implies that $FC_i \geq FC_j$ (i.e., around $T=0$), since $FC_i = \sum_\zeta \mathrm{P}(\zeta)\, h(\zeta)$, but this is a stronger requirement. However, this robustly captures the intuition and shows that even for non-marginal transfers, those who tend to be in components (or markets) that are smaller, more volatile, more important, or require more insurance are indeed deemed to be more central, even if the exact formulation is not analytically tractable.

Further, returning to endogenous participation, it is worth noting why our notion of centrality is defined with respect to injections happening ex-ante, before incomes are realized. Namely, our notion solves potential incentive problems. First, if income is privately observed, and liquidity is based on reported values, then agents may have incentives to report low income values in order to receive higher liquidity injections. Second, consider the case of moral hazard in income production. If income has to be produced (by investing or applying effort) and agents know that they will receive insurance from the planner, this dampens incentives for production, as in the standard moral hazard problem. Though making liquidity provision non-contingent may not be the optimal mechanism, it is robust in its ability to resolve the potential incentive problems in more general environments without having to spell out all the details of the model.

7. Discussion

In a number of economic environments, agents share risk, but there is heterogeneity in who participates and is able to exchange directly or indirectly with others. This is true of financial markets with search frictions, matching with limited and stochastic market participation, and in some monetary models. This is observed in risk-sharing village networks, among other settings. A common, standard model which we extend to a stochastic financial network (the distribution over components of the subgraphs induced by exogenous or endogenous participation shocks) is used to address the question of how one measures an agent's importance in such settings. We define the financial centrality of an agent as the marginal social value of providing a small liquid asset to that agent. So, we characterize financial centrality as measuring the price of a personalized bond: i.e., an asset that pays whenever agent i is able to trade, and anyone can trade in that asset. Therefore, centrality can be measured using classical asset pricing techniques, once the equilibrium pricing kernel is estimated. We show that the most valued agents are not only those who trade often, but trade when there are few traders, when income risk is high, when income shocks are positively correlated, when attitudes toward risk are more sensitive in the aggregate, when there are distressed institutions, and when there are tail risks. From a financial networks perspective, we provide a new contribution to the literature: an agent is more central, holding fixed frequency of trade, the fewer links or transaction partners they have.

Additionally, we look at a different decentralized environment, where agents engage in ex-ante cooperative bargaining, which determines the Pareto weights. We show the resulting weights depend on exactly the same features as financial centrality. This allows us to study financial centrality in the data without observing it directly. In a simple empirical example, we turn to a setting, where we have the requisite data: rural Thai villages. We provide observational evidence from village risk-sharing network data, consistent with our model, that the agents that receive the greatest share of the pie are indeed those who are not simply well-connected, but are active precisely when the market is otherwise thin in number of participants or consisting of participants with high ex-ante volatility of income.

The framework extends to both endogenous participation models, as in private information, moral hazard, or team production models where financial centrality may or may not have an extra component. In some contexts, endogenous participation has an identical form as exogenous participation; the liquidity does not generate a change in the participation distribution per se, so formally the participation decision can be thought of as exogenous. In other cases, endogenous participation leads to a change in the composition of participants in equilibrium due to the asset.

Finally, normative analysis is straightforward with the intuitions from the small asset case carrying through exactly—in the case of inertness—to the case with large transfers by the policymaker to potentially a set of agents. Moral hazard concerns rationalize why we have taken an ex-ante perspective. The provision of liquidity and its characterization can be generalized to environments in which the policymaker has limited controls.

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Appendix A. Proofs

A.1. Financial Centrality as Value of Marginal of Liquidity Injection.

Proof of Proposition 3.1. Since $V(t)$ is concave, this program is convex, and satisfies Slater's condition if $T > 0$, and hence the Kuhn-Tucker conditions of this program are both necessary and sufficient. The Lagrangian of program (6.2) is $\mathcal{L}(t, \eta, \nu) = V(t) + \eta\left(T - \sum_{j\in J} t_j\right) + \nu_j t_j$. Kuhn-tucker conditions are

(1) $V_j(t) = \eta - \nu_j$ for all $j \in J$, where $V_j = \partial V/\partial t_j$
(2) $\nu_j t_j = 0$ for all $j \in J$
(3) $\nu_j \geq 0$ for all $j \in J$
(4) $\eta\left(T - \sum_{j\in J} t_j\right) = 0$ and $\eta \geq 0$

If at an optimum t^* we have that $t_i^* > 0$ then $V_i(t^*) = \nu$. If $V_j(t^*) < V_i(t^*) = \nu$ then we must have

$$\nu_j = V_i(t^*) - V_j(t^*) > 0$$

implying that $t_j^* = 0$.

To show (2), Propose the following solution: $t_i^* = T$, $t_j^* = 0$ for all $j \neq i$, $\eta = V_i(t^*)$ and $\nu_j = \eta - V_j(t^*)$. Since V is differentiable, its partial derivatives are continuous around $t = 0$. Therefore, $\exists \hat{T}_J > 0$ such that for all $t \in \tau = \left\{t : \sum_{j\in J} t_j < \hat{T}_J \text{ and } t_j \geq 0 \text{ for all } j \in J\right\}$ we have $V_i(t) \geq V_j(t)$ for all $j \in J \sim \{i\}$ (since $FC_i \geq FC_j$). Therefore, if $T < \hat{T}$, a solution $t^* \in \tau$, and therefore we have $V_i(t^*) > V_j(t^*)$ for all j, and hence $\nu_j = \eta - V_j(t^*) = V_i(t^*) - V_j(t^*) > 0$; i.e. t^* satisfies the Kuhn-Tucker conditions. To prove uniqueness, suppose there exists another solution $\hat{t} : \sum_{j\in J} \hat{t}_j < \hat{T}$ and $\exists k \neq i$ with $\hat{t}_k > 0$. If that was the case, then $\eta = V_k(t)$. But because $\sum_{j\in J} \hat{t}_j < \hat{T}$ we also have that $V_i\left(\hat{t}\right) > V_k\left(\hat{t}\right)$. Therefore, $V_i\left(\hat{t}\right) + \nu_i \geq V_i\left(\hat{t}\right) > V_j\left(\hat{t}\right) = \eta$, violating condition (1). Therefore, the only solution to (6.2) is $t = t^*$. □

A.2. Characterizing Financial Centrality.

In what follows, we will explore the properties of which agents are more financially central as a function of fundamentals such as propensity to be an active trader, composition of those who are active when the agent is active, variances and covariances of incomes of active traders, risk preferences, and so on.

We next develop a useful formulation of financial centrality in terms of the multipliers of the maximization problem in (3.2).

Let $\widehat{q}(y, \zeta)$ be the Lagrange multiplier for the first condition and define an auxiliary multiplier vector

$$q(y, \zeta) : \ \widehat{q}(y, \zeta) := q(y, \zeta)\,\mathrm{P}(y, \zeta)\,, \tag{A.1}$$

and let $\gamma_i(s)$ be the corresponding Lagrange multiplier for the non negativity constraint $c_i \geq 0$. The Lagrangian for (3.2) is then

$$\mathcal{L} = \mathbb{E}_s\left\{\sum_{i\in I} \lambda_i u_i\left[c_i(s)\right] + q(s)\,\zeta_i\left[y_i - c_i(s)\right] + \gamma_i(s)\,c_i(s)\right\}. \tag{A.2}$$

In the baseline model, we assume market participation is independent of income draws. In this case, financial centrality can be expressed using the envelope theorem on program (3.2).

Lemma A.1. *Suppose the environment is inert to infinitesimal liquidity injection, and let $q(s)$ and $\gamma_i(s)$ be the multipliers of Lagrangian (A.2) for program (3.2) Then*

$$FC_i = \mathbb{E}_s \{\zeta_i q(s)\}. \tag{A.3}$$

Proof. We use the classical envelope theorem on a variation of program (3.2), changing the income of agent i to $\hat{y}_i = y_i + \zeta_i \epsilon_i$. Then, the envelope theorem implies

$$\frac{\partial V}{\partial \epsilon_i} |_{\epsilon=0} = \mathbb{E}_s \left[\zeta_i \frac{\partial \mathcal{L}}{\partial y_i} \times \frac{\partial y_i}{\partial \epsilon_i} \right] = \mathbb{E}_s \{\zeta_i q(s)\}.$$

proving the desired result. □

The multiplier $q(s)$ is, of course, the marginal value of consumption, at the (constrained) efficient allocation $c_i(\cdot)$. As we will see below, when defining a Walrasian equilibrium in an Arrow Debreu economy defined on this environment, $q(s)$ will correspond to the equilibrium price of the Arrow Debreu security that pays only at state s. As such, equation (A.3) is effectively the price of a fictitious asset that pays 1 consumption unit whenever $\zeta_i = 1$, using $q(s)$ as its pricing kernel. Next, we can consider the case where the participation distribution is responsive to infinitesimal liquidity injection. A key feature in responsive settings is that participation and income become correlated.

Proof of Proposition 4.1. For simplicity of exposition, assume a finite state space (i.e., y is a discrete random variable), so the Lagrangian is

$$\mathcal{L} = \sum_{y \in Y} \sum_{\zeta \in \{0,1\}^n} \left[\sum_{j \in I} \lambda_j u_j(c_j) + \hat{q}(y,\zeta) \sum_{j \in I} \zeta_j (y_j - c_j) \right] \mathrm{P}(\zeta \mid y) \mathrm{P}(y).$$

Using the envelope theorem, we get that

$$\begin{aligned} FC_i &= \frac{\partial \mathcal{L}}{\partial y_i} \\ &= \sum_{y \in Y} \sum_{\zeta \in \{0,1\}^n} \zeta_i \hat{q}(y,\zeta) \mathrm{P}(\zeta \mid y) \mathrm{P}(y) \\ &+ \sum_{y \in Y} \sum_{\zeta \in \{0,1\}^n} \left[\sum_{j \in I} \lambda_j u_j(c_j) + \hat{q}(y,\zeta) \sum_{j \in I} \zeta_j (y_j - c_j) \right] \frac{\partial \mathrm{P}(\zeta \mid y_i)}{\partial y_i} \mathrm{P}(y) \end{aligned}$$

and using the facts that $q(y,\zeta) = \hat{q}(y,\zeta) / \mathrm{P}(y,\zeta)$ and complementary slackness implies

$$\hat{q}(y,\zeta) \sum_{i \in V} \zeta_i (y_i - c_i) = 0$$

for all (y,ζ). We can simplify this expression as

$$FC_i = \mathbb{E}_{y,\zeta} \{\zeta_i q(y,\zeta)\} + \mathbb{E}_{y,\zeta} \left\{ \sum_{j \in I} \lambda_j u_j(c_j) \underbrace{\frac{\partial \mathrm{P}(\zeta \mid y_i)}{\partial y_i} \frac{1}{\mathrm{P}(\zeta \mid y_i)}}_{:= \mathbf{S}_i(\zeta \mid y_i)} \right\}$$

proving the desired result. □

Proof of Proposition 4.2. The first order conditions of program (3.2) with Lagrangian defined in (A.2) with respect to $c_i(s)$ whenever $\zeta_i = 1$ is $\lambda_i u_i' [c_i(s)] = q(s)$ (without taking into account the non-negativity constraint over consumption). Therefore, if $\lambda_i = \lambda_j = 1/n$ for all $i, j \in I$ and $u_i = u$ for all i, we then get that if $\zeta_i = \zeta_j = 1$ then $c_i(s) = c_j(s)$ (i.e., all agents participating in the market have equal consumption). Therefore, using the resource constraint, we obtain $c_i(s) = \overline{y}(s)$ whenever $\zeta_i = 1$, and obviously $c_i(s) = y_i$ otherwise. The first order condition also implies then that $q(s) = u'[\overline{y}(s)]$.

To obtain the approximation, we first make a second order Taylor approximation $g(y) := u'(y)$ around $y = \mathbb{E}(y) = \mu$:

$$u'(\overline{y}) \approx u'(\mu) + u''(\mu)(\overline{y} - \mu) + \frac{u'''(\mu)}{2}(\overline{y} - \mu)^2$$

and then taking expectations, we have

$$\mathbb{E}[u'(\overline{y}(s)) \mid \zeta] \approx u'(\mu) + u''(\mu)\mathbb{E}(\overline{y} - \mu \mid \zeta) + \frac{1}{2}u'''(\mu)\mathbb{E}\left[(\overline{y} - \mu)^2 \mid \zeta\right] = u'(\mu) + \frac{1}{2}u'''(\mu)\sigma^2/n(\zeta),$$

using the facts that $\mathbb{E}(\overline{y}) = \mu$ and that $\mathbb{E}(\overline{y} - \mu)^2 = \sigma^2/n(\zeta)$ if income draws are i.i.d. Reorganizing this expression, we get the desired result. □

The above result can be considerably generalized. Let us consider an extension that naturally correlates participation with income and allows for considerable heterogeneity in incomes, Pareto weights, tastes for risk, and so on. We suppose CARA utility and a jointly normal income distribution with heterogeneous mean, variance, and covarying income draws. Assume that agents observe shocks to income volatility and expected income to certain agents. To begin with, there is an aggregate (fundamental) shock $z \in Z$ with some distribution $G(z)$. This fundamental shock affects preferences expected income $\mu(z)$, income variance $\Sigma(z)$, preferences $u_i(c, z) = -\frac{1}{r_i(z)}\exp(-r_i(z)c)$, and even the planner's preferences $\lambda(z)$.

Formally, we assume $y \mid z \sim \mathcal{N}(\mu(z), \Sigma(z))$ and $\zeta \sim F(\zeta \mid z)$, and are such that $(y \mid z) \perp (\zeta \mid z)$ This is without loss of generality and nests the case where participation is endogenous prior to observing the realized income, but after observing the realized shock z. With these assumptions, income is Gaussian and $(y \mid z) \perp (\zeta \mid z)$. The conditional independence will buy us a simple characterization.

Let $\overline{\mu}_\zeta := \frac{1}{n_\zeta}\sum_i \zeta_i \mu_i$ be the average expected income of those trading, $\overline{\sigma}_\zeta^2 := \frac{1}{n_\zeta}\sum_{i,j}\zeta_i\zeta_j\sigma_{i,j}$ be the average volatility, $\overline{r}_\zeta := \left(\frac{1}{n_\zeta}\sum_{i:\zeta_i=1}\frac{1}{r_i}\right)^{-1}$ be the mean of the risk parameter of those trading, and $\overline{\lambda}_\zeta := \left(\prod_{i:\zeta_i=1}\lambda_i^{\overline{r}_\zeta/r_i}\right)^{\frac{1}{n_\zeta}}$ be the risk-weighted geometric mean of Pareto weights.

Proposition A.1. *Under the above assumptions, we have that*

$$\mathbb{E}[q(y, \zeta) \mid z, \zeta] = \exp\left[-\overline{r}_\zeta(z)\overline{\mu}_\zeta(z)\right] \times \overline{\lambda}_\zeta(z) \times \exp\left[\frac{\overline{r}_\zeta^2(z)}{2} \times \frac{\overline{\sigma}_\zeta^2(z)}{n_\zeta}\right]$$

so financial centrality is given by

$$FC_i = \mathbb{E}_{z,\zeta}\left\{\zeta_i \times \exp\left[-\overline{r}_\zeta(z)\,\overline{\mu}_\zeta(z)\right] \times \overline{\lambda}_\zeta(z) \times \exp\left[\frac{\overline{r}_\zeta^2(z)}{2} \times \frac{\overline{\sigma}_\zeta^2(z)}{n_\zeta}\right]\right\}$$

where the market averages $\left(\overline{r}_\zeta, \overline{\mu}_\zeta, \overline{\lambda}_\zeta, \overline{\sigma}_\zeta^2\right)$ *are functions of market fundamentals* $z \in Z$.

Proof of Proposition A.1. Since shock $z \sim G(z)$ is realized after the liquidity injection is realized, the Lagrangian used by the planner is

$$\mathcal{L} = \mathbb{E}_z\left[\mathbb{E}_s\left\{\sum_{i\in I}\lambda_i(z)\,u_i[c_i(s)] + q(s)\,\zeta_i[y_i - c_i(s)] \mid z\right\}\right]$$

hence

$$\mathcal{L} = \mathbb{E}_{z,s}\left\{\sum_{i\in I}\lambda_i(z)\,u_i[c_i(s,z),z] + q(s,z)\,\zeta_i[y_i - c_i(s,z)]\right\}$$

So FOCs are

$$\lambda_i(z)\frac{\partial u_i}{\partial c}[c_i(s,z),z] = q(s,z)$$

and in the CARA case,

$$\lambda_i(z)\exp(-r_i(z)\,c_i(s,z)) = q(s,z)$$

and, using the same results as before, we see than given z, we have

$$q(s,z) = \hat{\lambda}_\zeta(z)\exp\left(-\overline{r}_\zeta(z)\,\overline{y}_\zeta\right)$$

where $\hat{\lambda}_\zeta(z) := \exp\left[\frac{1}{n_\zeta}\sum\zeta_j\frac{\overline{r}_\zeta}{r_j}\ln(\lambda_j(z))\right]$ and $\overline{r}_\zeta(z) := \left(\frac{1}{n_\zeta}\sum\zeta_j\frac{1}{r_j(z)}\right)^{-1}$. Now, because we know that $y \mid z \sim \mathcal{N}(\mu(z), \Sigma(z))$ we can also see that

$$\mathbb{E}_s[\zeta_i q(s,z) \mid z] = \mathbb{E}_s\left[\zeta_i\hat{\lambda}_\zeta(z)\exp\left(-\overline{r}_\zeta(z)\,\overline{y}_\zeta\right) \mid z\right]$$

and using the independence assumptions we then know that

$$\mathbb{E}_s[\zeta_i q(s,z) \mid z] = \mathbb{E}_s(\zeta_i \mid z) \times \exp\left[-\overline{r}_\zeta(z)\,\overline{\mu}_\zeta(z)\right] \times \overline{\lambda}_\zeta(z) \times \exp\left[\frac{\overline{r}_\zeta^2(z)}{2} \times \frac{\overline{\sigma}_\zeta^2(z)}{n_\zeta}\right]$$

integrating with respect to z gives us the desired result. □

A.3. **Bargaining Foundations.**

A.3.1. *Nash Bargaining.*

Proof of Proposition 5.3. From the first order conditions of the planner's problem we have

$$\lambda_i\exp(-r_i c_i) = \overline{\lambda}_\zeta\exp(-\overline{r}_\zeta\overline{y})$$

so

$$-\frac{1}{r_i}\exp(-r_i c_i) = -\frac{\overline{\lambda}_\zeta}{r_i\lambda_i}\exp(-\overline{r}_\zeta\overline{y}).$$

Hence

$$\mathbb{E}_s\left[\zeta_i u_i\left(c_i\left(s\right)\right)\right] = -\frac{1}{r_i\lambda_i}\mathbb{E}_s\left[\zeta_i\overline{\lambda}_\zeta\exp\left(-\overline{r}_\zeta\overline{y}\right)\right] = -\frac{1}{r_i\lambda_i}\mathbb{E}_\zeta\left[\zeta_i\overline{\lambda}_\zeta\exp\left(-\overline{r}_\zeta\overline{\mu}+\frac{\overline{r}_\zeta^2}{2n_\zeta}\overline{\sigma}_\zeta^2\right)\right]$$
$$= -\frac{1}{r_i\lambda_i}FC_i.$$

Moreover

$$\mathbb{E}_s\left[\zeta_i u_i\left(y_i\right)\right] = -\frac{1}{r_i}\mathbb{E}_s\left[\exp\left(-r_iy_i\right)\right] = -\frac{1}{r_i}\mathbb{E}_\zeta\left[\zeta_i\exp\left(-r_i\mu_i+\frac{r_i^2}{2}\sigma_i^2\right)\right]$$
$$= -\frac{1}{r_i}\underbrace{\mathrm{P}\left(\zeta_i=1\right)}_{:=p_i}\exp\left(-r_i\mu_i+\frac{r_i^2}{2}\sigma_i^2\right).$$

This means

$$\mathbb{E}_s\left\{\zeta_i\left[u_i\left(c_i\left(s\right)\right)-u_i\left(y_i\right)\right]\right\} = \frac{1}{r_i}p_i\exp\left(-r\mu_i+\frac{r_i^2}{2}\sigma^2\right)-\frac{1}{r_i\lambda_i}FC_i$$

and so

$$\lambda_i = \frac{\alpha_i r_i}{\frac{\lambda_i}{\lambda_i}p_i\exp\left(-r_i\mu_i+\frac{r_i^2}{2}\sigma_i^2\right)-\frac{1}{\lambda_i}FC_i}$$

if and only if

$$\lambda_i = \frac{\lambda_i\alpha_i r_i}{\lambda_i p_i\exp\left(-r_i\mu_i+\frac{r_i^2}{2}\sigma_i^2\right)-FC_i} \iff \lambda_i p_i\exp\left(-r_i\mu_i+\frac{r_i^2}{2}\sigma_i^2\right)-FC_i=\alpha_i r_i$$

if and only if

$$\lambda_i = \frac{\alpha_i r_i + FC_i\left(\lambda\right)}{p_i\exp\left(-r_i\mu_i+\frac{r_i^2}{2}\sigma_i^2\right)}$$

as we wanted to show. □

A.3.2. *Kalai-Smorodinsky Bargaining.* The second most used bargaining solution in the literature is the Kalai-Smorodinsky solution. It also gives closed form solutions to Pareto weights and the weights are expressed as a function of fundamentals of the environment, rather than a fixed point equation.

The most important parameter in the *bliss point.* The bliss point for agent i, $\overline{U}_i$, is defined as the utility they would achieve if they consumed all the available income in the market in every state where they can trade and only her own income otherwise:

$$\overline{U}_i := \mathbb{E}_{\zeta,y}\left[\zeta_i u_i\left(\sum_j\zeta_j y_j\right)+\left(1-\zeta_i\right)u_i\left(y_i\right)\right]$$

and $\overline{U} = \left(\overline{U}_1,\overline{U}_2,\ldots,\overline{U}_n\right)$. Likewise, the disagreement point $\underline{U}_i$ is the value of autarky in this environment for each agent

$$\underline{U}_i := \mathbb{E}_{y_i}\left[u_i\left(y_i\right)\right]$$

and $\underline{U} := (\underline{U}_1, \underline{U}_2, \ldots, \underline{U}_n)$. The Kalai-Smorodinsky solution consists on finding the linear combination of $\overline{U}$ and the $\underline{U}$ that lies on the Pareto frontier of the utility possibility set; i.e, find $\alpha \in [0,1]$ such that $\alpha\overline{U} + (1-\alpha)\underline{U} \in \mathbb{P}(\mathcal{U})$, and the solution is $U^* = \alpha\overline{U} + (1-\alpha)\underline{U}$. Since $\overline{U} > \underline{U}$, the Kalai-Smorodinsky solution here would be

$$\max_{\alpha\in[0,1],\{c_i(y,\zeta)\}_{i\in I}} \alpha$$

subject to

$$\begin{cases} \mathbb{E}_{y,\zeta}\left[\zeta_i u_i\left(c_i\left(y,\zeta\right)\right) + \left(1-\zeta_i\right)u_i\left(y_i\right)\right] \geq \alpha\overline{U}_i + (1-\alpha)\underline{U}_i & \text{for all } i \\ \sum \zeta_i c_i\left(y,\zeta\right) \leq \sum \zeta_i y_i & \text{for all } \left(\zeta, y\right). \end{cases}$$

One of the most attractive properties of the Kalai-Smorodinsky solution is that the Pareto weights derived from it have a closed form formula and is not a fixed point equation (as in the Nash Bargaining solution case).

Proposition A.2. *If the risk sharing contract is the Kalai-Smorodinsky solution over the utility possibility set, then the Pareto weights associated with the solution are*

$$\lambda_i = \frac{1}{\mathbb{E}_s\left\{\zeta_i\left[u_i\left(Y\left(s\right)\right) - u_i\left(y_i\right)\right]\right\}}$$

where $Y(s) = \sum_j \zeta_j y_j$ is the aggregate income in state $s = (y,\zeta)$. If $u_i(c) = -r_i^{-1}\exp(-r_i c)$ and $y \sim \mathcal{N}(\mu,\Sigma)$, then

$$\lambda_i = \frac{\beta}{p_i \times \mathbb{E}_\zeta\left\{\exp\left(-r\mu + \frac{\sigma^2}{2}\right) - \exp\left[n_\zeta\left(-r\mu + \frac{\sigma^2}{2}\right)\right] \mid \zeta_i = 1\right\}}$$

where $\beta = r/\left(-r\mu + \sigma^2/2\right)$.

The following corollary is an immediate consequence.

Corollary A.1. *In the CARA-Normal model, with homogeneous preferences and i.i.d. income shocks, if $\mu > \frac{r}{2}\sigma^2$, then Lagrange multipliers are decreasing (in the FOSD sense) in market size.*

Unlike Nash bargaining, Pareto weights in this environment have a closed form solution, so comparative statics are easier to interpret, and the comparative statics are the same as the one suggested by the Nash Bargaining fixed point equations. The most important feature, of course, is that the elements determining λ are the same as those that determine our measure of financial centrality. This correspondence allows us to operationalize financial centrality in empirical analysis.

Proof of Proposition A.2. The Lagrangian is

$$\mathcal{L} = \alpha + \sum \mu_i \left\{\mathbb{E}_{y,\zeta}\left[\zeta_i u_i\left(c_i\right) + \left(1-\zeta_i\right)u_i\left(y_i\right)\right] - \alpha\overline{U}_i - (1-\alpha)\underline{U}_i\right\} + \sum_{\zeta,y} q\left(y,\zeta\right)\zeta_i\left(y_i - c_i\right)\mathrm{P}\left(y,\zeta\right)$$

with multipliers $(\mu_i)_{i=1:n}$ and $\left(q\left(y,\zeta\right)\mathrm{P}\left(y,\zeta\right)\right)_{y,\zeta}$. First order conditions are

$$\frac{\partial\mathcal{L}}{\partial\alpha} = 1 - \mu_i\left(\overline{U}_i - \underline{U}_i\right)$$

since $\alpha \in (0,1)$ (the bliss point cannot be feasible) then, to get an interior solution, we must have $\frac{\partial \mathcal{L}}{\partial \alpha} = 0 \iff \mu_i = 1/\left(\overline{U}_i - \underline{U}_i\right)$. The first order conditions with respect to consumption are

$$\frac{\partial \mathcal{L}}{\partial c_i (y, \zeta)} \mid_{\zeta_i = 1} = 0 \iff \mu_i u_i' (c_i) \, \mathrm{P}(y, \zeta) = q(y, \zeta) \, \mathrm{P}(y, \zeta)$$

therefore, in the planner representation, this is equivalent to the Pareto weights being

$$\lambda_i = \mu_i = \frac{1}{\overline{U}_i - \underline{U}_i}.$$

In the CARA-Normal model, let $Y := \sum \zeta_j y_j$. Since $y \sim \mathcal{N}(\mu, \Sigma)$, we have $Y \mid \zeta \sim \mathcal{N}\left(\sum \zeta_j \mu_j, \sum_{i,j} \zeta_i \zeta_j \sigma_{ij}\right)$. Therefore ha

$$\mathbb{E}_y \left[u_i \left(\sum_i \zeta_j y_j \right) \mid \zeta_i = 1 \right] = -\frac{1}{r} \mathbb{E}\left[\exp(-rY)\right] = -\frac{1}{r} M_Y(-r)$$

and $M_Y(t) = \exp\left(\mu_Y t + \frac{t^2}{2}\sigma_Y^2\right) = \exp\left(-r\mu_Y + \frac{r^2}{2}\sigma^2\right) = \exp\left(-r \times n_\zeta \overline{\mu}_\zeta + \frac{r^2}{2}\sum \sigma_{ij}\right)$. In the i.i.d. case, $M_Y(t) = \exp\left(-r n_\zeta \mu + \frac{r^2}{2}\sigma^2 n_\zeta\right) = \exp\left[n_\zeta(-r\mu + \gamma)\right]$ and the autarky value is $\mathbb{E}_y[u(y_i)] = \frac{-1}{r} M_{y_i}(-r) = \exp\left(-r\mu + \frac{r^2}{2}\sigma^2\right)$

Therefore

$$\begin{aligned} \overline{U}_i - \underline{U}_i &= \mathbb{E}\left\{ \zeta_i \left[u_i \left(\sum_{j:\zeta_j = 1} y_j \right) - u_i(y_i) \right] \right\} = p_i \mathbb{E}_{y,\zeta} \left\{ u_i \left(\sum_{j:\zeta_j=1} y_j \right) - u_i(y_i) \mid \zeta_i = 1 \right\} \\ &= p_i \times \left\{ \exp\left[n_\zeta \left(-r\mu + \frac{r^2}{2}\sigma^2 \right) \right] - \exp\left(-r\mu + \frac{r^2}{2}\sigma^2 \right) \right\} \end{aligned}$$

proving the desired result. □

A.4. **Endogenous Participation.**

Proof of Proposition 6.1. We first need to show the existence of the equilibrium $\overline{\zeta}_i(k_i)$ such that (a) $\zeta_i(k_i) = 0$ for all k_i is the lowest participation equilibrium and (b) there exist thresholds $\overline{k} = \left(\overline{k}_i\right)_{i \in [n]}$ and an equilibrium $\overline{\zeta}(k)$ such that $\overline{\zeta}_i(k_i) = 1 \iff k_i \leq \overline{k}_i$, and moreover, $\overline{\zeta}_i(k_i) \geq \zeta_i^*(k_i)$ for all $k \in K^n$, all agents $i \in [n]$, and for any other equilibrium participation $\zeta^*(k)$.

For this, define the incomplete information game

$$\Gamma = \left\{ A_i = \{\zeta_i \in \{0,1\}\}, U_i(\zeta, k_i) := \zeta_i \mathbb{E}_y \left\{ u\left(\overline{y}_{1 + \sum_{j \neq i} \zeta_j} \right) - u(y_i) - k_i \right\} + \mathbb{E}(u(y_i)) \right\}.$$

Because $y_i \sim \mathcal{N}(\mu, \sigma^2)$ and u is increasing and concave, it is easy to show that Γ is supermodular in ζ_i, ζ_{-i} and supermodular in $\eta = -k_i$ (if k_i were common knowledge). This, together with the FOSD ordering assumption, makes Γ a monotone supermodular game of incomplete information (as in Van Zandt and Vives (2007)), which ensures the existence of monotone BNE $\underline{\zeta}, \overline{\zeta}$ such that for any other equilibria $\zeta^*(k)$, we have $\underline{\zeta}_i(k_i) \leq \zeta_i^*(k_i) \leq \overline{\zeta}_i(k_i)$ for all $i, k_i \in K_i$. Since $\zeta_i \in \{0,1\}$, both $\underline{\zeta}, \overline{\zeta}$ are threshold strategies, $\overline{\zeta}_i(k_i) = 1 \iff k_i \leq \overline{k}_i$ and $\underline{\zeta}_i(k_i) = 1 \iff k_i < \underline{k}_i$. Since $k_i \geq 0$, it is easy to show that the profile where no one attends the market is a BNE of this game and is clearly the lowest. The highest must prescribe market participation at the threshold, which gives us the fix point equation,

Using the implicit function theorem, we know that if $\det(\mathbf{J}_k) \neq 0$ then any solution to fix point of equation $\overline{k} = \Psi\left(\overline{k}, \epsilon_i\right)$ satisfies that

$$\frac{\partial \overline{k}}{\partial \epsilon_i} \mid_{\epsilon_i = 0} = \left(\frac{\partial \overline{k}_j}{\partial \epsilon_i} \mid_{\epsilon_i = 0} \right)_{j \in [n]} = (I - \mathbf{J}_k)^{-1} \times \mathbf{F}^{(i)}$$

where $\mathbf{F}^{(i)} = \left(\frac{\partial \Psi_j}{\partial \epsilon_i} \mid \overline{k}, \epsilon = 0 \right)_{j \in [n]}$. For this, knowing that $u(\cdot)$ is differentiable, we have that

$$u(\overline{y}_m + \zeta_i \epsilon / m) = u(\overline{y}_m) + u'(\overline{y}_m)\, \zeta_i \frac{\epsilon}{m} + \frac{u''(\xi)}{2} \frac{\epsilon^2}{m^2}$$

for some $\xi \in [0, \frac{\epsilon}{m}]$. This implies that

$$k_m^*(\epsilon) := \mathbb{E}_y \left[u(\overline{y}_m + \zeta_i \epsilon / m) - u(y_i) \right]$$

$$\mathbb{E}_y \left[u(\overline{y}_m) - u(y_i) \right] + \mathbb{E}_y \left[\zeta_i \underbrace{u'(\overline{y}_m)}_{= q(s)} \right] \frac{\epsilon}{m} + \mathbb{E}\left[\frac{u''(\xi)}{2} \right] \frac{\epsilon^2}{m^2}$$

and therefore

$$\frac{\partial k_m^*}{\partial \epsilon} \mid_{\epsilon = 0} = \lim_{\epsilon \to 0} \frac{k_m^*(\epsilon) - k_m^*}{\epsilon} = \mathbb{E}_y \left[\zeta_i \frac{q(s)}{m} \right].$$

Thus,

$$\begin{aligned}
\mathbf{F}_{ij} := \mathbf{F}_j^{(i)} &= \frac{\partial}{\partial \epsilon} \left[\sum_{m \leq n} \mathbb{E}_y \left[k_m^*(\epsilon) \right] \times \pi\left(m, \overline{k} \mid \overline{k}_j\right) \right] = \sum_{m \leq n} \mathbb{E}_y \left\{ \frac{\partial k_m^*(\epsilon)}{\partial \epsilon} \mid_{\epsilon = 0} \right\} \times \pi\left(m, \overline{k} \mid \overline{k}_j\right) \\
&= \sum_{m \leq n} \mathbb{E}_y \left[\zeta_i \frac{q(s)}{m} \right] \pi\left(m, \overline{k} \mid \overline{k}_j\right) = \mathbb{E}\left[\zeta_i \zeta_j \frac{q(s)}{n_\zeta} \mid \overline{k}_j \right].
\end{aligned}$$

Finally, the participation effect in this model is

$$PE = \sum_{m=1}^{n} m k_m^* \sum_{j=1}^{n} \frac{\partial \pi\left(m, \overline{k}\right)}{\partial \overline{k}_j} \times \frac{\partial \overline{k}_j}{\partial \epsilon_i} = \sum_{j=1}^{n} \frac{\partial \overline{k}_j}{\partial \epsilon_i} \times \underbrace{\left(\sum_{m=1}^{n} m k_m^* \frac{\partial \pi\left(m, \overline{k}\right)}{\partial \overline{k}_j} \right)}_{:= \Lambda_j \geq 0 \text{ from FOSD assumption}}$$

proving the desired result. Moreover,

$$\frac{\partial \Psi_j}{\partial k_h} \mid_{\overline{k}, \epsilon = 0} = \frac{\partial}{\partial k_h} \left[\sum_{m \leq n} \mathbb{E}_y k_m^* \times \pi\left(m, \overline{k} \mid \overline{k}_j\right) \right] = \sum_{m \leq n} \mathbb{E}_y k_m^* \times \frac{\partial \pi\left(m, \overline{k} \mid \overline{k}_h\right)}{\partial k_h} \geq 0$$

and $\frac{\partial \Psi_j}{\partial k_j} \mid_{\overline{k}, \epsilon = 0} = 0$, again using the fact that $k_{-h} \mid k_h$ FOSDs $k_{-h} \mid k_h'$ whenever $k_h \geq k_h'$. □

A.5. **Large Transfers.**

Proof of Proposition 6.2. For part 1, the Lagrangian for this problem (given a vector of transfers $t^* \in \mathbb{R}^n_+$, and assuming $c_i(s) > 0$ in the optimum) is

$$\mathcal{L} = \mathbb{E}_s \left\{ \sum_i \lambda_i u_i(c_i) + q(s) \left[\sum_i \zeta_i (y_i + t_i - c_i) \right] \right\}$$

The first order conditions under the assumption that $y \perp \zeta$, homogeneous CARA preferences and Gaussian income draws are the same as before, with $c_i = r^{-1} \ln(\lambda_i) - r^{-1} \ln[q(s)]$, but now $q(s)$ satisfies

$$r^{-1} \sum_{i=1}^n \zeta_i \{ \ln(\lambda_i) - \ln[q(s)] \} = \sum_{i=1}^n \zeta_i (y_i + t_i) \iff$$

$$\ln\left(\overline{\lambda}_\zeta\right) - \ln[q(s)] = \overline{y}_\zeta + \overline{t}_\zeta \iff q(s) = \overline{\lambda}_\zeta \exp\left(-r\overline{y}_\zeta\right) \exp\left(-r\overline{t}_\zeta\right)$$

which then implies that $\mathbb{E}_y[q(s) \mid \zeta] = \mathbb{E}\left[\overline{\lambda}_\zeta \exp\left(-r\overline{y}_\zeta\right) \exp\left(-r\overline{t}_\zeta\right)\right] = h(\zeta) \exp\left(-r\overline{t}_\zeta\right)$, where $h(\zeta) = \overline{\lambda}_\zeta \exp\left(-r\overline{\mu}_\zeta\right) \exp\left(r^2 \overline{\sigma}^2_\zeta / n_\zeta\right)$. Therefore,

$$V_i(t) = \frac{\partial \mathcal{L}}{\partial t_i} \mid_{t=t^*} = \mathbb{E}_s \{ \zeta_i q(s) \} = \mathbb{E}_\zeta \left[\zeta_i h(\zeta) \exp\left(-r\overline{t}_\zeta\right) \right].$$

For part 2, suppose, by contradiction, that $t^*_j > t^*_i$. Based on Proposition 3.1, since $t^*_i > 0$ then $V_i(t^*) = V_j(t^*) = v^*$. We can rewrite $V_i(t^*)$ as

$$\begin{aligned} V_i(t^*) = & \sum_{A \subseteq I \setminus \{i,j\}} \mathrm{P}\left(\zeta^{i,A}\right) h\left(\zeta^{i,A}\right) \exp\left(-\frac{r}{1+|A|} \sum_{k \in A} t^*_k\right) \exp\left(-\frac{r}{1+|A|} t^*_i\right) \\ & + \sum_{B \supseteq \{i,j\}} \mathrm{P}\left(\zeta^{i,A}\right) h\left(\zeta^{i,A}\right) \exp\left(-r\overline{t}_\zeta\right) \end{aligned}$$

and analogously

$$\begin{aligned} V_j(t^*) = & \sum_{A \subseteq I \setminus \{i,j\}} \mathrm{P}\left(\zeta^{j,A}\right) h\left(\zeta^{j,A}\right) \exp\left(-\frac{r}{1+|A|} \sum_{k \in A} t^*_k\right) \exp\left(-\frac{r}{1+|A|} t^*_j\right) \\ & + \sum_{B \supseteq \{i,j\}} \mathrm{P}\left(\zeta^{i,A}\right) h\left(\zeta^{i,A}\right) \exp\left(-r\overline{t}_\zeta\right). \end{aligned}$$

Therefore

$$\begin{aligned} V_j(t^*) - V_i(t^*) = & \sum_{A \subseteq I \setminus \{i,j\}} \exp\left(-\frac{r}{1+|A|} \sum_{k \in A} t^*_k\right) [\mathrm{P}\left(\zeta^{j,A}\right) h\left(\zeta^{j,A}\right) \exp\left(-\frac{r}{1+|A|} t^*_j\right) \\ & - \mathrm{P}\left(\zeta^{i,A}\right) h\left(\zeta^{i,A}\right) \exp\left(-\frac{r}{1+|A|} t^*_i\right)] \\ & \underbrace{\leq}_{(i)} \sum_{A \subseteq I \setminus \{i,j\}} \exp\left(-\frac{r}{1+|A|} \sum_{k \in A} t^*_k\right) \mathrm{P}\left(\zeta^{i,A}\right) h\left(\zeta^{i,A}\right) \left[\exp\left(-\frac{r}{1+|A|} t^*_j\right) - \exp\left(-\frac{r}{1+|A|} t^*_i\right)\right] \\ & \underbrace{<}_{(ii)} 0 \end{aligned}$$

using in (i) that $\mathrm{P}\left(\zeta^{i,A}\right) h\left(\zeta^{i,A}\right) > \mathrm{P}\left(\zeta^{j,A}\right) h\left(\zeta^{j,A}\right)$ for all $A \subseteq I \backslash \{i, j\}$ and in (ii) the initial assumption that $t_j^* > t_i^*$. For the second result, if $t_i^* = t_j^*$ then we should have $\mathrm{P}\left(\zeta^{i,A}\right) h\left(\zeta^{i,A}\right) = \mathrm{P}\left(\zeta^{j,A}\right) h\left(\zeta^{j,A}\right)$ for all $A \subseteq I \backslash \{i, j\}$. As long as one such subset exists with strict inequality, gives the desired result that $t_i^* > t_j^*$. $\square$

Online Appendix: Not for Publication

Appendix B. Interpretations of Environment

B.1. **Example: Production Networks.** Agents running small and medium sized business have production technologies $z_i = a_i f(x_i, k_i)$ specifying inputs x_i including material inputs labor and outputs z_i, subject to idiosyncratic and aggregate shocks impacting a_i. In the context of a village economy, as one example, one can think of agents as households and think of prices of inputs (w) and outputs (p), respectively, as exogenously determined, outside the model. Profits, or losses, in terms of the obvious fiat money numeraire are $y_i = pz_i - wx_i$ are thus stochastically determined. Specifically within a period, previously accumulated capital k_i is as a given endowment. Then given current shocks a_i, a household running a firm decides on hiring labor and purchasing intermediate inputs to produce output z_i at the end of the period, subject to potential collateral constraints on financing (not written out here). This gives maximized within-period profits, which typically are linear in k_i. Profits are thus the random incomes that correspond with the primitives y_i of the basic model. Households are risk averse with indirect utility over potentially-smoothed end-of-period incomes, depending on mechanisms available, in the village economy.

However, participation in these networks is subject to shocks. The set of producers from whom intermediate inputs can be purchased, and the set of purchasers for sale are each a subset of all agents and further, subject to shocks. Links in supply chains for example, get broken. One mechanism to hedge variable profits is trade credit, for smoothing: if low, a firm can extend the date due for debiting the budget, negotiating an account payable, or be less generous on extending credit on accounts receivable, or, if profits are high, the opposite.

B.2. **Example: Income as Portfolio Returns.** Agents have an initial random endowment $e \in \mathbb{R}^n$, jointly distributed Gaussian: $e \sim \mathcal{N}(\mu_e, \Sigma_e)$. After observing their endowment, they have access to a set of $K \geq 1$ risky assets, with random linear returns, and a safe asset with a gross rate of return of 1. Formally, if an agent invests $w_{ik} \in \mathbb{R}$ units in asset k, they will get a gross return of $(1 + R_k) w_{ik}$, where $R \sim \mathcal{N}(\rho, \Lambda)$ where $\rho \in \mathbb{R}^K$ and Λ is a $K \times K$ symmetric and positive definite covariance matrix. Their endogenous income is then $y_i = e_i + w_i' R$. The only point of departure with the usual risk sharing environment is on trading opportunities or *market participation*. Not every agent is present in the market in every state; only a random set of agents gets access to the market, which can be thought of as a meeting place where they can trade. If agents do not have access to this market, they are in autarky and have to consume their endowment.

B.3. **Generalizations.** The basic setup can be generalized considerably. First of all, we can index by time, with long or even infinite horizon. We can entertain Markov process on shocks. Our timeline can be divided into sub-periods: traders meet in a market for two or more periods before the next market participation draw (and we allow both implementation via bilateral links of a multi-person outcome as well as borrowing and lending with risk contingencies within the longer period). Though dynamics could easily be incorporated throughout most of the paper, we spare the reader the requisite notation.

We are featuring one good but we can easily generalize the notation and allow commodity vectors over goods. Then there would be a sequence of resource constraints (market clearing), one for each good; utility functions still strictly concave though. Likewise we can reinterpret goods as securities and endowments as portfolios.

Trivially, our setting could be partial equilibrium with prices of all goods, or assets, fixed outside, as in a small open economy, one market at a time, or one village at a time. In this case value functions would be strictly concave over a selected numeraire good, taking outside prices as given. It is also easy to allow preference shocks rather than endowment shocks.

Moreover, we can generalize this to many cliques of agents meeting or, in other words, many segmented markets that are drawn in parallel, with ζ now being an $n \times k$ matrix and ζ_i^m is a dummy for whether i participates in market m. We study this in Online Appendix F.1.

Notice that our model does not force agents to interact with the same collection of agents in every period or every state. So, for example, an agent of type A may interact with those of type B in one state. But in another state perhaps agents of type A interact with agents of type C.

Appendix C. Responsiveness to Liquidity Injections

We explore agents' choices to determine whether or not they participate in contexts wherein the infinitesimal liquidity injection affects their participation distribution. This not only correlates y and ζ through this endogenous decision making process (which alone does not necessarily make the model responsive to injections), but changes the financial centrality expression as we noted in Proposition 4.1. Centrality now captures how marginally increasing income in states that the agent trades in, increases both the likelihood that the agent trades and the concurrent market participation decisions of other agents. This stands in contrast to endogenous participation models wherein the participation distribution is inert to infinitesimal liquidity injections.

C.1. **Overview.** Let us begin by clarifying the role of endogenous participation. There are two cases. In the first, every agent can decide whether or not to participate, given a distribution of participation opportunities. For instance, in a given state of the world, agents $i_1, \ldots, i_m$ may have the opportunity to enter the market, but not all necessarily decide to participate. If this decision in equilibrium is unchanged by an ϵ liquidity injection to any agent, then we may as well imagine the participation distribution as being exogenous. Such an example was given in the introduction. Financial centrality is identical even in the case where agents choose to participate; after all, the choice has no meaningful bearing on altering this distribution, so the planner evaluates the injection in the same manner as if this participation distribution was indeed exogenous.

In contrast, in what follows below, we focus on cases where the injection affects the participation decision itself. This allows us to study the participation effect, which thus far has not been a factor in our analysis of financial centrality. Recall Proposition 4.1, wherein financial centrality included an extra term

$$\mathbb{E}_s \left\{ \left[\sum_{j \in I} \lambda_j u_j \left(c_j \left(s \right) \right) \right] \mathbf{S}_i \left(\zeta \mid y \right) \right\}$$

which we called the participation effect. The goal of this section is to study this term beyond the example presented in the body. So we present two additional models that are responsive to infinitesimal liquidity injection, but with no income effect (i.e., $S_i(y) = 0$ almost surely). We also present for clarification and contrast, a fourth model in which participation and income are correlated and yet the response is inert.

C.2. **Private Information about Income Shocks.** In this example, the consumption allocation is also common knowledge, but agents can only observe (objective) private information about both income shocks $y \in \mathbb{R}^n$, and about other agents information. We encode beliefs and higher order beliefs about income shocks and information using a *type space* structure, a modeling device introduced by Harsanyi (1967). Formally, we model agents' beliefs with a *signal structure* (or a *common prior type space*) $\mathcal{Z} = \left\{(Z_i, \beta_i : Z_i \to \Delta(Y \times Z_{-i}))_{i \in I}, \beta_0\right\}$ where $z_i \in Z_i$ is the agent's signal (or *type*). Here this represents the information they observe before observing the draw of $s = (y, \zeta)$. $\beta_0 \in \Delta(Y \times \prod_i Z_i)$ is a common prior distribution over income shocks and signals and $\beta_i(\cdot \mid z_i)$ is the conditional belief distribution over income shocks and signals of other agents, derived from β_0 using Bayes rule.[12] Because Y is assumed to be finite and the choice set for every agent is binary, we can focus also only on finite signal spaces. We also add the constraint that $\mathrm{marg}_Y \beta_0 = F$ (i.e., the marginal distribution over income shocks coincide with the true distribution of shocks). Based on its type, agent i decides whether or not to access the market.

The timing is as follows:

(1) Income shocks $y \in \mathbb{R}^n_+$ is drawn according to $F(y)$.
(2) Agents observe only $z_i \in Z_i$, which are jointly drawn with probability

$$\mathrm{P}(z \mid y) = \beta_0(y, z) / \sum_{\hat{y} \in Y} \beta_0(\hat{y}, z). \tag{C.1}$$

(3) Agents decide whether to access the market ($\zeta_i = 1$) or not (which may be costly, with commonly known participation costs k_i) given their private information $z_i \in Z_i$.
(4) State $s = (y, \zeta)$ is publicly observed, and agents consume according to allocation $c(s)$.

To characterize the agents' market participation decisions, they need to form beliefs over the vector of income draws and market participations. We will model this as a game, where agent's strategies are the mappings from information to market participation. The natural solution concept here is the Bayesian Nash Equilibrium (BNE): a profile of functions $\zeta_i^* : Z_i \to \{0,1\}$[13] is a BNE if and only if, for all $i \in I$ and all $z_i \in Z_i$

$$\text{if } \zeta_i^*(z_i) = 1 \Longrightarrow \mathbb{E}_s\{u_i[c_i(s)] \mid \zeta_i = 1, z_i\} - k_i \geq \mathbb{E}_s\{u_i(y_i) \mid z_i\}$$

where the expectations for each agent is taken with respect to the probability measure

$$\mathrm{P}(s = (y, \zeta) \mid z_i) := \sum_{y \in Y} \sum_{j \neq i} \left[\sum_{z_j \in Z_j : \zeta_j^*(z_j) = \zeta_j} \beta_i(y, z_{-i} \mid z_i) \right].$$

[12]That is, for all (y, z_i, z_{-i}) we have $\beta_i(y, z_{-i} \mid z_i) = \frac{\beta_0[y,(z_i,z-i)]}{\sum_{\hat{t}_i} \beta_0[y,(\hat{z}_i,z_{-i})]}$.

[13]Without loss of generality, we focus on pure strategy equilibria.

Given a signal structure $\mathcal{Z}$ and a BNE profile $\zeta^* = (\zeta_i^*(\cdot))_{i\in I}$, we can then derive an ex-ante equilibrium distribution over states $s = (y, \zeta)$ as

$$\mathrm{P}(s = (y,\zeta)) = \mathrm{P}(y) \sum_{z\in Z:\zeta_i^*(z_i)=\zeta_i \forall i\in I} \mathrm{P}(z \mid y),$$

using (C.1). This would be the measure used by the social planner when measuring financial centrality, since they have to integrate over agents' signals from an ex-ante perspective, according to the assumed common prior distribution β_0.

In the model proposed in Section C.2, we assume that the credit line policy $t = (t_j)_{j\in J}$ is common knowledge among agents, and hence the policy has no effect on the information agents have access to. It does, however, affect the relative utility of market access. That is, the market access strategy (given transfer $t_i \geq 0$) is

$$\zeta_i^*(\theta_i \mid t) = 1 \iff \mathbb{E}_s \{u_i[c_i(y_i + t_i, y_{-i}, \zeta)] - u_i(y_i) \mid \theta_i\} \geq k_i. \tag{C.2}$$

If $c_i(\cdot)$ is a weakly increasing in own endowment (e.g., $c_i(s) = \overline{y}_\zeta$ in an environment with an utilitarian planner, and agents with homogeneous preferences) the transfer t_i acts as a subsidy for market participation, increasing the set of signals θ_i for which condition (C.2) is satisfied. However, since the transfer policy is assumed to be common knowledge, this also affects the marker participation decisions of other agents. If c_i is weakly increasing for all agents (e.g., also $c_i(s) = \overline{y}_\zeta$) then other agents also have higher incentives to access the market, since it is more likely that i will be trading, and i is more valuable, since i increases aggregate income whenever they trade. We summarize this result in the following corollary.

Corollary C.1. *Consider the above model described in Section C.2 and $\lambda \in \Delta^n$. If the allocation $c(\cdot)$ solving (3.2) is non-decreasing in y, then $FC_i > \mathbb{E}_s[\zeta_i q(s)]$.*

C.3. **Moral Hazard and Effort in Accessing the Market.** We briefly set up another example of endogenous market participation, without fully analyzing it, which concerns moral hazard. This is a generalization of the model analyzed in the preceding section. We take the exact same signal structure as before. The only difference is that instead of being a binary decision (whether to access the market or not) here we have a continuum of choices.

Assume that y is realized and every agent i observes only z_i, an imperfect signal about y (i.e., $z_i \sim \pi_i(z_i \mid y)$ for some conditional cdf π_i). Given this private information, agents simultaneously choose the probability of accessing the market, denoted by $p_i(z_i) \in [0,1] = \mathrm{P}(\zeta_i = 1)$. Agents have to pay a disutility cost $\psi(p)$, where ψ is strictly increasing and convex.

Given the profile of functions $(p_i : Z_i \to [0,1])_{i=1}^n$, the joint probability of market participation, given income draws, is given by

$$\mathrm{P}(\zeta \mid z) = \prod_{i=1}^n [p_i(z_i)]^{\zeta_i} [1 - p_i(z_i)]^{1-\zeta_i}.$$

Then consumption is realized according to a feasible consumption allocation $\hat{c}(s) = \zeta_i c_i(s) + (1-\zeta_i) y_i$, where $c_i(\cdot)$ is an (equilibrium) feasible allocation. For this example, we leave unspecified the choice of the consumption allocation, and it is only assumed that the consumption allocation as a function of the state $s = (y, \zeta)$ is common knowledge among agents.

Agents preferences (given $p_i(\cdot)$) are

$$U_i\left(y_i, p_i \mid (p_j(\cdot))_{j\neq i}\right) = p_i \mathbb{E}_{t_{-i},s}\left\{\sum_{\zeta_{-i}}\prod_{j\neq i}\left[p_j(z_j)\right]^{\zeta_j}\left[1-p_j(z_j)\right]^{1-\zeta_j} u_i\left[c_i\left(y_i, y_{-i}, \zeta_i = 1, \zeta_{-i}\right)\right] \mid z_i\right\}$$
$$+ (1-p_i)\, u_i(y_i) - \psi(p_i).$$

As in the private information example above, the solution concept once again is the BNE, with $p^*(t) = (p_i^*(z_i))_{i\in N}$ such that for all i and all $y_i \in Y$:

$$p_i^*(z_i) \in \underset{p_i\in[0,1]}{\operatorname{argmax}}\, U_i\left(y_i, p_i \mid \left\{p_j^*(\cdot)\right\}_{j\neq i}\right).$$

C.4. **Model with Investment in Risky Assets.** Consider the model in Section B.2 , with no participation effect (i.e. $\mathbf{S}_i(\zeta \mid y) = 0$ a.e) where agents choose their portfolios $w_i \in \mathbb{R}^K$ after the liquidity injection. We will show that in the case with CARA preferences and gaussian returns, there is no income effect (i.e. $\mathbf{S}_i(y) = 0$ a.s.), although this would typically not happen with different preferences.

The timing in this model is then (1) endowments are drawn; $e \sim \mathcal{N}(\mu_e, \Sigma_e)$ (2) Agents observe (e, ζ) and choose portfolios w_i (contingent on being in the market) (3) Returns are drawn $R \sim \mathcal{N}(\rho, \Lambda)$ and incomes are realized as $y_i = e_i + w_i' R$ and (4) Consumption is realized according to allocation. In general, an increase in the liquid asset could have effects on the demands of the risky and risk free assets, changing marginally their portfolio choice, and hence, the distribution of income. However, in the case of CARA preferences, this will not be the case.

Proposition C.1. *Take the model with CARA preferences and gaussian returns. After the realization of e_i, the portfolio choice is $w_i = (r_i\Lambda)^{-1}\rho$ (i.e. is independent of the realization). Therefore, income is distributed as $y_i = e_i + \rho'(r_i\Lambda)^{-1} R$.*

Proof. See Online Appendix H. □

Corollary C.2. *In the investment model with CARA preferences and gaussian returns, there is no income effect; i.e., $\mathbf{S}_i(y) = 0$ a.s.*

The reason behind the corollary is that the investment in the risky assets are independent of the endowment level, and hence, will also be independent of the liquidity injection, since it is equivalent ex-post to an increment in the available endowment. This is of course, a consequence of the absence of income effects exhibited by CARA preferences.

An important corollary of this model is how this model obtains income draws more positively correlated than the endowments process. More specifically, the result of Proposition C.1 implies that the income distribution has a covariance structure given by

$$\operatorname{cov}(y_i, y_j) = (r_i r_j)^{-1}\rho'\Lambda^{-1}\rho + \operatorname{cov}(e_i, e_j)$$

where the first component term, always positive, is the correlation resulting from the fact that all agents hold different amounts of a market portfolio $P = \Lambda^{-1}\rho$. This effect exacerbates aggregate risk, both with inert or responsive environments in market participation. This source of contagion is studied in Jackson and Pernoud (2019).

C.5. **Team Production Environments.** Finally, we illustrate that simply having income and participation being correlated does not mean a model exhibits responsiveness. We consider a setting where market participation shocks are determined exogenously first and then the income distributions for agents with market access depend on the identities of those trading. Formally, the timing on the resolution of uncertainty would be as follows: (1) Market participation ζ is drawn according a distribution $G(\zeta)$; (2) Income distribution is drawn from $y \sim F(y \mid \zeta)$. A leading example of such an environment is one of *team production*. Agents without market access draw income from their autarky income distribution $y_i \sim F_i(y_i)$. However, once agents are drawn together to form a market, income is drawn jointly, and then agents can divide aggregate income draws amongst them in any feasible consumption allocation.

Models like this also show correlation between market participation and income. However, it is straightforward to see that in such models, financial centrality is simply $FC_i = \mathbb{E}_s\{\zeta_i q(s)\}$ as before. This is simply because the injection policy of giving an injection to agent i has no effect on the market participation distribution, since it is assumed here to be exogenous to income draws. Clearly, this setting is one that is inert to infinitesimal liquidity injection. But unlike the baseline model, income and market participation are now not independent, and the expectation has to be calculated over market participation and income shocks jointly.

Appendix D. A Tractable Example of a Market Formation Process

Until now our discussion of the stochastic financial network has been rather abstract. It has been a fairly unrestricted distribution over the space of all subsets of agents: the realized market can be comprised of any subset of agents and then there is a distribution over each possibility. It is nonetheless instructive to examine specific examples that may micro-found the stochastic financial network distribution.

To make matters simple, consider the homogenous parameter case. Since

$$FC_i \propto \mathbb{E}_\zeta\left\{\zeta_i\left(1+\frac{\gamma}{n(\zeta)}\right)\right\} \tag{D.1}$$

we need to calculate $\mathbb{E}\left\{\frac{1}{n(\zeta)} \mid \zeta_i = 1\right\}$ and $\mathrm{P}(\zeta_i = 1)$.

D.1. **Generalized Poisson Model.** We generalize the degree model presented in the body. Let $z_i \in [0,1]$ denote the probability that an agent gets selected as the host. Then let $\mathbf{p}$ denote a matrix with entries $\mathbf{p}_{i,j}$ denoting the probability that j is in the market when i is the host, which is independent across j. We set $\mathbf{p}_{i,i} = 1$.

It is useful to define an individual specific parameter, which is the expected number of individuals in the trading room when i is selected as host, ν_i. This can be computed as $\nu_i := \sum_j \mathbf{p}_{ij}$. To characterize financial centrality, we need to know the expected sizes of the trade rooms when i is host and conditional on i being in the room, integrating across the other possible hosts. Two auxiliary random variables will be very useful in the rest of the section: $\mathbf{n}_{-i} = (n_\zeta - 1) \mid i$ is host, and $\mathbf{n}_{-j,i} = (n_\zeta - 2) \mid j$ hosts & $\zeta_i = 1$. This means that whenever i hosts, market size is $n_\zeta = 1 + \mathbf{n}_{-i}$, and whenever j hosts, and we condition on i accessing the market, then market size is $n_\zeta = 2 + \mathbf{n}_{-j,i}$. This auxiliary random variables have range from 0 to $k \in \{n-1, n-2\}$, and based on our assumptions, we have

$$\mathbf{n}_{-i} \sim \sum_{k \neq i} \text{Bernoulli}\left(\mathbf{p}_{i,k}\right) \text{ and } \mathbf{n}_{-ji} \sim \sum_{k \notin \{j,i\}} \text{Bernoulli}\left(\mathbf{p}_{j,k}\right),$$

where these Bernoulli distributions are independent, with success probabilities strictly less than 1.[14] These distributions are also called *Poisson Binomial distributions* and have been extensively studied in the literature. We will write $X \sim PB\left(\mathbf{p}\right)$ with $p = (p_1, p_2, \ldots, p_k)$ the vector of success probabilities of each Bernoulli trial. This distribution, in some cases, can be well approximated by a Poisson distribution.[15] In this model, we have that $\mathbf{n}_{-i} \sim PB\left(\mathbf{P}_i\right)$ and $\mathbf{n}_{-ji} \sim PB\left(\mathbf{P}_{-j,i}\right)$ where $\mathbf{P}_i = \left(\mathbf{p}_{i,k}\right)_{k \neq i} \in [0,1]^{n-1}$ and $\mathbf{P}_{-j,i} = \left(\mathbf{p}_{j,k}\right)_{k \notin \{i,j\}} \in [0,1]^{n-2}$. These random variables are useful to write our approximation to financial centrality as

$$FC_i \approx \hat{FC}_i := \mathrm{P}\left(\zeta_i = 1\right) \times \left[1 + \gamma \mathbb{E}\left(\frac{1}{n_\zeta} \mid \zeta\right)\right]$$

$$= p_i \left[1 + \gamma z_i \mathbb{E}\left(\frac{1}{1 + \mathbf{n}_{-i}}\right) + \gamma \sum_{j \neq i} z_j \mathbb{E}\left(\frac{1}{2 + \mathbf{n}_{-ji}}\right)\right],$$

where $p_i = \mathrm{P}\left(\zeta_i = 1\right)$. Therefore, we need to calculate the inverse moments $\mathbb{E}\left(1/(1+X)\right)$ and $\mathbb{E}\left(1/(2+X)\right)$ for n Poisson Binomial random variables; $X = \mathbf{n}_{-i}$ and $X = \mathbf{n}_{-ji}$ for all $j \neq i$. Hong (2013) provides a general survey on the commonly used methods to calculate explicitly the probability function of Poisson Binomial distributions using either recursive or Discrete Fourier Transform methods, which are fairly fast even with large n.[16] We also survey results (starting with Le Cam (1960)) that show that if the expected number of successes of a Poisson Binomial distribution is sufficiently high (corresponding in this case with higher expected market sizes), then it can be well approximated by a Poisson distribution. [17] In the context of this model, it means that if $\mathbb{E}\left(\mathbf{n}_{-i}\right) = \nu_i - 1$ is small (relative to n), then we can approximate $\mathbf{n}_{-i} \sim \text{Poisson}\left(\nu_i - 1\right)$ and $\mathbf{n}_{-ji} \sim \text{Poisson}\left(\nu_{-ji}\right)$, where $\nu_{-ji} = \mathbb{E}\left(n_{-ji}\right) = \sum_{k \notin \{i,j\}} \mathbf{p}_{jk} = \nu_j - \mathbf{p}_{ji} - 1$.

If $X \sim \text{Poisson}\left(\nu - 1\right)$, then $\mathbb{E}\left(1+X\right)^{-1} = m_1\left(\nu\right) := \left[1 - \exp\left(1-\nu\right)\right] / \left(\nu - 1\right)$ and $\mathbb{E}\left(2+X\right)^{-1} = m_2\left(\nu\right) := \left[1 - m_1\left(\nu\right)\right] / \left(\nu - 1\right)$, both strictly decreasing functions of $\nu \geq 1$. Using these formulas,

[14] Exact and approximation methods for calculating expectations of market sizes are sensitive to the assumption of interior (i.e., in $(0,1)$) success probabilities. This is the reason for the need to define the random variables $\mathbf{n}_{-i}$ and $\mathbf{n}_{-ji}$.

[15] Le Cam (1960) provided bounds on the error of approximation, which were improved by Stein (1986); Chen (1975), and Barbour and Hall (1984); Sason (2013) show that if $\hat{X}$ is the Poisson approximation with mean $\lambda_n = \sum_{i=1}^n p_{i,n}$, then $d_{TV}\left(X, \hat{X}\right) \leq \left(1 - e^{-\lambda_n}\right) \sum_i p_{i,n}^2 / \lambda_n$, where $d_{TV}\left(\cdot\right)$ denotes the total variation distance. This approximation will then typically be valid (for large n) if and only if $\lim_{n \to \infty} \lambda_n = \infty$; i.e. when market sizes ν_i grow without bound as the number of agents increases.

[16] Chen and Liu (1997) show stable (i.e. non-alternating) methods are $O\left(n^2\right)$, which would make the calculation of financial centrality of a given agent be $O\left(n^3\right)$. Discrete Fourier Methods are usually much faster (Fernández and Williams (2010)). See Hong (2013) for a general survey on the existing exact and approximating methods.

[17] This is not the only approximation studied in the literature. In models where the expected market size is high, Gaussian approximations behave rather well (see Volkova (1996), Hong (2013)). If success probabilities are similar (i.e., the variance $\sigma_p^2 := n^{-1} \sum_i \left(p_i - \overline{p}\right)^2$ is small enough) then approximation to a Binomial distribution is fairly accurate (Ehm (1991); Barbour et al. (1992)).

we can then approximate $\hat{FC}_i$ by:

$$\hat{FC}_i \approx p_i \times \left\{ 1 + \gamma z_i m_1(\nu_i) + \gamma \sum_{j \neq i} z_j m_2(\nu_j - \mathbf{p}_{j,i}) \right\}.$$

This shows the following. First, nodes with a larger expected reach as measured by ν_i are more central (as long as n is large enough relative to γ). Second, nodes that have larger expected inverse room size when they are hosts are more central. Third, i is more central when $\mathbf{p}_{j,i}$ increases, particularly when ν_j is small. So when j tend to invite small rooms as hosts, but i is likely to be in such a j's room, then i is more valuable.

A special case are symmetric models, where $z_i = 1/n$ for all i and $\mathbf{p}_{i,j} = \mathbf{p}_{j,i}$ (e.g., the model $\mathbf{p}_{i,j} = \alpha^{\delta(i,j)}$, since distance is symmetric). In this case

$$FC_i = \frac{1}{n}\nu_i \left\{ 1 + \gamma \times \frac{1}{n} m_1(\nu_i) + \gamma \sum_{j \neq i} \frac{1}{n} m_2(\nu_j - \mathbf{p}_{j,i}) \right\}.$$

This has the advantage that the centrality of agent i depends solely on the expected market size of each agent (as a host) that they get connected to, and the probability that i conects to them. The marginal value of the inverse room size effect when i is the host, proportional to $\nu_i \times m_1(\nu_i)$, declines in ν_i if and only if $\nu_i \geq 2.79$ (there is a positive effect in $\mathrm{P}(\zeta_i = 1)$, but an offsetting negative effect in $m_1(\nu_i)$).

If we want to calculate centrality exactly, we can still use the calculation of the exact pdf of $\mathbf{n}_{-i}$ and $\mathbf{n}_{-ji}$ to get the exact financial centrality. For example, in the CARA-Normal model with homogeneous preferences and independent and identically distributed income draws, we know that $FC_i = \mathbb{E}_\zeta \{\zeta_i \exp(\gamma / n_\zeta)\}$, which can be decomposed as

$$FC_i = p_i \left\{ z_i \mathbb{E}\left[\exp\left(\frac{\gamma}{1 + \mathbf{n}_{-i}}\right)\right] + \sum_{j \neq i} z_j \mathbb{E}\left[\exp\left(\frac{\gamma}{2 + \mathbf{n}_{-ji}}\right)\right] \right\}$$

and then be calculated explicitly using the distributions for $\mathbf{n}_{-i}$ and $\mathbf{n}_{-ji}$.

D.2. **Sequential Market Formation.** In the Poisson models, for $j \neq k \neq i$, note that $\zeta_j \perp \zeta_k$ conditional on i hosting. But trading groups may be determined sequentially, along a chain of meetings. In this case the study of random walks on graphs provides the right vocabulary to capture this.

We can model this in a simple way, though the analytic characterization is hard to come by. Let $(z_i)_{i \in I}$ denote the probabilities that each node is the host, let $(p_{ij})_{i,j \in I}$ denote the probability that i meets j, and let β be the probability that at each stage the chain continues. With complementary probability $1 - \beta$, the chain terminates exogenously. However, the chain also terminates if an agent is revisited (and hence no new agents are added to the market).

This process, at termination, determines the size of the trading room. While it is easy to describe, and easy to simulate, it is hard to analytically compute moments for the distribution of $\frac{1}{n_\zeta}$ (Aldous and Fill, 2002; Durrett, 2007), even if chains are not terminated upon revisiting an agent. This is because what matters is the number of distinct agents in the market, not just the number of steps the chain makes (which, in that case, would simply follow a geometric random variable). In the special case with large n, $z_i = 1/n$, $\beta = 1$ (no random exogenous termination) and $p_{ij} = 1/d_i$ (i.e., uniform random walk, with equal probability among first degree neighbors) and g comes from an Erdös-Renyi process, Tishby et al. (2017) get closed form expressions for the distribution of chain length (or market size in our setup), showing that it follows a product of an exponential and a Rayleigh distribution.[18]

D.3. **Market Participation Shocks as Transaction Chains.** Now we give an alternative interpretation of the market participation shocks. Any market participation shock can be interpreted as a realization of a chain of bilateral transactions among a subset of agents in the economy, which are allowed to run short-run deficits. Formally, a simple transaction chain is a set of agents that can only trade with adjacent agents. Namely, there is a set of agents $J = \{i_1, i_2, \ldots, i_k\} \in I$ (which are selected randomly), such that i_j can trade only with agents i_{j-1} and i_{j+1}, for $j \in \{0, 1, \ldots, k\}$ (except for the first agent i_1, who can only trade with i_2, and the last member i_k, who can only trade with i_{k-1}). Agent j can make or receive transfers $\hat{T}_{j,h} \in \mathbb{R}$ for $h \in \{j-1, j+1\}$, which might be such that $\hat{T}_{j,h} + y_j < 0$ (i.e., giving agent h more than the endowment they have at the moment of the transaction). If $\hat{T}_{j,h} > 0$ it means that j sends resources to agent h, while $\hat{T}_{j,h} < 0$ means that j receives resources from k. The budget constraint that j faces is then $T_{j,j-1} + T_{j-1,j} + T_{j,j+1} + T_{j+1,j} \leq y_j$. Defining $T_{j,h}$ as net transfers instead of gross transfers, we then have that $T_{j,j+1} = -T_{j+1,j}$. Therefore, we can work only with the net transfers $T_j = T_{j,j+1}$ for agents $j = 1, 2, \ldots k-1$, and the simplified budget constraint for each agent is

$$T_j \leq y_j + T_{j-1}$$

for every $j = 1, \ldots, k-1$. There is a clearing house that, at the end of the day, settle all transactions. That is, agents can have short run deficits, but at the end of the period, payments are settled simultaneously, once all transactions are agreed upon. Without loss of generality, let's assume $i_j = j$, so that $C = \{1, 2, \ldots k\}$. A consumption profile of the agents in the chain C, is a description of consumption amounts $c = (c_1, c_2 \ldots c_k)$. A consumption allocation is feasible if and only if $\sum_{i=1}^{k} c_i = \sum_{i=1}^{k} y_i$. We say that a consumption bundle is *transfer-feasible* if and only if it is feasible and there exist transfers $\{T_{i,j}\}_{i=1}^{n}$ such that

(1) $c_j = y_j + T_{j-1} - T_j \geq 0$
(2) $\sum_{j=1}^{k-1} (T_{j-1} - T_j) = 0$.

In order to be able to define this objects for all j, we set $T_{1-1} = T_{k,k+1} = 0$. Therefore, for $i = 1$ we have $c_1 = y_1 - T_2$ and for $i = k \in \{1, \ldots n\}$ we have $c_k = y_k + T_{k-1}$. For such a consumption allocation, we say the sequence of net transfers $\{T_j\}$ *implements the allocation* c. The (rather obvious) result is that the set of feasible consumption profiles is equal to the set of transfer feasible allocations. This then implies that by modeling the interactions among agents as trades as if everyone was trading with each other is just an useful representation.

So, the basic assumptions in this environment are that (1) agents can only trade bilaterally with adjacent agents (with a predetermined order) in the chain and (2) promises to pay (i.e., net transfers) have to be settled jointly, after all trades have been agreed upon. This is the most

[18] This, of course, can be adapted by allowing $\beta \in (0, 1)$.

important assumption which abstracts away from leverage or run-away constraints (which would limit the short-run deficits agents can have in any given moment). In Proposition D.1 we show that, if we allow agents to run short-run deficits until the end of the day, when all transactions are settled, then any feasible consumption allocation among k agents can be implemented by a trading chain (in no particular order of agents).

Proposition D.1. *Let $c = (c_i)_{i=1}^{i=k}$ be a feasible consumption allocation (so $\sum_i c_i = \sum_i y_i$). Then, the net transfers T_j defined as*

$$T_j = T_{j \to j+1} := \sum_{i=1}^{i=j} (y_i - c_i) \tag{D.2}$$

implement c. Moreover, the following gross transfers implement c

$$\hat{T}_{j \to j+1} = \max\{0, T_j\} \text{ and } \hat{T}_{j+1 \to j} = \max\{0, -T_j\}$$

so either $\hat{T}_{j \to j+1} = T_j > 0$ and $\hat{T}_{j+1 \to j} = 0$, or $\hat{T}_{j \to j+1} = 0$ and $\hat{T}_{j+1 \to j} = -T_j \geq 0$.

Proof of Proposition D.1. The fact that $\sum_{j=1}^{k} (T_{j-1} - T_j)$ comes from equation (D.2): we have

$$T_{j-1} - T_j = \sum_{i=1}^{i=j-1} (y_i - c_i) - \sum_{i=1}^{i=j} (y_i - c_i) = c_j - y_j$$

and hence

$$\sum_{j=1}^{k} (T_{j-1} - T_j) = \sum_{j=1}^{k} (c_j - y_j) = 0$$

since c is feasible. The consumption attained for each agent is

$$\hat{c}_j = y_j + T_{j-1} - T_j = y_j + (c_j - y_j) = c_j$$

i.e., it achieves the target consumption allocation. $\square$

Appendix E. Decentralization as Arrow Debreu Economies

Proof of Proposition 5.1. We will focus on allocations where $c_i(s) > 0$ for all $s = (y, \zeta) : \zeta_i = 1$ for simplicity. Since (5.1) is a convex optimization problem and $u(\cdot)$ is strictly concave and differentiable, Kuhn-Tucker conditions are necessary and sufficient to characterize the optimum. This is also true for the planner's problem (3.2). Let $\mu_i > 0$ be the Lagrange multiplier of the AD budget constraint in (5.2) (this constraint will always be binding). The first order conditions of the consumer problem with respect to $a_i(s)$ at states $s = (y, \zeta) : \zeta_i = 1$

$$u_i'[c_i(s)]\,\mathrm{P}(s) = \mu_i r(s) \text{ for all } s : \zeta_i = 1 \text{ where } c_i(s) = y_i(s) + a_i(s) \tag{E.1}$$

where $c_i(s) = y_i(s) + a_i(s)$. Also see that the choice of $a_i(s)$ is superfluous in the consumer's problem if $c_i(s) = 0$ for all $s : \zeta_i = 0$, and that the budget constraint can be written as $\sum \zeta_i c_i(s)\, r(s) \leq \sum \zeta_i y_i r(s)$. Hence $c = (c_i(s))_{i \in I, s \in S}$ is a Walrasian Equilibrium with transfers allocation if $\exists \mu_i > 0 \forall i \in I$ such that conditions (E.1) and the resource constraint (3.3) are satisfied, and such that $c_i(s) = y_i$ for all $s : \zeta_i = 0$. The corresponding Walrasian Equilibrium has $a_i(s) = c_i(s) - y_i$, $r(s) = (1/\mu_i)\, u_i'[c_i(s)]\,\mathrm{P}(s) > 0$ and $\tau_i = \sum_s a_i(s)\, r(s) = (1/\mu_i)\,\mathbb{E}_s\{[c_i(s) - y_i]\, u_i'[c_i(s)]\}$.

Doing the same exercise for the planner's problem (3.2) , we get that a consumption allocation $c_i(s)$ solves the planner's problem with Pareto weights $\lambda \in \Delta$ if and only if it satisfies the resource constraint (3.3) for all $s \in S$, $c_i(s) = y_i$ for all $s : \zeta_i = 0$ and all $i \in I$, and satisfies for all $i \in I$:

$$\lambda_i u_i'[c_i(s)] = q(s) \text{ for all } s : \zeta_i = 1 \tag{E.2}$$

where $q(s)$ is the (normalized) Lagrange multiplier of the resource constraint at state s.

Therefore, a Walrasian Equilibrium with transfers consumption allocation c will also be the solution to the planner's problem (3.2) with Pareto weights $\lambda_i = 1/\mu_i$. Likewise, for given $\lambda \in \Delta$, the solution to the planner's problem (3.2) will be a Walrasian Equilibrium with transfers if we take $\mu_i = 1/\lambda_i$. Moreover, the implementing price function $r(s)$ and transfers τ_i satisfy:

$$\begin{aligned} r(s) &= (1/\mu_i)\, u_i'[c_i(s)]\,\mathrm{P}(s) = q(s)\,\mathrm{P}(s) \\ \tau_i &= \mathbb{E}_s\{[c_i(s) - y_i]\, q(s)\} \end{aligned} \tag{E.3}$$

since $1/\mu_i = \lambda_i$. $\square$

Of course, there is a mapping between a Walrasian Equilibrium without lump-sum transfers and its corresponding utilitarian planner representation, with its Pareto weight vector λ. Two special cases are of interest. In the benchmark case of the CARA-Normal model, assuming constrained efficient allocations are implemented without lump sum transfers, we obtain a fixed point equation mapping the primitives of the model (income distribution moments and preferences) to the Pareto weights of the planner's problem which we derive in Online Appendix G.

We also show that in the case where the planner has uniform Pareto weights (i.e., $\lambda_i = 1/n$ for all i), preferences are identical and shocks are i.i.d. Gaussian variables, then the planner's problem can be implemented by a Walrasian Equilibrium with no transfers with $q(s) = \exp\left(-r\overline{y}_\zeta\right)$ and $c_i(s) = \zeta_i \overline{y}_\zeta + (1 - \zeta_i)\, y_i$, where $\overline{y}_\zeta := \frac{1}{n_\zeta} \sum_{j \in I} \zeta_j y_j$ is the mean income of agents in the market, and $n_\zeta := \sum_{j\in I}\zeta_j$ is the market size at state s. Moreover, the price of personalized debt is simply $FC_i = \mathbb{E}_\zeta\left\{\zeta_i \exp\left(\gamma\frac{\sigma^2}{2}\right)\right\}$.

Appendix F. Extensions

In this section we study two extensions that depart from the class of environments above. One of the most seemingly important restrictions on the models studied so far is the existence of *centralized markets*. That is, agents either are in autarky or have market access and can trade with any other agent that also has market access. While the bilateral trading chains introduced above relaxes this interpretation, it maintains the possibility that any agent is reachable by any other through a finite sequence of trades, as long as both have market access. In Section F.1 we introduce a generalization of the basic environment, allowing for the existence of several *segmented markets* working in parallel, where agents can only trade among a subset of all agents who have market access. That is, a draw from the stochastic financial network consists of a collection of subgraphs (cliques). For example $ijkl$ and $mnop$ may be two cliques of four who can exchange with each other in some state of the world. But in another state of the world, perhaps the cliques are ij, kl, mno, and p (a singleton). Each clique is a segmented market. We show that the basic definitions and formulas of financial centrality still hold, if we reinterpret having "market access" to be present in the market where the agent being injected with liquidity is trading at.

Another important assumption maintained throughout this paper is that the social planner evaluating the marginal value of injected liquidity also is able to implement the allocation $c(\cdot)$ that maximizes her expected utility. However, a relevant case is one where the planner can only influence the economy by the liquidity injection policies and cannot directly choose the allocation herself. This would be the case when the allocation is chosen according to some other solution concept, like Walrasian Equilibrium, multi-player bargaining games, and so on. In such situations, the social planner would have to take the consumption allocation as given when measuring the marginal effects of injecting liquidity in this economy. In Section F.2 we study financial centrality under the assumption that the consumption allocation is Pareto optimal, which implies that there exist some representing social preferences (i.e., Pareto weights) for which it would be optimal. We then obtain similar expressions for financial centrality, which now incorporates a term relating the Pareto weights of the social planner with the representative Pareto weights of the allocation.

F.1. **Segmented Markets.** We consider an environment with the same income shocks and preferences, but one where agents may gain access to random, segmented markets. Formally, a *market segmentation* is a partition $\pi = \{m_1, m_2, \ldots m_r\}$ over the set of agents I; i.e., $\cup_{m\in\pi} m = I$ and $m \cap m' = \emptyset$ for all $m \neq m'$. In this alternative environment, the relevant state of nature is now $s = (y, \pi)$, where π is the *market segmentation state*, with probability distribution $\mathrm{P}(s)$. We refer to each $m \in \pi$ as *a market* at state s. Let $\mathcal{P}$ be the set of all partitions of I that have positive probability under $\mathrm{P}(s)$. We denote $m(i, \pi) \in \pi$ to be the market (at segmentation π) where i is able to trade. If $m(i,\pi) = \{i\}$, we say i is in autarky at π, and otherwise we say i has *market access* at π.

Segmented markets now modify the definition of feasibility of allocations. We say that an allocation $c = (c_i(s))_{i\in I}$ is feasible if and only if, for all $s = (y, \pi)$ and all $m \in \pi$ we have

$\sum_{i \in m} c_i(s) \leq \sum_{i \in m} y_i$. Clearly, the class of environments embeds the single market environments studied before—i.e., markets where any partition π in the support is made up of a single multi-agent market $m_u(\pi) \subseteq I$ with $|m_u(\pi)| \geq 1$, and everyone else being in autarky. Hence we can summarize the state by $s = (y, \zeta)$ where $\zeta_i = 1$ if and only if $i \in m_u(\pi)$. In general, for a given partition π we write $\zeta_i^m \in \{0, 1\}$ for the indicator of whether i has access to market m.

Given Pareto weights $\lambda \in \Delta^n$ and agent $i \in I$, the planner's problem value function of injecting liquidity $t_i \geq 0$ to agent i is $V^s(t_i) := \max_{(c_j(y,\pi))_{j \in I}} \mathbb{E}_s \left\{ \sum_{j \in I} \lambda_j u_j [c_j(s)] \right\}$ subject to $\sum_{j \in m} c_j(s) \leq \sum_{j \in m} y_j + t_i \zeta_i^m$ for all $s = (y, \pi)$ and all $m \in \pi$. Financial centrality is now defined as before. Intuitively, a planner needs to integrate also over all possible market segmentations in order to assess the marginal value of the liquidity injection policy for agent i, since the shadow value of the injection will depend on the market agent i is trading at. We show that the financial centrality measure follows the same formula as in the centralized markets environments, in a "virtual single market economy" where having market access is understood as being able to trade with the agent of interest.

Definition F.1. Take a segmented market economy $\mathcal{E}$, with distribution over states $\mathbf{P}(y, \pi)$. Define $\mathcal{E}_i$ to be a *virtual single market economy* where all agents have identical preferences over consumption, and the distribution over outcomes $\tilde{\mathbf{P}}(y, \zeta)$ is given by:

$$\tilde{\mathbf{P}}(y,\zeta) = \mathbf{P}\left((y,\pi) \in Y \times \mathcal{P} : \begin{cases} (1) : j \in m(i,\pi) \text{ for all } j : \zeta_j = 1 & \text{and} \\ (2) : \# m(i,\pi) > 1 \end{cases} \right), \tag{F.1}$$

i.e., an agent $j \neq i$ has market access on economy $\mathcal{E}_i$ only when they are able to trade (i.e., in the same market) with agent i in $\mathcal{E}$.

Proposition F.1 asserts that financial centrality in a segmented markets economy follows the same "asset pricing formula" we had in Proposition A.3, but on the virtual single market economy $\mathcal{E}_i$. The proof is quite straightforward, and simply generalizes the proof of Proposition A.3 and is therefore omitted.

Proposition F.1. . *Let $\mathcal{E}$ be a segmented markets economy, and suppose it is inert to infinitesimal provisions of liquid assets. Then, for $i \in I$ and any $\lambda \in \Delta$, the financial centrality for agent i coincides with the financial centrality of agent i in the virtual single market economy $\mathcal{E}_i$. That is,*

$$FC_i := \frac{\partial V^s(t)}{\partial t_i} \Big|_{t=0} = \mathbb{E}^{\tilde{\mathbf{P}}}_{s=(y,\zeta)} \{\zeta_i q(s)\}$$

where $\mathbb{E}^{\tilde{\mathbf{P}}}(\cdot)$ is the expectation taken w.r.t measure $\tilde{\mathbf{P}}$ defined in F.1.

Intuitively, financial centrality only deals with the effect of the increase in agent i's endowment, which can only impact those agents who can trade with her. Because of separability of the planner's preferences over different agents consumptions, the marginal welfare effect on the segmented markets i is trading on have no effect on the welfare evaluation of other segmented markets at the same time. Therefore, whether agents not trading with i are either trading among themselves, or in autarky, is irrelevant when evaluating the policy. Moreover, any two states which generate the same segmented market for agent i are equivalent from the point of view of the planner when evaluating this policy. This result is easily generalized for endogenous market participation economies.

F.2. **Passive Planners.** In this section, we consider the original environment, but assume the consumption allocation is a primitive of the model (e.g., being determined by a Walrasian Equilibrium or a bargaining protocol). In this setup, the social planner can only influence the allocation by making the proposed liquidity injections. If the social planer has preferences given by $V = \mathbb{E}\left[\sum \lambda_i u_i\left(c_i\right)\right]$, and agents consume according to a (differentiable) allocation $c(\cdot)$, financial centrality is defined as

$$FC_i = \mathbb{E}_s\left\{\zeta_i \sum_{j:\zeta_j=1} \lambda_j u_j'\left[c_j\left(s\right)\right] \frac{\partial c_j}{\partial y_i}\left(s\right)\right\}.$$

An important case is where $c(\cdot)$ is a (constrained) Pareto optimal allocation; i.e., there exists a *representing Pareto weight* vector φ such that $c(\cdot)$ solves problem (3.2) with φ instead of λ. Also, let $q(s)$ be the usual normalized Lagrange multiplier of the resource constraint at state s, for this $\varphi-$ planner problem. It is easy to show (see below) that financial centrality in this setting is

$$FC_i = \mathbb{E}_s\left\{\zeta_i q\left(s\right)\left[\sum_{j:\zeta_j=1} \rho_j \frac{\partial c_j\left(s\right)}{\partial y_i}\right]\right\}, \tag{F.2}$$

where $\rho_j := \lambda_j/\varphi_j$[19].

A special case is when the consumption allocation satisfies $\partial c_j/\partial y_i = n_\zeta^{-1}$ whenever $\zeta_j = \zeta_i = 1$. This is the case in the CARA model with homogeneous preferences, even if income draws are not normal (see below). Whenever this happens, equation F.2 can be simplified to

$$FC_i = \mathbb{E}_s\left\{\zeta_i q\left(s\right) \times \bar{\rho}_\zeta\right\},$$

where $\overline{\rho}_\zeta = n_\zeta^{-1}\sum \zeta_j \rho_j$ is the arithmetic mean of the Pareto weights ratio, and $q(s)$ is the Lagrange multiplier in the Pareto problem with weights φ. In the CARA-Normal model this then translates into

$$FC_i = \mathbb{E}_\zeta\left\{\zeta_i \overline{\varphi}_\zeta \exp\left(-r\overline{\mu}_\zeta\right) \exp\left(\frac{r^2}{2}\frac{\overline{\sigma}_\zeta^2}{n_\zeta}\right) \times \overline{\rho}_\zeta\right\},$$

which is the same formula as before, but with an extra term, $\overline{\rho}_\zeta := n_\zeta^{-1}\sum \zeta_j\left(\lambda_j/\varphi_j\right)$ which is the mean of relative Pareto weights. Another important case where $\partial c_j/\partial y_i = n_\zeta^{-1}$ is an environment where agents have homogeneous preferences and identical and independently distributed random draws. If the allocation comes from a Walrasian equilibrium, we know that the representing Pareto weight is $\varphi_j = 1$ for all j (see Proposition G.2), and therefore $FC_i = \exp\left(-r\mu\right)\mathbb{E}_\zeta\left\{\zeta_i \exp\left(\frac{r^2}{2}\frac{\sigma^2}{n_\zeta}\right) \times \overline{\lambda}_\zeta\right\}$, where $\overline{\lambda}_\zeta := n_\zeta^{-1}\sum \zeta_j \lambda_i$ is now the mean of the Pareto weight of the social planner. In the baseline case of with homogeneous preferences, i.i.d. income draws and a representing Pareto weight $\varphi_j = 1$ for all j (so $c_i = \overline{y}$ if $\zeta_i = 1$) we can approximate the centrality measure to $FC_i \approx \mathbb{E}_\zeta\left\{\zeta_i\left(1 + \gamma\frac{\sigma^2}{n_\zeta}\right)\overline{\lambda}_\zeta\right\}$, which resembles the centrality measure obtained in Subsection G for CES and CARA preferences.

[19] Of course, when $\lambda = \varphi$ we have $\rho_j = 1$ for all j, and since $\sum_{j:\zeta_j=1} \partial c_j\left(s\right)/\partial y_i = 1$ for all $s : \zeta_i = 1$, we recover the usual formula in this case.

Proof. First, we want to show equation F.2. For that, we use again the first order conditions of planner's problem 3.2 but representing Pareto weights $\varphi \geq 0$: $\varphi_j u_j'(c_j(s)) = q(s) \iff \lambda_j u_j'(c_j(s)) = \rho_j q(s)$ where $\rho_j = \lambda_j/\varphi_j$. Using this in the original definition of centrality in this setup, we get

$$FC_i = \mathbb{E}_\zeta \left\{ \zeta_i \sum_{j:\zeta_j=1} \lambda_j u'(c_j(s)) \frac{\partial c_j(s)}{\partial y_i} \right\} = \mathbb{E}_\zeta \left\{ \zeta_i \sum_{j:\zeta_j=1} \rho_j q(s) \frac{\partial c_j(s)}{\partial y_i} \right\}$$
$$= \mathbb{E}_\zeta \left\{ \zeta_i q(s) \sum_{j:\zeta_j=1} \rho_j \frac{\partial c_j(s)}{\partial y_i} \right\},$$

showing the desired result. Also, because the resource constraint is always binding at every state s, we have the identity $\sum_{j:\zeta_j=1} c_j(s) = \sum_{j:\zeta_j=1} y_j$, which at states $s : \zeta_i = 1$ implies that $\sum_{j:\zeta_j=1} \partial c_j(s)/\partial y_i = 1$. Therefore, if $\lambda = \varphi$, then $\rho_j = 1 \forall j$, $q(s)$ is the multiplier for the Pareto problem with Pareto weights $\lambda = \varphi$ and hence, $FC_i = \{\zeta_i q(s)\}$, like we had above. □

We now study the special case of the CARA-Normal model with homogeneous preferences and a representing Pareto weight vector φ. We know (see Online Appendix G) that in this model, $c_j(s) = r^{-1} \ln\left(\varphi_j/\overline{\varphi}_\zeta\right) + \overline{y}$, where $\overline{\varphi}_\zeta = \exp\left(n_\zeta^{-1} \sum \zeta_j \ln \varphi_j\right)$. This then means that whenever $\zeta_i = \zeta_j = 1$, we have $\partial c_j(s)/\partial y_i = n_\zeta^{-1}$. Moreover, we also showed that in this environment, $q(s) = \overline{\varphi}_\zeta \exp(-r\overline{y})$. Therefore, using F.2 we get $FC_i = \mathbb{E}_s \left\{\zeta_i \overline{\varphi}_\zeta \exp(-r\overline{y}) \times \overline{\rho}_\zeta\right\}$, where now $\overline{\rho}_\zeta := n_\zeta^{-1} \sum \zeta_j \rho_j$ is the arithmetic mean of relative Pareto weights. Using the assumption $y \perp \zeta$, we can then rewrite it as

$$\mathbb{E}_s \left\{ \zeta_i \overline{\varphi}_\zeta \exp\left(-r\overline{\mu}_\zeta\right) \exp\left(\gamma \frac{\overline{\sigma}_\zeta^2}{n_\zeta}\right) \times \overline{\rho}_\zeta \right\}.$$

Appendix G. Walrasian Equilibrium without Transfers

Following the definitions in Subsection E, and given a (normalized) price function $r \in \Delta(S)$, we can simplify the consumer's problem by just choosing consumption to maximize utility, given only one "expected" budget constraint. Formally, agent $i \in \{1, \ldots, n\}$ solves

$$V_i(q) := \max \mathbb{E}_s \{\zeta_i u_i [c_i(s)] + (1 - \zeta_i) u_i(y_i)\} \tag{G.1}$$

$$\text{subject to: } \mathbb{E}_s [\zeta_i c_i(s) r(s)] \leq \mathbb{E}_s [\zeta_i y_i r(s)].$$

As we did when defining the Lagrange multipliers for the planning problem, we normalize the price function as $q(s) \mathrm{P}(s) = \hat{q}(s)$, where $\hat{q}$ is the actual price measure. A Walrasian equilibrium is a pair $(c, q) = \left(\{c_i(s)\}_{i \in I, s \in S}, \{q(s)\}_{s \in S}\right)$ such that

- $\{c_i(s)\}_{s \in S}$ solves (G.1) given prices $q(s)$
- and markets clear at all states: $\sum_i \zeta_i c_i(s) \leq \sum_i \zeta_i y_i$. for all $s = (y, \zeta)$.

Proposition 5.1 implies there exists a vector λ such that the equilibrium allocation solves the planning problem (3.2), and such that the normalized prices satisfy $r(s) = q(s)$, where $q(s)$ are the normalized Lagrange multipliers of the resource constraint at state s. Following Negishi (1960) and more recently Echenique and Wierman (2012), we can then solve for the equilibrium allocation by finding the Pareto weights that satisfy the budget constraints for all agents. Formally, let $c_i^*(s \mid \lambda)$ be the optimal consumption allocation in the planning problem with weights λ, and $q^*(s \mid \lambda)$ the Lagrange multipliers (normalized by the probabilities of each state). Then, a Pareto weight vector λ corresponds to a Walrasian equilibrium allocation if and only if

$$\mathbb{E}_s [\zeta_i c_i^*(s \mid \lambda) q^*(s \mid \lambda)] = \mathbb{E}_\zeta [\zeta_i y_i q^*(s \mid \lambda)] \text{ for all } i = 1, 2 \ldots, n. \tag{G.2}$$

The next proposition characterizes the Pareto weights equation for the CARA-Normal case.

Proposition G.1. *Suppose $u_i(c) = -r_i^{-1} \exp(-r_i c)$ and $y \sim \mathcal{N}(\mu, \Sigma)$. Let $\overline{r}_\zeta := \left(\frac{1}{n_\zeta} \sum \zeta_i r_i^{-1}\right)^{-1}$ be the harmonic mean of risk aversion in market ζ, and $\overline{\lambda}_\zeta := \exp\left[\frac{1}{n_\zeta} \sum_i \zeta_i (\overline{r}_\zeta / r_i) \ln(\lambda_i)\right]$ be the average Pareto weight in the market, weighted by the relative risk aversion. Also, let $\Sigma_{i,\zeta} := \sum_j \zeta_j \sigma_{ij}$ Then the Pareto weight vector λ solving (G.2) satisfies*

$$\ln(\lambda_i) = \frac{\mathbb{E}_\zeta \left\{\zeta_i \left[\ln\left(\overline{\lambda}_\zeta\right) + \left(r_i \mu_i - \overline{r}_\zeta \overline{\mu}_\zeta\right) - \frac{\overline{r}_\zeta}{n_\zeta} \left(r_i \Sigma_{i,\zeta} - \overline{r}_\zeta \overline{\sigma}_\zeta^2\right)\right] \eta(s)\right\}}{\mathbb{E}_\zeta \{\zeta_i \eta(s)\}} \tag{G.3}$$

for $i = 1, \ldots, n$, *where* $\eta(s) := \overline{\lambda}_\zeta \exp\left(-\overline{r}_\zeta \overline{\mu}_\zeta + \frac{\overline{r}_\zeta^2}{2} \frac{\overline{\sigma}_\zeta^2}{n_\zeta}\right)$.

Proof. From the first order conditions under CARA preferences, we get

$$\lambda_i \exp(-r_i c_i) = q(s) \iff c_i = \frac{1}{r_i} \ln(\lambda_i) - \frac{1}{r_i} \ln(q(s)) \tag{G.4}$$

and that

$$q(s) = \overline{\lambda}_\zeta \exp(-\overline{r}_\zeta \overline{y}) = \overline{\lambda}_\zeta \exp(-\overline{r}_\zeta \overline{y}).$$

Using the first order conditions again, whenever $\zeta_i = 1$ we get

$$c_i(s) = \frac{\ln\left(\lambda_i/\overline{\lambda}_\zeta\right)}{r_i} + \frac{\overline{r}_\zeta}{r_i}\overline{y}(s). \tag{G.5}$$

Then, the value of the consumption allocation, at prices $q(s)$ is

$$\begin{aligned}\mathbb{E}_s\{\zeta_i c(s) q(s)\} &= \mathbb{E}_s\left[\zeta_i \frac{\ln(\lambda_i)}{r_i} q(s)\right] - \mathbb{E}_s\left\{\zeta_i \frac{\ln\left(\overline{\lambda}_\zeta\right)}{r_i} q(s)\right\} + \mathbb{E}_s\left\{\zeta_i \frac{\overline{r}_\zeta}{r_i}\overline{y}(s) q(s)\right\} \\ &= \frac{1}{r_i}\ln(\lambda_i) FC_i(\lambda) - \frac{1}{r_i}\mathbb{E}_s\left\{\zeta_i\overline{\lambda}_\zeta\left[\ln\left(\overline{\lambda}_\zeta\right) + \overline{r}_\zeta\overline{y}\right]\exp(-\overline{r}_\zeta\overline{y})\right\}\end{aligned}$$

where $\mathbb{E}_y(-\overline{r}_\zeta\overline{y}) = \exp\left(-\overline{r}_\zeta\overline{\mu}_\zeta + \frac{\overline{r}_\zeta}{2}\frac{\overline{\sigma}_\zeta}{n_\zeta}\right)$ as we have seen before. Moreover

$$\mathbb{E}_s\left[\overline{r}_\zeta\overline{y}\exp(-\overline{r}_\zeta\overline{y})\right] = \mathbb{E}_\zeta\left[\left(\overline{r}_\zeta\overline{\mu}_\zeta - \frac{\overline{r}_\zeta^2}{n_\zeta}\overline{\sigma}_\zeta^2\right)\exp\left(-\overline{r}_\zeta\overline{\mu}_\zeta + \frac{r_\zeta^2}{2}\frac{\overline{\sigma}^2}{n_\zeta}\right)\right].$$

On the other hand, the value of agent i' s income stream is

$$w_i = \mathbb{E}_\zeta\left[\zeta_i y_i q(s)\right] = \mathbb{E}_s\left\{\zeta_i\overline{\lambda}_\zeta y_i \exp\left[-\overline{r}_\zeta\overline{y}(s)\right]\right\}.$$

Using the moment generating function $M_y(t) = \mathbb{E}_y[\exp(t'y)] = \exp\left(t'\mu + \frac{1}{2}t'\Sigma t\right)$, we get $\mathbb{E}\left[y_i\exp\left(-\overline{r}_\zeta\overline{y}_\zeta\right)\right] = \frac{\partial M}{\partial t_i}\mid_{t=-\frac{\overline{y}_\zeta}{n_\zeta}\times\mathbf{1}}$ where $\mathbf{1}$ is a vector of 1's, so that $t'y = \overline{r}_\zeta\overline{y}$. This then implies that $\mathbb{E}_y[y_i\exp(-\overline{r}_\zeta\overline{y})] = \left(\mu_i - \frac{\overline{r}_\zeta}{n_\zeta}\Sigma_{i,\zeta}\right)\exp\left(-\overline{r}_\zeta\overline{\mu}_\zeta + \frac{\overline{r}_\zeta}{2n_\zeta}\overline{\sigma}_\zeta^2\right)$, where $\Sigma_{i,\zeta} := \sum_j \zeta_j\sigma_{ij}$. Putting all these results together, we can write the budget constraint as

$$\mathbb{E}_s\left[\zeta_i(c_i - y_i) q(s)\right] = 0$$

if and only if

$$\begin{aligned}&r_i^{-1}\ln(\lambda_i) FC_i(\lambda) - r_i^{-1}\mathbb{E}_\zeta\left\{\zeta_i\overline{\lambda}_\zeta\left[\ln\left(\overline{\lambda}_\zeta\right) - \overline{r}_\zeta\overline{\mu}_\zeta + \frac{\overline{r}_\zeta^2}{n_\zeta}\overline{\sigma}_\zeta^2\right]\exp\left(-\overline{r}_\zeta\overline{\mu}_\zeta + \frac{\overline{r}_\zeta^2}{2n_\zeta}\overline{\sigma}_\zeta^2\right)\right\} \\ &= \mathbb{E}_\zeta\left\{\zeta_i\overline{\lambda}_\zeta\left(\mu_i - \frac{\overline{r}_\zeta}{n_\zeta}\Sigma_{i,\zeta}\right)\exp\left(-\overline{r}_\zeta\overline{\mu}_\zeta + \frac{\overline{r}_\zeta^2}{2n_\zeta}\overline{\sigma}_\zeta^2\right)\right\}\end{aligned}$$

and so

$$\ln(\lambda_i) FC_i(\lambda) = \mathbb{E}_\zeta\left\{\zeta_i\overline{\lambda}_\zeta\left[\ln\left(\overline{\lambda}_\zeta\right) + \left(r_i\mu_i - \overline{r}_\zeta\overline{\mu}_\zeta\right) - \frac{\overline{r}_\zeta}{n_\zeta}\left(r_i\Sigma_{i,\zeta} - \overline{r}_\zeta^2\overline{\sigma}_\zeta^2\right)\right]\exp\left(-\overline{r}_\zeta\overline{\mu}_\zeta + \frac{\overline{r}_\zeta^2}{2n_\zeta}\overline{\sigma}_\zeta^2\right)\right\}.$$

□

Observe that the denominator has $\mathbb{E}_\zeta\{\zeta_i\eta(s)\} = FC_i$. Also, because $\lambda \in \Delta^n$, we have $\ln(\lambda_i)$ and $\ln\left(\overline{\lambda}_\zeta\right) < 0$, which implies that if we could, somehow, increase FC_i without affecting the numerator of the right hand side of (G.3), we would increase λ_i in the fixed point equation. An important corollary of Proposition G.1 is the proof of Proposition A.1, since we would have $\overline{r}_\zeta = r_i = r$ for all ζ, and the fact that incomes are identically distributed and independent imply $\overline{\sigma}_\zeta^2 = \sigma^2$, $\overline{\mu}_\zeta = \mu$

and $\Sigma_{i,\zeta} = \sigma^2$. This simplifies the fixed point equation as

$$\ln(\lambda_i) FC_i(\lambda) = \mathbb{E}_\zeta \left\{ \zeta_i \overline{\lambda}_\zeta \ln\left(\overline{\lambda}_\zeta\right) \exp\left(-r\mu + \frac{r^2}{2}\frac{\sigma^2}{n_\zeta}\right)\right\}$$

to which a solution is $\lambda_i = 1/n$. We summarize this result in Proposition G.2.

Proposition G.2. *Suppose* $u_i(c) = -r^{-1}\exp(-rc)$ *and* $y_i \sim_{i.i.d.} \mathcal{N}(\mu, \sigma^2)$. *Then* $\lambda_i = 1/n\ \forall i$ *solves G.2, and hence* $FC_i(\lambda) = \exp(-r\mu)\,\mathbb{E}_\zeta\left\{\zeta_i \exp\left(\frac{r^2}{2}\frac{\sigma^2}{n_\zeta}\right)\right\}$.

Appendix H. Economy with Portfolio Investments

H.1. **Setup.** We study the model in Section B.2, in the case with normally distributed returns and CARA preferences. The timing of the environment is as follows:

(1) Endowment vector is drawn according to $e \sim \mathcal{N}(\mu_e, \Sigma_e)$,
(2) Agents choose portfolio investments $w_i \in \mathbb{R}^K$,
(3) Returns are drawn: $R \sim \mathcal{N}(\rho, \Lambda)$ and income is determined as $y_i = e_i + w_i' R$.

We will assume that the realization of the endowment e_i is privately observed by agents at the moment of deciding the investment portfolio w_i. We will show that this will not affect the decision; i.e. ,$w_i(e_i) = w_i$ for all $e_i \in \mathbb{R}$

To set up the problem, remember that given the efficient Pareto optimal consumption allocation, agents will consume according to

$$c_i(s) = \begin{cases} a_i + \eta_{\zeta,i}\overline{y}_\zeta & \text{if } \zeta_i = 1 \\ y_i & \text{if } \zeta_i = 0 \end{cases}$$

where $a_i := \ln\left(\overline{\lambda}_\zeta/\lambda_i\right)/r_i$ and $\eta_i := \overline{r}_\zeta/r_i$. To simplify exposition, we will assume that $\lambda_i = 1$ for all i (this will not change the results in any way, as we will see) so $a_i = 0$ for all i. Also, since there are no strategic interaction between the agents' portfolio decisions, the planner would always choose to maximize ex-post welfare; i.e. for any profile of portfolio decisions $w = (w_1, \dots, w_n)$.

For a given realization of portfolio returns R, then aggregate tradable income given (ζ, R) is

$$Y(\zeta, R) = \sum_{i=1}^n \zeta_i (y_i + t_i) = \eta_\zeta \left(\overline{e}_\zeta + \overline{t}_\zeta + \overline{w}_\zeta' R\right)$$

where $\overline{e}_\zeta := n_\zeta^{-1}\sum_{i=1}^n \zeta_i y_i$, $\overline{t}_\zeta = n_\zeta^{-1}\sum_{i=1}^n \zeta_i t_i$ and $\overline{w}_\zeta := n_\zeta^{-1}\sum_{i=1}^n \zeta_i w_i$ respectively. This means that, conditional on $\zeta_i = 1$, the first and second moments of individual consumption are:

$$\mathbb{E}(c_i \mid e, \zeta) = \eta_{\zeta,i}\left(\overline{e}_\zeta + \overline{t}_\zeta + \overline{w}_\zeta' \rho\right)$$

and its variance:

$$\operatorname{var}(c_i \mid e, \zeta) = \eta_{\zeta,i}^2 \overline{w}_\zeta^T \Lambda \overline{w}_\zeta$$

which then means that, conditional on (e, ζ) (with $\zeta_i = 1$) we have that $c_i \sim \mathcal{N}\left(\eta_{\zeta,i}\left(\overline{e}_\zeta + \overline{t}_\zeta + \overline{w}_\zeta^T \rho\right), \eta_{\zeta,i}^2 \overline{w}_\zeta^T \Lambda \overline{w}_\zeta\right)$. This then means that expected utility for agent i (conditional on having market access) can be written as

$$\mathbb{E}[u(c_i) \mid e, \zeta] = -r_i^{-1}\exp\left(-r_i \eta_{\zeta,i}\left(\overline{e}_\zeta + \overline{t}_\zeta + \overline{w}_\zeta^T \rho\right) + \frac{r_i^2}{2}\eta_{\zeta,i}^2 \overline{w}_\zeta^T \Lambda \overline{w}_\zeta\right)$$

and using the fact that $\eta_{\zeta,i} = \overline{r}_\zeta / r_i$ we can simplify it to

$$\mathbb{E}\left[u\left(c_i\right) \mid e, \zeta\right] = -r_i^{-1} \exp\left(-\overline{r}_\zeta\left(\overline{e}_\zeta + \overline{t}_\zeta + \overline{w}_\zeta^T \rho\right) + \frac{\overline{r}_\zeta^2}{2} \overline{w}_\zeta' \Lambda \overline{w}_\zeta\right)$$

and if, instead we have $\zeta_i = 0$, then

$$\mathbb{E}\left[u\left(c_i\right) \mid e, \zeta\right] = -r_i^{-1} \exp\left(-r_i\left(e_i + w_i^T \rho\right) + \frac{r_i^2}{2} w_i' \Lambda w_i\right)$$

H.2. **Equilibrium.** We look for Bayesian Nash equilibria of the simultaneous move game where agents, taking the investment strategy $w_j\left(e_j\right)$ as given, decide optimally on their portfolio decisions. Formally, a profile of investment functions $w_i : \mathbb{R} \to \mathbb{R}^K$ is a Bayesian Nash Equilibrium (BNE) if, for all agents and all $e_i \in \mathbb{R}$ we have that

$$w_i\left(e_i\right) \in \underset{w_i \in \mathbb{R}^k}{\operatorname{argmax}} \mathbb{E}_{e,R,\zeta}\left[u_i\left(c_i\right) \mid w_j\left(\cdot\right)\right]$$

We get the following result:

Proposition H.1. *There exists a BNE of the investment game where $w_i\left(e_i\right) = \left(r_i \Lambda\right)^{-1} \rho$ for all e_i*

Proof. To prove this statement, we first write the first order conditions of the investment problem for agent i. The agent chooses w_i to solve

$$\sum_{\zeta : \zeta_i = 1} \zeta_i \frac{\partial \mathbb{E}\left[u_i\left(c_i\right) \mid e, \zeta\right]}{\partial w_i} \mathrm{P}\left(\zeta\right) + \left(1 - p_i\right) \frac{\partial \mathbb{E}\left[u_i\left(c_i\right) \mid e, \zeta_i = 0\right]}{\partial w_i} = 0$$

See that

$$\begin{aligned} \frac{\partial \mathbb{E}\left[u_i\left(c_i\right) \mid e, \zeta\right]}{\partial w_i} &= \frac{\partial \mathbb{E}\left[u\left(c_i\right) \mid e, \zeta\right]}{\partial \overline{w}_\zeta} \times \frac{\partial \overline{w}_\zeta}{\partial w_i} = \\ &= n_\zeta^{-1} \frac{\partial \mathbb{E}\left[u\left(c_i\right) \mid e, \zeta\right]}{\partial \overline{w}_\zeta} \end{aligned}$$

and, with some algebra, we can show that

$$\frac{\partial \mathbb{E}\left[u_i\left(c_i\right) \mid e, \zeta\right]}{\partial \overline{w}_\zeta} =$$

$$= -\eta_{\zeta,i} \exp\left[-\overline{r}_\zeta\left(\overline{e}_\zeta + \overline{t}_\zeta + \overline{w}_\zeta^T \rho\right) + \frac{\overline{r}_\zeta^2}{2} \overline{w}_\zeta' \Lambda \overline{w}_\zeta\right] \times \left(\overline{r}_\zeta \Lambda \overline{w}_\zeta - \rho\right)$$

Analogously, we can show that

$$\frac{\partial \mathbb{E}\left[u_i\left(c_i\right) \mid e, \zeta_i = 0\right]}{\partial w_i} = -\eta_{\zeta,i} \exp\left[-r_i\left(e_i + w_i^T \rho\right) + \frac{r_i^2}{2} w_i' \Lambda w_i\right] \times \left(r_i \Lambda w_i - \rho\right)$$

It is easy to check that if $w_j = \left(r_j \Lambda\right)^{-1} \rho$ then $w_i = \left(r_i \Lambda\right)^{-1} \rho$ maximizes the agents expected utility, for any realization of e □

H.3. **Income correlation Caused by Portfolio Choices.** Take the version where e is privately observed. We can use the above results to show that the resulting income distribution will actually give positively correlated incomes. The intuitive reason for this is the same as in Jackson and Pernoud (2019): agents have access to the same set of investment opportunities. Even with heterogeneous preferences, they still are risk averse, and therefore their investments will be positively correlated, which will be translated into a positive correlation of incomes, even if the underlying endowment process is not.

For this, we simply take the equilibrium portfolio decision for each agent, which gives that $y_i = e_i + r_i^{-1}\rho'\Lambda^{-1}R$. This then means that its conditional covariances are

$$\begin{aligned}\operatorname{cov}(y_i, y_j \mid e) &= \mathbb{E}_R\left\{\left[r_i^{-1}\rho'\Lambda^{-1}(R-\rho)\right] \times \left[r_j^{-1}\rho'\Lambda^{-1}(R-\rho)\right]\right\} = \\ &= (r_i r_j)^{-1}\rho'\Lambda^{-1}\mathbb{E}_R\left[(R-\rho)(R-\rho)'\right]\Lambda^{-1}\rho = \\ &= (r_i r_j)^{-1}\rho'\Lambda^{-1}\rho\end{aligned}$$

See that since Λ is positive definite and $r_i, r_j > 0$ we have that $\operatorname{cov}(y_i, y_j \mid e) > 0$. This result is independent of the assumption of endowments being independent. since then we can write the covariance between y_i and y_j as

$$\operatorname{cov}(y_i, y_j) = (r_i r_j)^{-1}\rho'\Lambda^{-1}\rho + \operatorname{cov}(e_i, e_j)$$

so, even if endowments are independent (i.e., $\operatorname{cov}(e_i, e_j) = 0$) then incomes would be correlated because of their similar portfolio decisions. This is relevant in our model, because we know that the value of a liquidity injection is increasing in the average cross-correlations of income draws.